Superposition

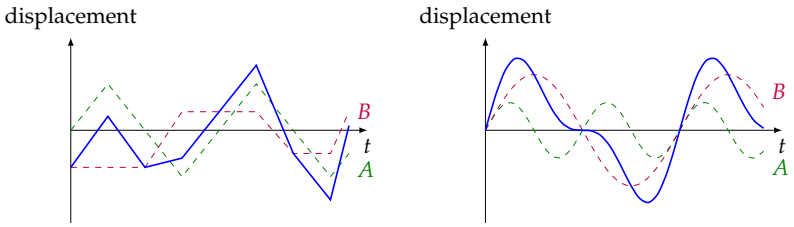
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By the end of this topic I can...

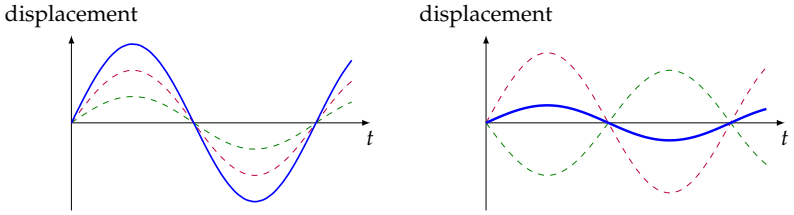
- state the principle of superposition and use path and phase difference
- explain interference and use the double-slit formula
- use the diffraction grating equation
- describe stationary waves, nodes and anti-nodes and find their wavelengths

Superposition of waves

The **principle of superposition**: when two or more waves meet, the resultant displacement is the vector sum of the individual displacements.



- **constructive**: peak meets peak (greatest amplitude)
- **destructive**: peak meets trough (amplitudes cancel)



Path difference and phase difference

- **path difference** ΔL: the difference in distance travelled by two waves to reach a point

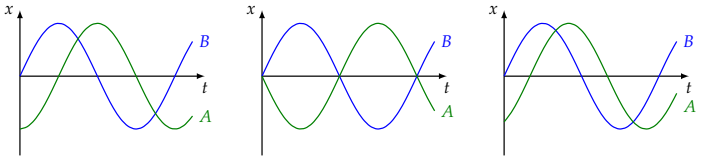
Constructive: $\Delta L =$ $0,\ \lambda,\ 2\lambda,\ \ldots$ Destructive: $\Delta L =$ $\frac{1}{2}\lambda,\ \frac{3}{2}\lambda,\ \ldots$

Example 10.1 Two loudspeakers are wired to produce identical sound signals in unison. The sound wave produced has a wavelength of 80 cm. Describe the volume you hear when you are at a distance of (a) 10 m from both speakers, (b) 10 m from one speaker and 12 m from the other?

path difference for case (a): $\Delta L = 10 - 10 = 0$
so constructive superposition, resultant amplitude is large, a loud sound is heard
path difference for case (b): $\Delta L = 12 - 10 = 2.0$ m $\Rightarrow$ $\Delta L = \frac{5}{2}\lambda$
so destructive superposition, resultant amplitude is small, sound is quiet

- **phase difference** $\Delta\phi$: how far one wave is ahead of another, in radians or degrees
- in phase: $\Delta\phi =$ $0, 2\pi, \ldots$; anti-phase: $\Delta\phi =$ $\pi, 3\pi, \ldots$

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta L}{\lambda} = \frac{\Delta t}{T}$$



Example 10.2 Each diagram shows the displacement of a point in two waves A and B of wavelength 30 cm travelling in the same direction. The time shift between them is (a) $\frac{1}{4}T$, (b) $\frac{1}{2}T$, (c) $\frac{1}{6}T$. Find the phase difference and path difference in each case.

