Work and Energy

AS Level Physics 9702

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By the end of this topic I can...

- calculate work done by a force, a gas and from a force–displacement graph
- use kinetic and gravitational potential energy
- apply $W = \Delta E$ and conservation of energy
- define power and efficiency and use $P = Fv$

Work done

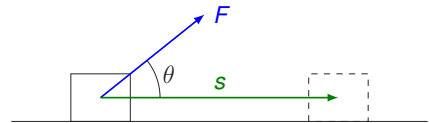
Work done

Force × **displacement in the direction of the force**.

- unit: N m = J; a **scalar**
- negative when the force opposes motion

$$W = Fs$$

$$W = Fs\cos\theta$$



Work done by a gas

Gas at constant pressure

Work done = pressure × change in volume.

- varying force: **area under the F–s graph**

$$W_{\text{gas}} = p\Delta V$$

Example: angled force

Example: angled force

$$W = Fs\cos\theta = Fvt\cos\theta = 20 \times 2.0 \times 30 \times \cos 60^\circ \quad \Rightarrow \quad W = 600\ \text{J}$$

Example: gas pump

Example: gas pump

$$W = p\Delta V = pA\Delta s = 8000 \times 600 \times 10^{-4} \times 30 \times 10^{-2} \quad \Rightarrow \quad W = 144 \text{ J}$$

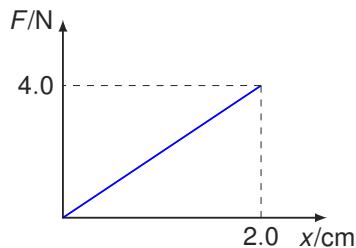
Example: compressing a spring

When a spring is compressed by 2.0 cm, the force applied increases uniformly from zero to 4.0 N. How much work is done by this force?

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F-*x* graph for the force is plotted as shown
work to compress spring equals area under *F*-*x* graph:
$W = \frac{1}{2} \times 4.0 \times 2.0 \times 10^{-2} = 0.040$ J



Kinetic energy

Kinetic energy

Energy due to **motion**.

- from $W = Fs = mas$ with $v^2 = u^2 + 2as$

$$E_\text{k} = \tfrac{1}{2}mv^2$$

Gravitational potential energy

G.P.E.

Energy due to **position** in a gravitational field.

- only the **change** matters
- other stores: elastic, chemical, nuclear, internal

$$\Delta E_p = mg\Delta h$$

Work–energy and conservation

Conservation of energy

Energy cannot be created or destroyed, only **transformed**.

- net work by external forces = change in total energy

$$W = \Delta E$$

Example: racing car

Example: racing car

The gain in K.E. equals work by driving force plus *negative* work against resistance

$$W_{\text{total}} = \Delta E_k \quad \Rightarrow \quad Fs - fs = \frac{1}{2}mv^2 - 0$$

$$(5000 - 1500) \times 50 = \frac{1}{2} \times 800 \times v^2 \quad \Rightarrow \quad v \approx 20.9 \text{ m s}^{-1}$$

Example: crane lifting a cube

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The work done by crane equals gain in G.P.E. of cube

$$W = \Delta E_p$$

$$\Rightarrow \quad W = mg\Delta h = \rho V g \Delta h = 2400 \times 0.50^3 \times 9.81 \times 4.0 \approx 1.18 \times 10^4 \text{ J}$$

Example: falling object

Example: falling object

$$\text{G.P.E loss} = \text{K.E gain} \quad \Rightarrow \quad mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gh}$$

Example: marble thrown upwards

Example: marble thrown upwards

K.E. loss $=$ G.P.E. gain $+$ energy loss due to resistance

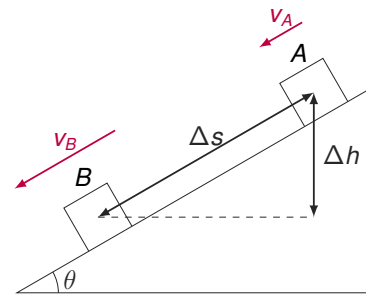
$$\frac{1}{2}mu^2 - 0 = mgH_{\max} + fH_{\max} \quad \Rightarrow \quad H_{\max} = \frac{mu^2}{2(mg+f)}$$

Example: box on a slope

A box of mass m slides down along a slope that is inclined at an angle θ to the horizontal. There is a constant friction f acting on the box. When the box has moved through a distance of Δs down the slope from A to B, write down an equation relating its velocities v_A and v_B by applying the law of conservation of energy.

