Solids

AS Level Physics 9702

J. Rowing • Vijay International School Praslin

By the end of this topic I can. . .

- use Hooke's law, spring constant and elastic potential energy
- combine springs in series and in parallel
- define stress, strain and the Young modulus
- interpret stress–strain graphs

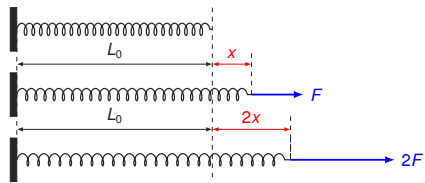
Hooke's law

Hooke's law

Extension is **proportional** to the load, below the limit of proportionality.

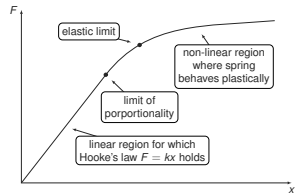
- k: spring constant, N m^{-1}
- stiff spring = large k

$$F = kx$$



Limits

- **limit of proportionality**: F no longer $\propto x$
- **elastic limit**: beyond it the spring shows permanent set



Example: finding k

Example: finding k

$$k = \frac{F}{x} = \frac{mg}{L - L_0} = \frac{0.250 \times 9.81}{(26.0 - 20.0) \times 10^{-2}} \quad \Rightarrow \quad k \approx 40.9 \text{ N m}^{-1}$$

Example: acceleration on a spring

Example: acceleration on a spring

$$F_{\text{net}} = F - mg = kx - mg = ma \quad \Rightarrow$$

$$a = \frac{kx - mg}{m} = \frac{270 \times 5.0 \times 10^{-2} - 1.2 \times 9.81}{1.2} \approx 1.44 \text{ m s}^{-2}$$

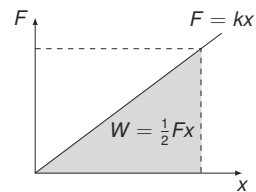
Elastic potential energy

Strain energy

Area under the F–x graph.

- force varies, so no $W = Fx$

$$E_\text{p} = \tfrac{1}{2}Fx = \tfrac{1}{2}kx^2$$



Example: work to stretch

Example: work to stretch

Work done needed equals the increase in elastic potential energy:

$$W = \Delta E_p = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = \frac{1}{2} \times 2000 \times (0.050^2 - 0.030^2) = 1.6 \text{ J}$$

Example: spring launches a trolley

Example: spring launches a trolley

Elastic potential energy in spring $\Rightarrow$ kinetic energy of trolley

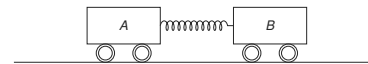
$$\frac{1}{2}Fx = \frac{1}{2}mv^2 \quad \Rightarrow \quad \frac{1}{2} \times 20 \times 0.050 = \frac{1}{2} \times 0.40 \times v^2 \quad \Rightarrow \quad v \approx 1.58 \text{ m s}^{-1}$$

Example: two trolleys

The same spring (compressed 5.0 cm by a 20 N force) is set between two trolleys *A* and *B* of masses 400 g and 600 g. After release, what are their final speeds?

Example: two trolleys

The same spring (compressed 5.0 cm by a 20 N force) is set between two trolleys A and B of masses 400 g and 600 g. After release, what are their final speeds?



Energy: $\frac{1}{2}Fx = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 \quad \Rightarrow \quad 0.50 = 0.20v_A^2 + 0.30v_B^2$.

Momentum (started at rest): $0.40v_A = 0.60v_B \quad \Rightarrow \quad v_A = 1.5v_B$.

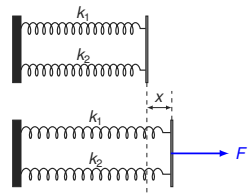
Substituting: $0.20(2.25v_B^2) + 0.30v_B^2 = 0.75v_B^2 = 0.50 \quad \Rightarrow \quad v_B \approx 0.82\ \text{m s}^{-1}$, $v_A \approx 1.2\ \text{m s}^{-1}$.

Springs in parallel

Parallel

Same **extension**; forces add.

$$k = k_1 + k_2 + \cdots$$

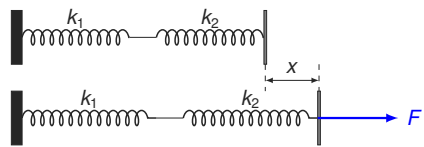


Springs in series

Series

Same **force**; extensions add.

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots$$

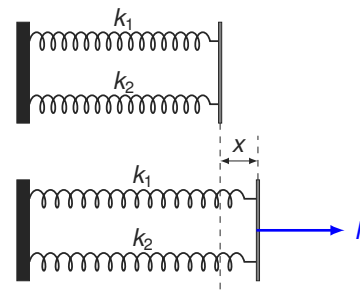


Example: combined springs

(a) A spring with $k_1 = 20$ N cm^{-1} is connected in series with a second spring with $k_2 = 30$ N cm^{-1}. When a force of 60 N is applied, what is the total extension of the combination? (b) The same two springs are now connected in parallel. When a force of 50 N is applied on the combination, what is the extension?