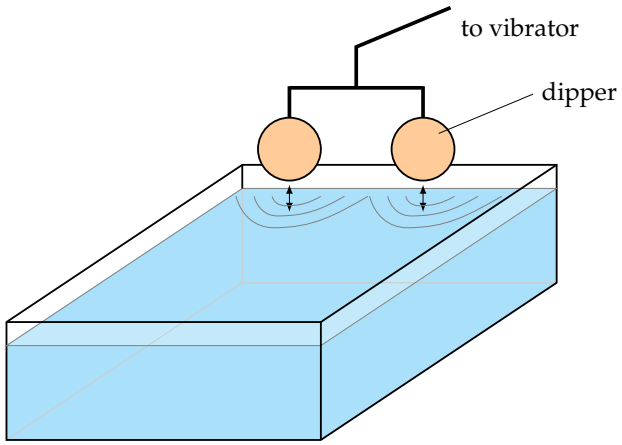
Use $\frac{\Delta\phi}{2\pi} = \frac{\Delta L}{\lambda} = \frac{\Delta t}{T}$.
(a) $\Delta\phi = \frac{1}{4} \times 2\pi = \frac{\pi}{2}$, $\Delta L = \frac{1}{4} \times 30 = 7.5$ cm.
(b) $\Delta\phi = \pi$, $\Delta L = 15$ cm.
(c) $\Delta\phi = \frac{\pi}{3}$, $\Delta L = 5.0$ cm.

Interference

Interference is the formation of a stable pattern of ___maxima___ and ___minima___ when waves of ___constant phase difference___ (___coherent___ waves) overlap.

• ___coherent___ sources have a constant phase difference; the easiest way to get them is to use ___one source___ (a laser through two slits, two speakers on one signal generator, dippers on one beam)

• maxima where $\Delta L = n\lambda$; minima where $\Delta L = (n + \frac{1}{2})\lambda$



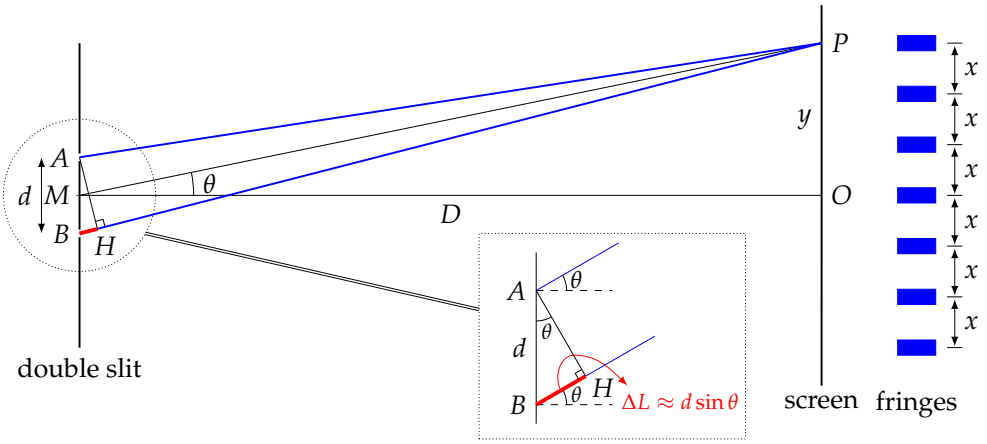
Example 10.3 Two coherent waves meet in space, one of which has an amplitude of 0.30 cm and the other of 0.20 cm. How does the intensity of maxima compare with the intensity of minima?