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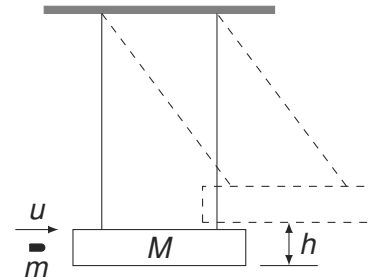
$$
\begin{aligned}
\text{Total work done} &= \text{change in total energy} \\
-W_f &= \Delta E_k + \Delta E_p \\
-fs &= \left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right) + (mgh_B - mgh_A) \\
-fs &= \left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right) - mg\Delta h
\end{aligned}
$$

Example: ballistic pendulum

A *ballistic pendulum*: a bullet of mass m is fired into a block of mass M hanging from light strings. The block and bullet swing up so that their centre of mass rises a height h. Find the bullet's initial speed u.

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Collision (momentum conserved):

$$mu = (M+m)v \quad \Rightarrow \quad v = \frac{mu}{M+m}.$$

Swing (energy conserved):

$$\tfrac{1}{2}(M+m)v^2 = (M+m)gh \quad \Rightarrow \quad v = \sqrt{2gh}.$$

Combine: $u = \left(1 + \dfrac{M}{m}\right)\sqrt{2gh}.$

Power

Power

Work done per unit time.

- unit: W = J s^{-1}
- $P = Fv$ for a force moving at speed v

$$P = \frac{\Delta W}{\Delta t}$$

$$P = Fv$$

Example: running up steps

Example: running up steps

$$P = \frac{\Delta E_p}{\Delta t} = \frac{mg\Delta h}{\Delta t} = \frac{72 \times 9.81 \times (150 \times 0.25)}{120} \quad \Rightarrow \quad P \approx 221 \text{ W}$$

Example: wind turbine

Example: wind turbine

$$P = \frac{\Delta E_k}{\Delta t} = \frac{\frac{1}{2}\Delta m v^2}{\Delta t} = \frac{\frac{1}{2}\rho \Delta V v^2}{\Delta t} = \frac{\frac{1}{2}\rho A \Delta x v^2}{\Delta t} \quad \Rightarrow \quad P = \frac{1}{2}\rho A v^3$$

$$P = \frac{1}{2} \times 1.3 \times 500 \times 10^3 \approx 3.25 \times 10^5 \text{ W}$$

Example: ship at constant speed

Example: ship at constant speed

Constant speed so equilibrium between driving force and resistive force

$$P = Fv = fv = 9000 \times 15 \quad \Rightarrow \quad P = 1.35 \times 10^5 \text{ W}$$

Example: car at constant power

Example: car at constant power

The equation of motion for the car is: $F_{\text{net}} = F - f = ma \quad \Rightarrow \quad \frac{P}{v} - \alpha v = ma$
at $v = 15$ m s^{-1}: $\frac{24000}{15} - 16 \times 15 = 800a \quad \Rightarrow \quad a = 1.7$ m s^{-2}
as car's velocity v increases, driving force $F = \frac{P}{v}$ decreases, resistive force $f = \alpha v$ increases
so resultant force will decrease, the car will accelerate at a decreasing rate
eventually it reaches an equilibrium state where $F = f$, the car then travels at constant speed v_t

$$F = f \quad \Rightarrow \quad \frac{P}{v_t} = \alpha v_t \quad \Rightarrow \quad \frac{24000}{v_t} = 16 v_t \quad \Rightarrow \quad v_t \approx 38.7 \text{ m s}^{-1}$$

Efficiency

Efficiency

Useful output divided by **total input**.

$$\eta = \frac{W_{\text{useful}}}{W_{\text{total}}} = \frac{P_{\text{useful}}}{P_{\text{total}}}$$

Example: water pump

Example: water pump

$$\eta = \frac{\Delta E_p}{\Delta E_{\text{in}}} = \frac{\Delta mgh}{P_{\text{in}} \Delta t} = \frac{1500 \times 9.81 \times 8.0}{3000 \times 60} \quad \Rightarrow \quad \eta \approx 65.4\%$$

Example: hydroelectric turbine

Example: hydroelectric turbine

$$P_{\text{out}} = \eta P_{\text{in}} = \eta \frac{\Delta E_p}{\Delta t} = \eta \frac{\Delta mgh}{\Delta t} = 85\% \times \frac{2500 \times 9.81 \times 70}{60}$$

$$\Rightarrow \quad P_{\text{out}} \approx 2.43 \times 10^4 \text{ W}$$

Key equations

work $W = Fs\cos\theta$

power $P = Fv$

kinetic energy $E_\text{k} = \frac{1}{2}mv^2$

efficiency $\eta = \dfrac{W_\text{useful}}{W_\text{total}} = \dfrac{P_\text{useful}}{P_\text{total}}$

G.P.E. $\Delta E_\text{p} = mg\Delta h$