Work and Energy

Name _____ Date _____

By the end of this topic I can…

- calculate work done by a force, including at an angle, by a gas, and from a force–displacement graph
- use kinetic and gravitational potential energy
- apply $W = \Delta E$ and conservation of energy
- define power and efficiency and use $P = Fv$

Work

Work done by a force is the product of the force and the displacement in the direction of the force:

$$W = Fs \qquad [W] = \text{N m} = \text{J}$$

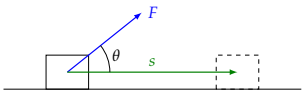
If the force acts at angle θ to the displacement, only the component along the displacement counts:

$$W = Fs\cos\theta$$

- work is a scalar: it has no direction, but can be positive or negative
- work done against resistive forces (friction, drag) is negative: $\theta = 180^\circ$ and $\cos\theta = -1$
- for a gas at constant pressure: $W = pA\Delta s$, so

$$W_{\text{gas}} = p\Delta V$$

- the area under a force–displacement graph is the work done (for a varying force)



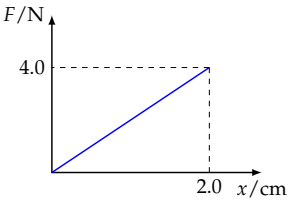
Example 6.1 A 20 N force is applied at 60° to the horizontal to move a 1.0 kg object at a constant speed of 2.0 m s^{-1} for 30 s. How much work is done by the force?

$$W = Fs\cos\theta = Fvt\cos\theta = 20 \times 2.0 \times 30 \times \cos 60^\circ \quad \Rightarrow \quad W = 600 \text{ J}$$

Example 6.2 A piston in a gas pump has an area of 600 cm^2. During one stroke, the pump moves a distance of 30 cm against a constant pressure of 8000 Pa. How much work is done?

$$W = p\Delta V = pA\Delta s = 8000 \times 600 \times 10^{-4} \times 30 \times 10^{-2} \quad \Rightarrow \quad W = 144\ \text{J}$$

Example 6.3 When a spring is compressed by 2.0 cm, the force applied increases uniformly from zero to 4.0 N. How much work is done by this force?



F-x graph for the force is plotted as shown
work to compress spring equals area under F-x graph: $W = \frac{1}{2} \times 4.0 \times 2.0 \times 10^{-2} = 0.040$ J

Energy

Kinetic energy

For a constant force: $W = Fs = mas = m\dfrac{v^2 - u^2}{2}$, so work done = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$.

Kinetic energy is the energy an object has because of its motion:

$$E_\text{k} = \tfrac{1}{2}mv^2$$

Gravitational potential energy

Gravitational potential energy is the energy a body has because of its position in a gravitational field. Near the Earth's surface the change in G.P.E. is

$$\Delta E_\text{p} = mg\Delta h$$

- only the change in G.P.E. matters; the zero level is chosen for convenience

Other forms

- elastic (strain) potential energy : stored in a deformed spring or other elastic body
- others: chemical, nuclear, internal (thermal), electric potential, electromagnetic

Work–energy and conservation of energy

The change in the total energy of an object equals the net work done on it by external forces:

$$W = \Delta E$$

The **principle of conservation of energy**: energy cannot be created or destroyed, only transformed from one form to another; the total stays constant.

Example 6.4 A racing car of 800 kg starts off from rest. If the driving force is 5000 N, and the car experiences a constant resistive force of 1500 N, what is its speed after it has travelled 50 m?

The gain in K.E. equals work by driving force plus *negative* work against resistance

$$W_{\text{total}} = \Delta E_k \quad \Rightarrow \quad Fs - fs = \frac{1}{2}mv^2 - 0$$

$$(5000 - 1500) \times 50 = \frac{1}{2} \times 800 \times v^2 \quad \Rightarrow \quad v \approx 20.9 \text{ m s}^{-1}$$

Example 6.5 A concrete cube of side 0.50 m and density 2400 kg m^{-3} is lifted 4.0 m by a crane. How much work is done?

The work done by crane equals gain in G.P.E. of cube

$$W = \Delta E_p$$

$$\Rightarrow \quad W = mg\Delta h = \rho V g \Delta h = 2400 \times 0.50^3 \times 9.81 \times 4.0 \approx 1.18 \times 10^4 \text{ J}$$

Example 6.6 For an object falling from rest due to gravity, if air resistance is negligible, what is its speed when it has fallen through a distance of h?