Amplitude of maxima: $A_{max} = A_1 + A_2 = 0.30 + 0.20 = 0.50$ cm
Amplitude of minima: $A_{min} = A_1 - A_2 = 0.30 - 0.20 = 0.10$ cm
Ratio of intensities: $\frac{I_{max}}{I_{min}} = \left(\frac{A_{max}}{A_{min}}\right)^2 = \left(\frac{0.50}{0.10}\right)^2 \quad \Rightarrow \quad \frac{I_{max}}{I_{min}} = 25$

Young's double-slit experiment

For slit separation d and slit-to-screen distance D ($D \gg d$), the ___fringe spacing___ is

$$x = \frac{\lambda D}{d}$$



• bright fringes where $d \sin\theta = n\lambda$; for small angles $\sin\theta \approx \tan\theta = y/D$, so $y_n = n\lambda D/d$

• longer wavelength, larger D or smaller d gives wider fringes

Example 10.4 A teacher demonstrates the double-slit experiment with a beam of red light produced from a helium-neon laser. The beam of wavelength of 632 nm is sent through a double-slit separated by 0.30 mm and passed onto a wall at about 2.0 m from the slits. (a) What is the fringe separation? (b) If the experiment is carried out using a green laser, what changes do you expect?

Fringe separation: $x = \frac{\lambda D}{d} = \frac{632\times10^{-9}\times2.0}{0.30\times10^{-3}} \Rightarrow x \approx 4.2\times10^{-3}$ m
if replaced by green light, wavelength becomes shorter, so smaller fringe separation

Example 10.5 Coherent light passes through a double slit. Initially, the light intensity from each slit is the same. State the change to the appearance of the fringes if (a) the separation between the slits is increased, (b) The width of one of the slits is reduced.

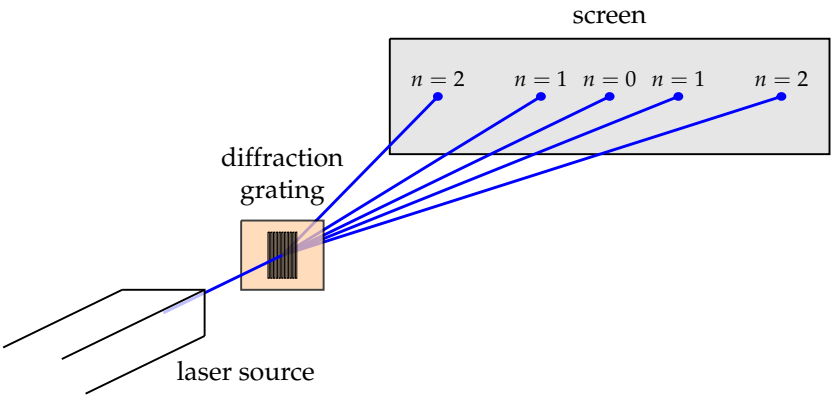
(a) From $x = \frac{\lambda D}{d}$, shorter λ results in smaller fringe separation
but brightness of the fringes remains unchanged
(b) Reducing width of slit causes amplitude of light from that slit to decrease
at maxima, resultant amplitude decreases, so bright fringes become less bright
at minima, amplitudes do not cancel completely, so dark fringes become a bit brighter
but no change to separation between fringes

Diffraction gratings

A **diffraction grating** has hundreds of slits per mm, producing sharper and brighter maxima than a double slit:

$$d \sin\theta = n\lambda$$

where d is the slit separation and n the order; light is incident normally.



- the highest order: $n_{\max} < \dfrac{d}{\lambda}$ (round down); total maxima $= 2n_{\max} + 1$
- white light: the central maximum is white; at higher orders a spectrum forms with red at the greater angle

Example 10.6 Light of 632 nm wavelength produces first-order maxima at 16° when passing through a diffraction grating at right angles. How many lines per millimetre are there in the diffraction grating?

slit separation: $d = \frac{n\lambda}{\sin\theta} = \frac{1\times632\times10^{-9}}{\sin 16^\circ} \Rightarrow d \approx 2.29\times10^{-6}$ m
number of lines in 1 mm: $N = \frac{1\text{ mm}}{2.29\times10^{-6}\text{ m}} = \frac{1\times10^{-3}}{2.29\times10^{-6}} \Rightarrow N \approx 436$

