Waves

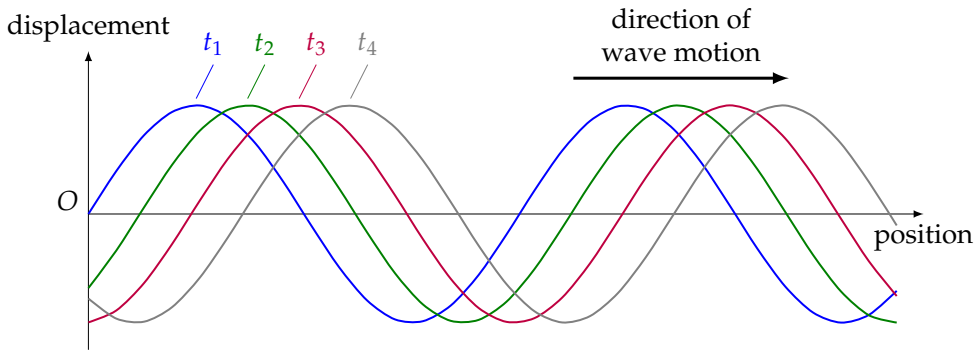
Name _____ Date _____

By the end of this topic I can…

- describe waves using amplitude, wavelength, period, frequency and speed
- distinguish transverse from longitudinal and mechanical from electromagnetic waves
- use intensity, Malus's law and the Doppler effect

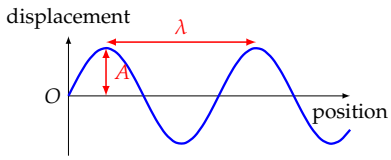
Describing waves

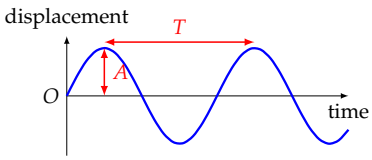
A wave transfers energy from place to place without transferring matter.



- **displacement**: distance of a point from its equilibrium position
- **amplitude** A: the maximum displacement
- **wavelength** λ: the distance between two adjacent points vibrating in phase (e.g. crest to crest)
- **period** T: the time for one complete oscillation of a point; **frequency** f: the number of oscillations per unit time

$$f = \frac{1}{T} \qquad [f] = \text{Hz} = \text{s}^{-1}$$





In one period the wave moves forward one wavelength, so the **wave speed** is

$$v = \frac{\lambda}{T} \qquad v = f\lambda$$

Example 9.1 When a wave travels on a water surface, the maximum depth of water is 21 cm and the minimum depth is 18 cm. What is the amplitude of the wave?

The amplitude is half the end-to-end distance: $A = \frac{1}{2}(21 - 18) = 1.5$ cm

Example 9.2 A wave travelling at 4.0 m s^{-1} has a wavelength of 50 cm, what is its period?

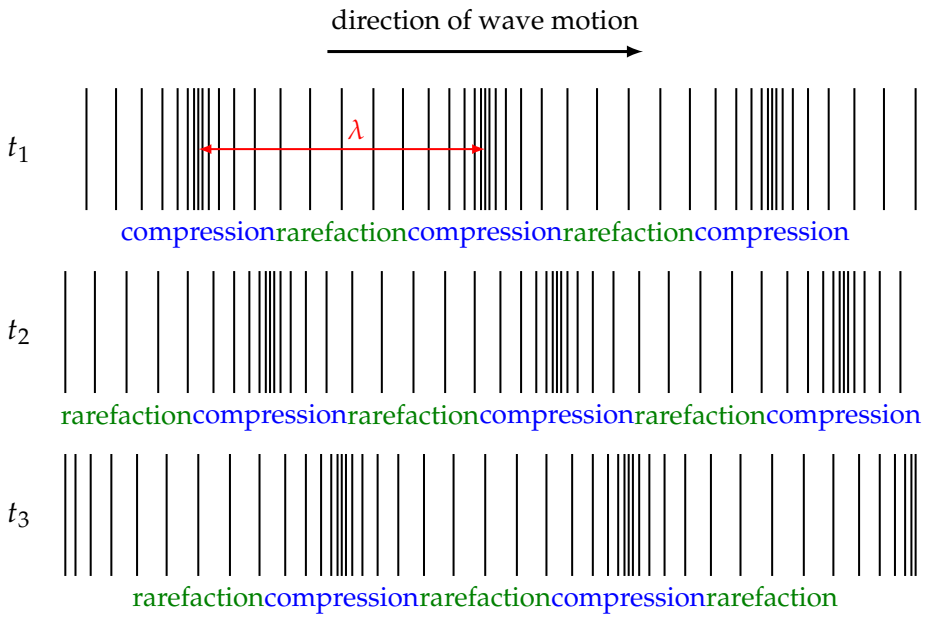
$$v = \frac{\lambda}{T} \quad \Rightarrow \quad T = \frac{\lambda}{v} = \frac{0.50}{4.0} = 0.125 \text{ s}$$