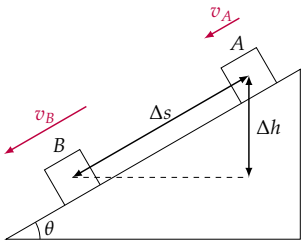
$$\text{G.P.E loss} = \text{K.E gain} \quad \Rightarrow \quad mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gh}$$

Example 6.7 A marble is projected vertically upwards with an initial velocity u. The average resistive force acting is f. How do you determine the maximum height reached by the marble?

$$\text{K.E. loss} = \text{G.P.E. gain} + \text{energy loss due to resistance}$$

$$\frac{1}{2}mu^2 - 0 = mgH_{\max} + fH_{\max} \quad \Rightarrow \quad H_{\max} = \frac{mu^2}{2(mg+f)}$$

Example 6.8 A box of mass m slides down along a slope that is inclined at an angle θ to the horizontal. There is a constant friction f acting on the box. When the box has moved through a distance of Δs down the slope from A to B, write down an equation relating its velocities v_A and v_B by applying the law of conservation of energy.

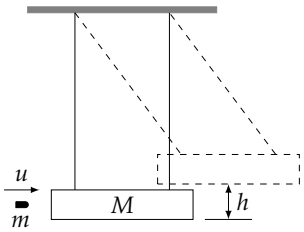


$$\begin{aligned}
\text{Total work done} &= \text{change in total energy} \\
-W_f &= \Delta E_k + \Delta E_p \\
-fs &= \left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right) + (mgh_B - mgh_A) \\
-fs &= \left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right) - mg\Delta h \\
v_B^2 &= v_A^2 + 2g\Delta s \sin\theta - \frac{2fs}{m}
\end{aligned}$$

or equivalently, we can write

$$\begin{aligned}
\text{G.P.E loss} &= \text{K.E gain} + \text{energy loss due to friction} \\
mg\Delta h &= \left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right) + fs \\
v_B^2 &= v_A^2 + 2g\Delta s \sin\theta - \frac{2fs}{m}
\end{aligned}$$

note that the two alternative ways of thinking produce the same result



Example 6.9 A *ballistic pendulum*: a bullet of mass m is fired into a block of mass M hanging from light strings. The block and bullet swing up so that their centre of mass rises a height h. Find the bullet's initial speed u.

Collision (momentum conserved): $mu = (M+m)v \quad \Rightarrow \quad v = \dfrac{mu}{M+m}$.

Swing (energy conserved): $\frac{1}{2}(M+m)v^2 = (M+m)gh \quad \Rightarrow \quad v = \sqrt{2gh}$.

Combine: $u = \left(1 + \dfrac{M}{m}\right)\sqrt{2gh}$.

Power and efficiency

Power is the work done per unit time :

$$P = \frac{\Delta W}{\Delta t} \qquad [P] = \mathrm{J\,s^{-1}} = \mathrm{W}$$

For a force moving at speed v in its own direction:

$$P = Fv$$

Example 6.10 There are 150 steps to the top of a tower, and the average height of each step is 25 cm. It takes a man of 72 kg two minutes to run up all the steps. What is his average power?

$$P = \frac{\Delta E_p}{\Delta t} = \frac{mg\Delta h}{\Delta t} = \frac{72 \times 9.81 \times (150 \times 0.25)}{120} \quad \Rightarrow \quad P \approx 221 \text{ W}$$

Example 6.11 A turbine is used to generate electrical power from the wind. Given that the blades of the turbine sweep an area of 500 m^2, the density of air is 1.3 kg m^{-3}, and the wind speed is 10 m s^{-1}. Assume no energy loss, find the power available from the wind.

$$P = \frac{\Delta E_k}{\Delta t} = \frac{\frac{1}{2}\Delta m v^2}{\Delta t} = \frac{\frac{1}{2}\rho \Delta V v^2}{\Delta t} = \frac{\frac{1}{2}\rho A \Delta x v^2}{\Delta t} \quad \Rightarrow \quad P = \frac{1}{2}\rho A v^3$$

$$P = \frac{1}{2} \times 1.3 \times 500 \times 10^3 \approx 3.25 \times 10^5 \text{ W}$$

Example 6.12 A ship is cruising a a constant speed of 15 m s^{-1}. The total resistive force acting is 9000 N. What is the output power of this ship?

Constant speed so equilibrium between driving force and resistive force

$$P = Fv = fv = 9000 \times 15 \quad \Rightarrow \quad P = 1.35 \times 10^5 \text{ W}$$

Example 6.13 A car of mass $800kg$ accelerates from rest on a horizontal road. Suppose the engine provides a constant power of 24000 W, and the resistive force can be given by $f = 16v$, where v is the speed of the car in m s^{-1}. (a) What is the acceleration of the car when it is travelling at 15 m s^{-1}? (b) What happens to the car if it maintains this driving power?