Example 10.7 Green light of 510 nm wavelength is incident normally on a diffraction grating with slit separation of 2.0 μm. (a) What is the highest order seen? (b) how many maxima are produced? (c) If blue light is used for the same diffraction grating experiment, suggest the effect on the diffraction pattern.

to find highest order, we have: $n_{\max} < \frac{d \sin 90^\circ}{\lambda} = \frac{2.0\times10^{-6}}{510\times10^{-9}} \approx 3.9 \quad \Rightarrow \quad n_{\max} = 3$
1st, 2nd, 3rd-order on either side and 0th-order at centre, so $2 \times 3 + 1 = 7$ maxima are formed
blue light has shorter wavelength, so more maxima will be produced with blue light, and separation between each maxima would be closer

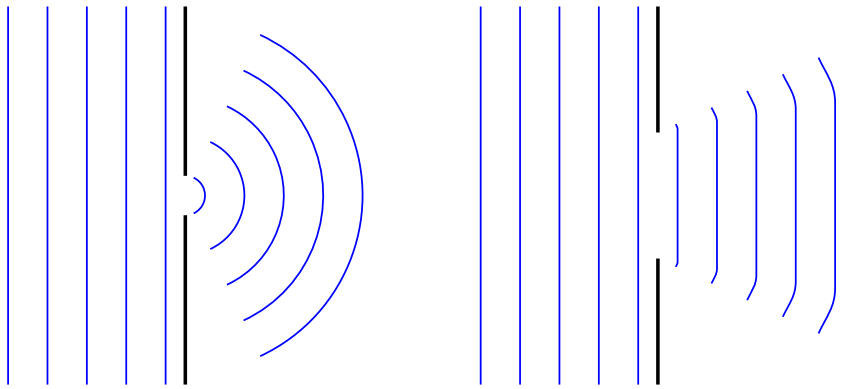
Example 10.8 Light of wavelength 630 nm produces a second-order maxima at angle of 60° when it is directed at a diffraction grating. If visible light of another wavelength can also give a maximum at the same angle, what is this wavelength?

using diffraction grating equation: $d \sin\theta = n\lambda$, we find $n\lambda$ is constant for fixed θ
from information given in the question, this constant is $2 \times 630 = 1260$ nm
if $n = 1$, $\lambda = 1260$ nm, which would be infra-red
if $n = 3$, $\lambda = 420$ nm, which would be visible (violet)
if $n \geq 4$, $\lambda \leq 315$ nm, which would be ultraviolet
so the desired wavelength is 420 nm

Diffraction

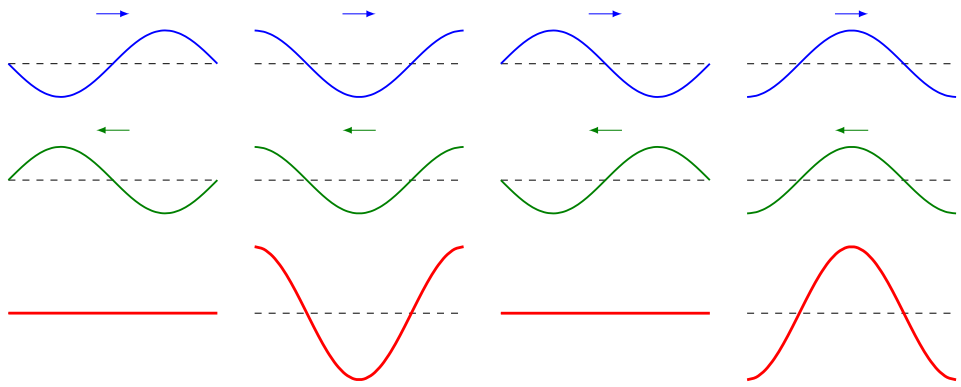
Diffraction is the spreading of a wave when it passes through a gap or around an obstacle.

- the effect is greatest when the gap width is comparable to the wavelength



Stationary waves

A **stationary (standing) wave** forms when two waves of the same frequency travelling in opposite directions superpose (often a wave and its reflection).