Types of wave

Transverse and longitudinal

A **transverse** wave has vibrations perpendicular to the direction of energy transfer.
A **longitudinal** wave has vibrations parallel to the direction of energy transfer.

- transverse examples: light, waves on a string, water surface waves
- longitudinal examples: sound, waves on a slinky; they have compressions and rarefactions



Example 9.3 What is the distance between a compression and a rarefaction for a sound wave of frequency 550 Hz? (Speed of sound in air is about 330 m s^{-1}.)

$$d = \frac{1}{2}\lambda = \frac{v}{2f} = \frac{330}{2 \times 550} \quad \Rightarrow \quad d = 0.30 \text{ m}$$

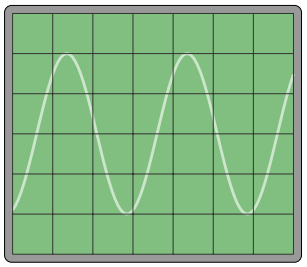
Sound waves

• sound is a longitudinal wave and needs a medium: it cannot travel through a vacuum

• the speed depends on the material (air ≈ 340 m s^{-1}, water ≈ 1500 m s^{-1}, steel ≈ 5000 m s^{-1}), not on frequency

• pitch depends on frequency; loudness depends on amplitude

• an **oscilloscope** displays voltage (y) against time (x); the time-base sets the time per division and the Y-gain the volts per division



Example 9.4 A sound wave is detected by a microphone and the trace is displayed on an oscilloscope as shown. If the time-base is set at 0.5 ms div^{-1} and the voltage gain is set at 2 V div^{-1}. What is the frequency and the amplitude of the signal?

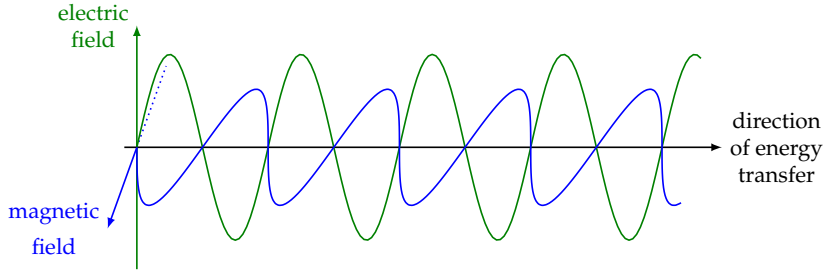
period: $T = 3 \times 0.5 \text{ ms} = 1.5 \times 10^{-3}$ s
frequency: $f = \frac{1}{T} = \frac{1}{1.5 \times 10^{-3} \text{ s}} \approx 667$ Hz
amplitude: $A = 2 \times 2 \text{ V} = 4.0 \text{ V}$

Electromagnetic waves

• **mechanical waves** need a medium; **electromagnetic waves** can travel through a vacuum

• EM waves are transverse: oscillating electric and magnetic fields at right angles to each other and to the direction of travel

