Superposition

AS Level Physics 9702

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By the end of this topic I can...

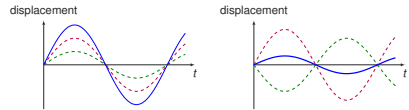
- state the principle of superposition; use path and phase difference
- explain interference and use $x = \lambda D/d$
- use the diffraction grating equation
- describe stationary waves and find their wavelengths

Principle of superposition

Superposition

Resultant displacement = **vector sum** of the individual displacements.

- constructive: peak meets peak
- destructive: peak meets trough



Path and phase difference

Phase and path

Constructive: $\Delta L = n\lambda$. Destructive: $\Delta L = (n + \frac{1}{2})\lambda$.

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta L}{\lambda} = \frac{\Delta t}{T}$$

Example: two loudspeakers

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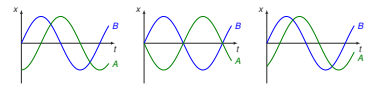
path difference for case (a): $\Delta L = 10 - 10 = 0$
so constructive superposition, resultant amplitude is large, a loud sound is heard
path difference for case (b): $\Delta L = 12 - 10 = 2.0$ m $\quad \Rightarrow \quad \Delta L = \frac{5}{2}\lambda$
so destructive superposition, resultant amplitude is small, sound is quiet

Example: phase differences

Each diagram shows the displacement of a point in two waves A and B of wavelength 30 cm travelling in the same direction. The time shift between them is (a) $\frac{1}{4}T$, (b) $\frac{1}{2}T$, (c) $\frac{1}{6}T$. Find the phase difference and path difference in each case.

Example: phase differences

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Use $\dfrac{\Delta\phi}{2\pi} = \dfrac{\Delta L}{\lambda} = \dfrac{\Delta t}{T}$.

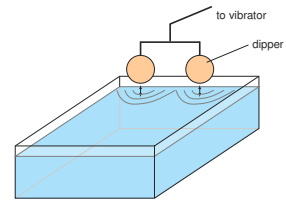
(a) $\Delta\phi = \frac{1}{4} \times 2\pi = \frac{\pi}{2}$, $\Delta L = \frac{1}{4} \times 30 = 7.5$ cm.

(b) $\Delta\phi = \pi$, $\Delta L = 15$ cm.

(c) $\Delta\phi = \frac{\pi}{3}$, $\Delta L = 5.0$ cm.

Interference

- stable pattern of maxima and minima
- needs **coherent** sources: constant phase difference
- one source split: double slit, two speakers on one generator



Example: maxima and minima

Example: maxima and minima

Amplitude of maxima: $A_{\text{max}} = A_1 + A_2 = 0.30 + 0.20 = 0.50$ cm

Amplitude of minima: $A_{\text{min}} = A_1 - A_2 = 0.30 - 0.20 = 0.10$ cm

Ratio of intensities: $\frac{I_{\text{max}}}{I_{\text{min}}} = \left(\frac{A_{\text{max}}}{A_{\text{min}}}\right)^2 = \left(\frac{0.50}{0.10}\right)^2 \quad \Rightarrow \quad \frac{I_{\text{max}}}{I_{\text{min}}} = 25$

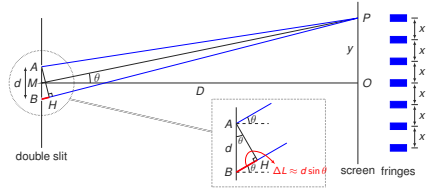
Young's double slit

Double slit

Fringe spacing for $D \gg d$.

- longer λ, larger D, smaller d: wider fringes

$$x = \frac{\lambda D}{d}$$



Example: red laser

Example: red laser

Fringe separation: $x = \frac{\lambda D}{d} = \frac{632\times10^{-9}\times2.0}{0.30\times10^{-3}} \quad \Rightarrow \quad x \approx 4.2 \times 10^{-3}$ m

if replaced by green light, wavelength becomes shorter, so smaller fringe separation

Example: changing the slits

Example: changing the slits

(a) From $x = \frac{\lambda D}{d}$, shorter λ results in smaller fringe separation
but brightness of the fringes remains unchanged
(b) Reducing width of slit causes amplitude of light from that slit to decrease
at maxima, resultant amplitude decreases, so bright fringes become less bright
at minima, amplitudes do not cancel completely, so dark fringes become a bit brighter
but no change to separation between fringes

Diffraction grating

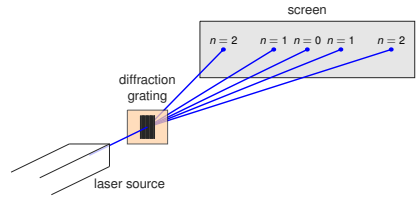
Grating

Maxima are **sharper and brighter** than for a double slit.