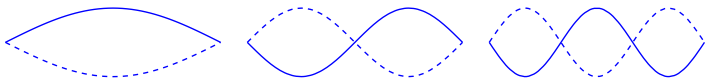
$t = 0$ $t = \frac{T}{4}$ $t = \frac{T}{2}$ $t = \frac{3T}{4}$

- **nodes**: points of zero amplitude; **anti-nodes**: points of maximum amplitude
- the distance between adjacent nodes is $\frac{1}{2}\lambda$, so

$$\lambda = 2 \times \text{(node to node distance)}$$

- a fixed (closed) end is a node; a free (open) end is an anti-node

Both ends fixed



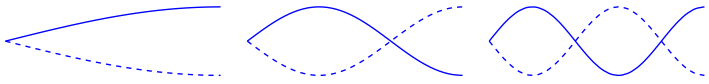
fundamental mode 2nd harmonic 3rd harmonic

$$\lambda_n = \frac{2L}{n}, \quad f_n = nf_1$$

Example 10.9 In Melde's experiment a string of length 80 cm vibrates with five loops. (a) Find the wavelength. (b) How could a stationary wave of longer wavelength be produced?

(a) Five loops are five half-wavelengths: $L = \frac{5}{2}\lambda \quad \Rightarrow \quad \lambda = \frac{2}{5} \times 80 = 32$ cm.
(b) Use a longer string, lower the oscillator frequency,
or increase the tension (faster wave speed).

One fixed (closed) end and one open end



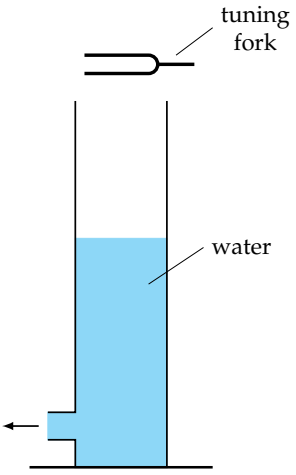
fundamental mode 2nd harmonic 3rd harmonic

$$\lambda_n = \frac{4L}{2n-1}, \quad f_n = (2n-1)f_1$$

Example 10.10 The graph shows a standing sound wave produced in an air column in a closed pipe of length 30 cm. The frequency of this sound wave is 825 Hz. (a) What is the wavelength of the sound wave? (b) Calculate the speed of sound. (c) What is the frequency of the fundamental mode for sound wave in this air column? (d) Describe the motion of an air molecule near the open end.

(a) wavelength of this excited mode: $\lambda = \frac{4}{3}L = \frac{4}{3} \times 30 \times 10^{-2} \quad \Rightarrow \quad \lambda = 0.40$ m
(b) wave speed: $v = \lambda f = 0.40 \times 825 \quad \Rightarrow \quad v = 330$ m s^{-1}
(c) wavelength of fundamental mode: $\lambda = 4L = 4 \times 30 \times 10^{-2} \quad \Rightarrow \quad \lambda = 1.2$ m
fundamental frequency: $f = \frac{v}{\lambda} = \frac{330}{1.2} \quad \Rightarrow \quad f = 275$ Hz
(d) note that an anti-node is formed near open end, also sound wave is longitudinal
so air molecule near open end vibrate *horizontally* with greatest amplitude

Example 10.11 A tuning fork is held above a tall cylinder which is initially filled with water. The water level, measured from the bottom of the cylinder, is lowered at a constant rate. A loud sound is first heard when the water level is at 60.0 cm, and the next loud sound is heard when the water level is at 26.8 cm. It is known that the speed of sound in air is 340 m s^{-1}. What is the frequency of the sound wave produced by the tuning fork?