• all EM waves travel at $c =$ 3.00×10^8 m s^{-1} in a vacuum



Region	Wavelength / m	Frequency / Hz
radio waves	$10^{-1} \sim 10^{6}$	$10^{2} \sim 10^{9}$
microwaves	$10^{-3} \sim 10^{-1}$	$10^{9} \sim 10^{11}$
infra-red	$7 \times 10^{-7} \sim 10^{-3}$	$10^{11} \sim 10^{14}$
visible light	$4 \times 10^{-7} \sim 7 \times 10^{-7}$	$10^{14} \sim 10^{15}$
ultraviolet	$10^{-9} \sim 4 \times 10^{-7}$	$10^{15} \sim 10^{17}$
X-rays	$10^{-13} \sim 10^{-9}$	$10^{17} \sim 10^{19}$
γ-rays	$10^{-16} \sim 10^{-11}$	$10^{19} \sim 10^{24}$

- order of increasing wavelength:
  γ, X-ray, UV, visible, IR, microwave, radio

Example 9.5 A beam of electromagnetic radiation is known to have a frequency of 25 THz in vacuum. What is its wavelength?

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}}{25 \times 10^{1}2} \quad \Rightarrow \quad \lambda = 1.2 \times 10^{-5}\ \text{m} \quad \text{(infra-red)}$$

Intensity

Intensity is the power per unit cross-sectional area :

$$I = \frac{P}{S} \qquad [I] = \text{W m}^{-2}$$

- intensity is proportional to the square of the amplitude :

$$I \propto A^{2}$$

- from a point source the energy spreads over $4\pi r^{2}$, so (inverse square law)

$$I \propto \frac{1}{r^{2}}$$

Example 9.6 If the amplitude of an incoming wave is increased by 50%, what is the increase in the wave intensity?

$I \propto A^{2} \quad \Rightarrow \quad \frac{I'}{I} = \left(\frac{A'}{A}\right)^{2} = (1 + 50\%)^{2} = 2.25 \quad \Rightarrow$ so intensity is increased by 125%

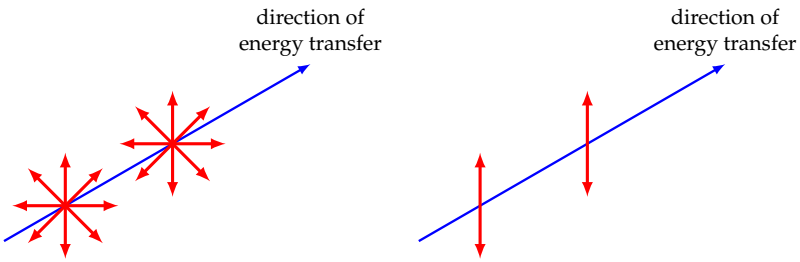
Example 9.7 Two observers A and B are at a distance r_A and r_B from a point source where $r_A = 2r_B$. (a) Find the ratio of their intensities $\frac{I_A}{I_B}$. (b) Find the ratio of their amplitudes $\frac{A_A}{B_B}$.

$$I \propto \tfrac{1}{r^2} \quad \Rightarrow \quad \tfrac{I_A}{I_B} = \left(\tfrac{r_B}{r_A}\right)^2 = \left(\tfrac{1}{2}\right)^2 \quad \Rightarrow \quad \tfrac{I_A}{I_B} = \tfrac{1}{4}$$

$$I \propto A^2 \quad \Rightarrow \quad \tfrac{A_A}{A_B} = \sqrt{\tfrac{I_A}{I_B}} = \sqrt{\tfrac{1}{4}} \quad \Rightarrow \quad \tfrac{A_A}{A_B} = \tfrac{1}{2}$$

Polarisation

A wave is **plane-polarised** if its vibrations are in ___one direction only___, perpendicular to the direction of energy transfer.