- round $n_{\max}$ down
- white light: white centre, spectra at higher orders

$$d \sin \theta = n\lambda$$

$$n_{\max} < \frac{d}{\lambda}$$



Example: lines per millimetre

Example: lines per millimetre

slit separation: $d = \frac{n\lambda}{\sin\theta} = \frac{1\times632\times10^{-9}}{\sin 16^\circ} \quad \Rightarrow \quad d \approx 2.29 \times 10^{-6}$ m

number of lines in 1 mm: $N = \frac{1\text{ mm}}{2.29\times10^{-6}\text{ m}} = \frac{1\times10^{-3}}{2.29\times10^{-6}} \quad \Rightarrow \quad N \approx 436$

Example: highest order

Example: highest order

to find highest order, we have: $n_{\max} < \frac{d \sin 90^\circ}{\lambda} = \frac{2.0 \times 10^{-6}}{510 \times 10^{-9}} \approx 3.9 \quad \Rightarrow \quad n_{\max} = 3$

1st, 2nd, 3rd-order on either side and 0th-order at centre, so $2 \times 3 + 1 = 7$ maxima are formed

blue light has shorter wavelength, so more maxima will be produced with blue light, and separation between each maxima would be closer

Example: same angle

Example: same angle

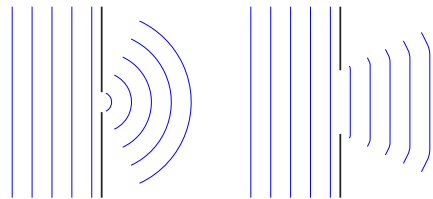
using diffraction grating equation: $d \sin\theta = n\lambda$, we find $n\lambda$ is constant for fixed θ
from information given in the question, this constant is $2 \times 630 = 1260$ nm
if $n = 1$, $\lambda = 1260$ nm, which would be infra-red
if $n = 3$, $\lambda = 420$ nm, which would be visible (violet)
if $n \geq 4$, $\lambda \leq 315$ nm, which would be ultraviolet
so the desired wavelength is 420 nm

Diffraction

Diffraction

A wave **spreads** through a gap or around an obstacle.

- greatest when gap width is comparable to the wavelength



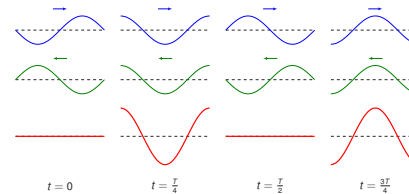
Stationary waves

Stationary wave

Two waves of the same frequency travelling in **opposite directions** superpose.

- nodes: zero amplitude; anti-nodes: maximum
- fixed end: node; open end: anti-node

$\lambda = 2 \times$ (node to node distance)

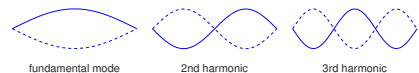


Both ends fixed

Fixed–fixed

Nodes at both ends.

$$\lambda_n = \frac{2L}{n}, \quad f_n = nf_1$$



Example: Melde's experiment

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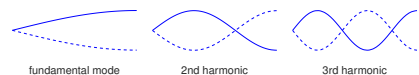
(a) Five loops are five half-wavelengths: $L = \frac{5}{2}\lambda \quad \Rightarrow \quad \lambda = \frac{2}{5} \times 80 = 32$ cm.
(b) Use a longer string, lower the oscillator frequency, or increase the tension (faster wave speed).

One end open

Closed–open

Node at one end, anti-node at the other.

$$\lambda_n = \frac{4L}{2n-1}, \quad f_n = (2n-1)f_1$$



Example: closed pipe

Example: closed pipe

(a) wavelength of this excited mode: $\lambda = \frac{4}{3}L = \frac{4}{3} \times 30 \times 10^{-2} \quad \Rightarrow \quad \lambda = 0.40$ m

(b) wave speed: $v = \lambda f = 0.40 \times 825 \quad \Rightarrow \quad v = 330$ m s^{-1}

(c) wavelength of fundamental mode: $\lambda = 4L = 4 \times 30 \times 10^{-2} \quad \Rightarrow \quad \lambda = 1.2$ m

fundamental frequency: $f = \frac{v}{\lambda} = \frac{330}{1.2} \quad \Rightarrow \quad f = 275$ Hz

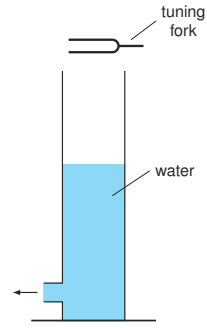
(d) note that an anti-node is formed near open end, also sound wave is longitudinal so air molecule near open end vibrate *horizontally* with greatest amplitude

Example: tuning fork and water

A tuning fork is held above a tall cylinder which is initially filled with water. The water level, measured from the bottom of the cylinder, is lowered at a constant rate. A loud sound is first heard when the water level is at 60.0 cm, and the next loud sound is heard when the water level is at 26.8 cm. It is known that the speed of sound in air is 340 m s^{-1}. What is the frequency of the sound wave produced by the tuning fork?

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loud sound is heard when stationary wave is set up
node is formed at surface of water
node-to-node distance is $60.0 - 26.8 = 33.2$ cm
wavelength of this wave: $\lambda = 2 \times 33.2$ cm $= 0.664$ m
frequency: $f = \frac{v}{\lambda} = \frac{340}{0.664} \quad \Rightarrow \quad f \approx 512$ Hz

Key equations

phase $\dfrac{\Delta\phi}{2\pi} = \dfrac{\Delta L}{\lambda} = \dfrac{\Delta t}{T}$

grating $d\sin\theta = n\lambda$

double slit $x = \dfrac{\lambda D}{d}$

fixed ends $\lambda_n = \dfrac{2L}{n}, \quad f_n = nf_1$