The equation of motion for the car is: $F_{\text{net}} = F - f = ma \quad \Rightarrow \quad \frac{P}{v} - \alpha v = ma$
at $v = 15$ m s^{-1}: $\frac{24000}{15} - 16 \times 15 = 800a \quad \Rightarrow \quad a = 1.7$ m s^{-2}
as car's velocity v increases, driving force $F = \frac{P}{v}$ decreases, resistive force $f = \alpha v$ increases
so resultant force will decrease, the car will accelerate at a decreasing rate
eventually it reaches an equilibrium state where $F = f$, the car then travels at constant speed v_t

$$F = f \quad \Rightarrow \quad \frac{P}{v_t} = \alpha v_t \quad \Rightarrow \quad \frac{24000}{v_t} = 16 v_t \quad \Rightarrow \quad v_t \approx 38.7 \text{ m s}^{-1}$$

Efficiency is the ratio of useful output to total input :

$$\eta = \frac{W_{\text{useful}}}{W_{\text{total}}} = \frac{P_{\text{useful}}}{P_{\text{total}}}$$

Example 6.14 A water pumping system uses 3.0 kW of electrical power to raise water from a well. The pump lifts 1500 kg of water per minute through a vertical height of 8.0 m. What is the efficiency of the sysem?

$$\eta = \frac{\Delta E_p}{\Delta E_{\text{in}}} = \frac{\Delta mgh}{P_{\text{in}}\Delta t} = \frac{1500 \times 9.81 \times 8.0}{3000 \times 60} \quad \Rightarrow \quad \eta \approx 65.4\%$$

Example 6.15 Water flows into a turbine from a reservoir at a vertical distance of 70 m above. The water flows through the turbine at a rate of 2500 kg per minute. What is the output power of the turbine if it is 85% efficient?

$$P_{\text{out}} = \eta P_{\text{in}} = \eta \frac{\Delta E_p}{\Delta t} = \eta \frac{\Delta mgh}{\Delta t} = 85\% \times \frac{2500 \times 9.81 \times 70}{60}$$

$$\Rightarrow \quad P_{\text{out}} \approx 2.43 \times 10^4 \text{ W}$$

Practice questions

Question 6.1 A trolley is pushed 2.0 m along a track by a force of 5.0 N. It experiences a constant frictional force of 3.0 N. Find the work done by (a) the 5.0 N force, (b) friction.

(a) $W = Fs = 5.0 \times 2.0 = 10$ J.
(b) $W = -3.0 \times 2.0 = -6.0$ J (negative: work is done against friction).

Question 6.2 A child of mass 40 kg slides down a slope from a height of 2.0 m above the ground. The slide is 6.0 m long. How much work is done by gravity?

Only the vertical drop matters: $W = mg\Delta h = 40 \times 9.81 \times 2.0 \approx 7.8 \times 10^2$ J.

Question 6.3 A fixed mass of gas at 1.50×10^5 Pa expands at constant pressure from 2.80×10^{-4} m^3 to 8.40×10^{-4} m^3. Find the work done by the gas.

$W = p\Delta V = 1.50 \times 10^5 \times (8.40 - 2.80) \times 10^{-4} = 84$ J.

Question 6.4 A stone is projected vertically upwards at 16 m s^{-1} from the ground. Air resistance is negligible. (a) Find the greatest height. (b) Find its speed at a height of 5.0 m. (c) If it is instead projected at an angle with the same speed, what is its speed at 5.0 m?

(a) $\frac{1}{2}mu^2 = mgH \quad \Rightarrow \quad H = \dfrac{16^2}{2 \times 9.81} \approx 13$ m.
(b) $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mgh \quad \Rightarrow \quad v^2 = 256 - 2 \times 9.81 \times 5.0 \quad \Rightarrow \quad v \approx 12.6$ m s^{-1}.
(c) Energy depends only on height, not path, so the speed is again ≈ 12.6 m s^{-1}.

Question 6.5 A hammer of mass 600 g hits a nail at 10 m s^{-1} and pushes it 3.0 mm into a plank. Find the average resistive force on the nail.

Work done against the resistive force equals the kinetic energy lost: $F \times 3.0 \times 10^{-3} = \frac{1}{2}(0.600)(10)^2 = 30$ J, so $F = 1.0 \times 10^4$ N.

Question 6.6 A 3.0 kg block is released from rest on a slope at $\theta = \sin^{-1}(1/5)$ to the horizontal. It travels 6.0 m down the slope against a frictional force of 5.0 N. Find its final speed.

Loss of G.P.E. $= 3.0 \times 9.81 \times (6.0 \times \frac{1}{5}) = 35.3$ J. Work against friction $= 5.0 \times 6.0 = 30$ J.
$\frac{1}{2}(3.0)v^2 = 35.3 - 30 = 5.3$ J $\quad \Rightarrow \quad v \approx 1.9$ m s^{-1}.

Question 6.7 A car engine exerts an average force of 400 N in moving the car 900 m in 200 s. What is the average power of the engine?

$W = Fs = 400 \times 900 = 3.6 \times 10^5$ J, so $P = \dfrac{W}{t} = \dfrac{3.6 \times 10^5}{200} = 1.8 \times 10^3$ W.
(Check: $v = 4.5$ m s^{-1}, $P = Fv = 1800$ W.)