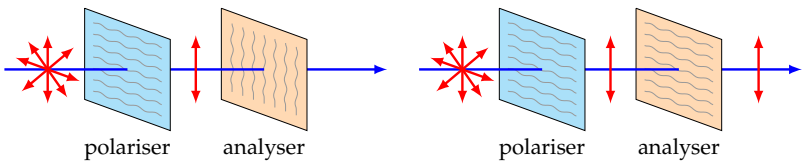


- only ___transverse___ waves can be polarised; sound cannot
- a **polariser** transmits only the component of vibration ___parallel to its axis___; unpolarised light emerges plane-polarised

Malus's law: plane-polarised light of intensity I_0 meeting an analyser at angle θ to its axis transmits

$$I = I_0 \cos^2 \theta$$

- amplitude transmitted: $A = A_0 \cos\theta$; for several filters apply the law ___to each in turn___, using the new plane of polarisation each time



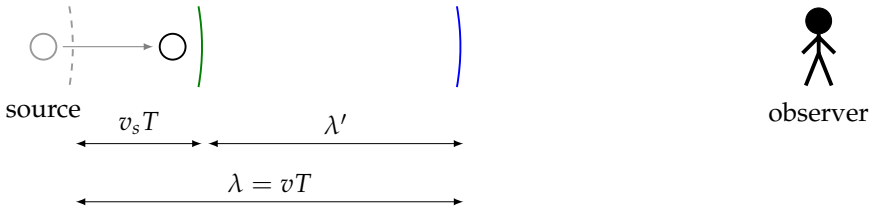
Example 9.8 A beam of light polarised in the vertical direction has an amplitude A and intensity I. It passes through a polarising filter whose axis of polarisation is at 45° to the vertical. (a) What is the amplitude and the direction of polarisation of the emerging beam? (b) If the emerging beam then enters another filter whose axis of polarisation is at 75° to the vertical, what is the amplitude and intensity of the emerging beam?

through first filter: $A_1 = A\cos\theta_1 = A\cos 45° \Rightarrow A_1 = \frac{1}{\sqrt{2}}A$ light is polarised in a direction at 60° to the vertical
through second filter: $A_2 = A_1\cos\theta_2 = \frac{1}{\sqrt{2}}A \times \cos(75° - 45°) \Rightarrow A_2 = \frac{\sqrt{6}}{4}A$
$I_2 = \left(\frac{\sqrt{6}}{4}\right)^2 I \Rightarrow I_2 = \frac{3}{8}I$
or, $I_2 = I\cos^2\theta_1\cos^2\theta_2 = I \times \cos^2 45° \times \cos^2(75° - 45°) \Rightarrow I_2 = \frac{3}{8}I$

Doppler effect

The **Doppler effect** is the change in observed frequency caused by relative motion between a wave source and an observer.

- source moving towards the observer: wavelength shorter, frequency higher; moving away: lower



In one period the source moves v_sT, so $\lambda' = \lambda \mp v_sT$ ($-$ for approach), which gives (source moving, observer at rest)

$$f' = f\frac{v}{v \mp v_s}$$

- use $-$ when the source moves towards the observer and $+$ when it moves away
- uses: speed cameras/Doppler radar, blood-flow imaging, red-shift of stars

Example 9.9 A police car moving towards you at 16 m s^{-1} sirens at 500 Hz. Given that the speed of sound in air is 340 m s^{-1}, at what frequency do you hear the siren?

$$f' = f\frac{v}{v - v_s} = 500 \times \frac{340}{340 - 16} \quad \Rightarrow \quad f' \approx 525 \text{ Hz}$$

Example 9.10 A star emits an H_α line of wavelength 656 nm. (a) What is the frequency of this H_α line? (b) An observer on earth detects a wavelength of 680 nm, what can we say about motion of the star? (c) Find the relative speed of the star with respect to the earth.

The original frequency of H_α: $f = \frac{c}{\lambda} = \frac{3.00\times10^8}{656\times10^{-9}} \approx 4.57 \times 10^{14}$ Hz
The observed wavelength is longer (redshift), this means the star is moving away, or receding.
To find speed of star, we can consider observed frequency: $f' = \frac{c}{\lambda'} = f\frac{c}{c+v_s}$

$$\frac{3.00 \times 10^8}{680 \times 10^{-9}} = 4.57 \times 10^{14} \times \frac{3.00 \times 10^8}{3.00 \times 10^8 + v_s} \quad \Rightarrow \quad v_s \approx 1.10 \times 10^7 \text{ m s}^{-1}$$

alternatively, we can consider observed wavelength: $\lambda' = \lambda + v_s T$, or: $\Delta\lambda = \lambda' - \lambda = \frac{v_s}{f}$

$$(680 - 656) \times 10^{-9} = \frac{v_s}{4.57 \times 10^{14}} \quad \Rightarrow \quad v_s \approx 1.10 \times 10^7 \text{ m s}^{-1}$$

Practice questions

Question 9.1 A bottle floating on water bobs up and down over a total distance of 20 cm twice each second. Find its amplitude and frequency.