loud sound is heard when stationary wave is set up
node is formed at surface of water
node-to-node distance is $60.0 - 26.8 = 33.2$ cm
wavelength of this wave: $\lambda = 2 \times 33.2$ cm $= 0.664$ m
frequency: $f = \frac{v}{\lambda} = \frac{340}{0.664} \quad \Rightarrow \quad f \approx 512$ Hz

Stationary and progressive waves

	Progressive wave	Stationary wave
energy	transferred along the wave	not transferred
amplitude	the same for all points	varies: zero at nodes, maximum at anti-nodes
phase difference	any value from 0 to 2π	0 or π only

Practice questions

Question 10.1 A wave of frequency 10 Hz travels at 4.0 m s^{-1}. What is the phase difference between two points 0.50 m apart?

$\lambda = \frac{v}{f} = 0.40$ m. $\Delta\phi = 2\pi\frac{\Delta L}{\lambda} = 2\pi \times \frac{0.50}{0.40} = 2.5\pi$ rad (equivalent to $\pi/2$).

Question 10.2 Yellow light of wavelength 580 nm passes through slits 0.40 mm apart onto a screen 2.5 m away. What is the distance between a bright fringe and its nearest dark fringe?

$x = \frac{\lambda D}{d} = \frac{580 \times 10^{-9} \times 2.5}{0.40 \times 10^{-3}} = 3.6 \times 10^{-3}$ m. A dark fringe is half a spacing away: 1.8 mm.

Question 10.3 A red laser of wavelength 630 nm gives fringes 15 mm apart on a screen 4.0 m from a double slit. Find the slit separation.

$d = \dfrac{\lambda D}{x} = \dfrac{630 \times 10^{-9} \times 4.0}{15 \times 10^{-3}} = 1.7 \times 10^{-4}$ m.

Question 10.4 Light falls normally on two slits 2.00 mm apart. The screen is 2.5 m away and the 4th bright spot is 3.2 mm from the central maximum. Find the wavelength.

$x = \dfrac{3.2}{4} = 0.80$ mm. $\lambda = \dfrac{xd}{D} = \dfrac{0.80 \times 10^{-3} \times 2.00 \times 10^{-3}}{2.5} = 6.4 \times 10^{-7}$ m $= 640$ nm.

Question 10.5 Light is incident normally on a grating with 1000 lines per mm. The angle between the two first-order maxima is 60°. Find the wavelength.

$d = \dfrac{1}{1000}$ mm $= 1.0 \times 10^{-6}$ m. Each first order is at 30°: $\lambda = d \sin\theta = 1.0 \times 10^{-6} \times \sin 30° = 5.0 \times 10^{-7}$ m $= 500$ nm.

Question 10.6 Light of wavelength 590 nm gives a first-order maximum at 20.0° from a grating. Find (a) the slit separation, (b) the angular separation of the first and second orders.

(a) $d = \dfrac{\lambda}{\sin 20.0°} = 1.73 \times 10^{-6}$ m.

(b) $\sin\theta_2 = \dfrac{2\lambda}{d} = 0.684 \quad \Rightarrow \quad \theta_2 = 43.2°$. Separation $= 43.2° - 20.0° = 23.2°$.

Question 10.7 Light of wavelength 480 nm is incident normally on a grating of slit separation 3.0 μm. How many maxima are observed?

$\frac{d}{\lambda} = \frac{3.0 \times 10^{-6}}{480 \times 10^{-9}} = 6.25 \quad \Rightarrow \quad n_{\max} = 6$. Number of maxima $= 2 \times 6 + 1 =$ 13.

Question 10.8 A stationary wave is three wavelengths long. How many nodes (including both ends) and anti-nodes are there?

Each wavelength has two half-wavelength loops, so 6 loops: __6 anti-nodes__ and __7 nodes__.

Question 10.9 A rope of length 150 cm is fixed at both ends. At what wavelengths can stationary waves form?

$\lambda = \frac{2L}{n} = \frac{300}{n}$ cm, so 300 cm, 150 cm, 100 cm, 75 cm, ...