Amplitude $= \frac{1}{2} \times 20 = 10$ cm. Frequency $= 2.0$ Hz.

Question 9.2 A horn produces a note of frequency 512 Hz. Sound travels at 330 m s^{-1} in air. Find the wavelength.

$$\lambda = \frac{v}{f} = \frac{330}{512} \approx 0.64 \text{ m}.$$

Question 9.3 On an oscilloscope with the time-base at 5.0 ms div^{-1}, 3 complete waveforms are seen over 5 divisions. Find (a) the period, (b) the frequency.

(a) Total time $= 5 \times 5.0 = 25$ ms for 3 waves: $T = \dfrac{25}{3} = 8.3$ ms.
(b) $f = \dfrac{1}{T} = \dfrac{1}{8.3 \times 10^{-3}} = 1.2 \times 10^{2}$ Hz.

Question 9.4 An electromagnetic wave has a period of 1.0 ps. Find (a) its wavelength, (b) the number of wavelengths in 1.0 m.

(a) $\lambda = cT = 3.0 \times 10^{8} \times 1.0 \times 10^{-12} = 3.0 \times 10^{-4}$ m.
(b) $N = \dfrac{1.0}{3.0 \times 10^{-4}} \approx 3.3 \times 10^{3}$.

Question 9.5 A sound wave has amplitude A_0 and intensity I_0. The amplitude increases to $3A_0$. Find the new intensity.

$I \propto A^2 \quad \Rightarrow \quad I = 3^2 I_0 = 9I_0$.

Question 9.6 Radiation of intensity 500 W m^{-2} arrives normally on a solar panel of effective area 4.0 m^2. How much energy arrives in one hour? What if the amplitude is doubled?

$P = IS = 500 \times 4.0 = 2000$ W, so $E = Pt = 2000 \times 3600 = 7.2 \times 10^{6}$ J.
Doubling the amplitude quadruples the intensity: $E = 4 \times 7.2 \times 10^{6} = 2.9 \times 10^{7}$ J.

Question 9.7 Horizontally polarised light of intensity I_0 passes through a filter with its plane of polarisation at 30° to the horizontal. Find the transmitted intensity.

$I = I_0 \cos^2 30^\circ = 0.75 I_0$.

Question 9.8 Vertically polarised light of intensity I_0 meets a filter B with a horizontal axis. (a) Find the emergent intensity. (b) A filter C at 45° to the vertical is inserted before B. Find the new emergent intensity.

(a) $\theta = 90°$, so $I = 0$.
(b) After C: $I_0 \cos^2 45° = \frac{1}{2} I_0$. C is 45° from B, so after B: $\frac{1}{2} I_0 \cos^2 45° = \frac{1}{4} I_0$.

Question 9.9 A car's horn sounds at 500 Hz but a roadside observer hears 450 Hz. Find the speed and direction of the car (speed of sound 340 m s^{-1}).

The frequency is lower, so the car is moving away : $450 = \dfrac{500 \times 340}{340 + v_s} \Rightarrow v_s = 340\left(\dfrac{500}{450} - 1\right) \approx 38$ m s^{-1}.

Question 9.10 An ambulance travelling at 25 m s^{-1} passes close to a stationary observer. Its signal has frequency 1500 Hz. What is the overall change in the frequency heard (speed of sound 340 m s^{-1})?

Approaching: $f' = \dfrac{1500 \times 340}{340 - 25} = 1619$ Hz. Receding: $f' = \dfrac{1500 \times 340}{340 + 25} = 1397$ Hz.
The frequency drops by $1619 - 1397 \approx 2.2 \times 10^2$ Hz.