Example: combined springs

(a) A spring with $k_1 = 20$ N cm^{-1} is connected in series with a second spring with $k_2 = 30$ N cm^{-1}. When a force of 60 N is applied, what is the total extension of the combination? (b) The same two springs are now connected in parallel. When a force of 50 N is applied on the combination, what is the extension?



For series connection:

$k = \left(\frac{1}{k_1} + \frac{1}{k_1}\right)^{-1} = \left(\frac{1}{20} + \frac{1}{30}\right)^{-1} = 12$ N cm^{-1} $\Rightarrow$

$x = \frac{F}{k} = \frac{60}{12} = 5.0$ cm

or sum up extension of each spring:

$x = x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2} = \frac{60}{20} + \frac{60}{30} = 5.0$ cm

for parallel connection: $k = k_1 + k_2 = 20 + 30 =$

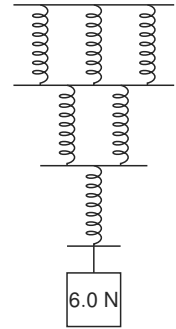
50 N cm^{-1} $\Rightarrow$ $x = \frac{F}{k} = \frac{50}{50} = 1.0$ cm $\square$

Example: network of springs

A set of identical springs are set up as shown. Each individual spring extends by 1.0 cm under a load of 1.0 N. Assume the limit of proportionality is not exceeded, what is the total extension for this combination when a load of 6.0 N is applied?

Example: network of springs

A set of identical springs are set up as shown. Each individual spring extends by 1.0 cm under a load of 1.0 N. Assume the limit of proportionality is not exceeded, what is the total extension for this combination when a load of 6.0 N is applied?



For a single spring: $k = 1.0\ \text{N cm}^{-1}$, the combined spring constant: $k_{\text{total}} = \left(\frac{1}{3k} + \frac{1}{2k} + \frac{1}{k}\right)^{-1}$

$$k_{\text{total}} = \left(\frac{1}{3.0} + \frac{1}{2.0} + \frac{1}{1.0}\right)^{-1} = \frac{6}{11} \approx 0.545\ \text{N cm}^{-1}$$

Total extension: $x = \frac{F}{k_{\text{total}}} = \frac{6.0}{0.545} = 11\ \text{cm}$

Alternatively, we can find and add extensions of each layer. In particular, the top layer withstands a force of 6.0 N shared by three springs, so each spring has a force of 2.0 N, extension is 2.0 cm. Similarly, the other two layers extend by 3.0 cm and 6.0 cm respectively,

Stress and strain

Stress and strain

Stress = **force per unit area**. Strain = **extension per unit length**.

- stress unit: Pa; strain has no unit

$$\sigma = \frac{F}{A}$$

$$\epsilon = \frac{x}{L}$$

Example: breaking force

Example: breaking force

$$\sigma = \frac{F}{A} \quad \Rightarrow \quad F = \sigma A = 2.0 \times 10^{8} \times 1.5 \times 10^{-6} = 3.0 \times 10^{2}\ \mathrm{N}$$

Young modulus

Young modulus

Ratio of **stress** to **strain** (below the limit of proportionality).

- property of the material, not the shape

$$E = \frac{\sigma}{\epsilon} = \frac{FL}{Ax}$$

$$k = \frac{EA}{L}$$

Example: steel wire

Example: steel wire

$$E = \frac{\mathrm{F/A}}{\mathrm{x/L}} = \frac{FL}{Ax} = \frac{200 \times 1.5}{\pi \times (0.30 \times 10^{-3})^2 \times 5.0 \times 10^{-3}} \quad \Rightarrow \quad E \approx 2.1 \times 10^{11}\ \mathrm{Pa}$$

Example: copper wire

Example: copper wire

strain: $\epsilon = \frac{\sigma}{E} = \frac{7.8 \times 10^7}{1.2 \times 10^{11}} \quad \Rightarrow \quad \epsilon = 6.5 \times 10^{-4} = 0.065\%$

extension: $x = \epsilon L = 6.5 \times 10^{-4} \times 2.0 \quad \Rightarrow \quad x = 1.3 \times 10^{-3}$ m

Example: bridge blocks

Example: bridge blocks

$$E = \frac{FL}{Ax} \quad \Rightarrow \quad F = \frac{EAx}{L} = \frac{2.1 \times 10^{11} \times (0.15 \times 0.15) \times 2.0 \times 10^{-3}}{0.30}$$

$$\Rightarrow \quad F \approx 3.15 \times 10^{7} \text{ N}$$

Example: comparing wires

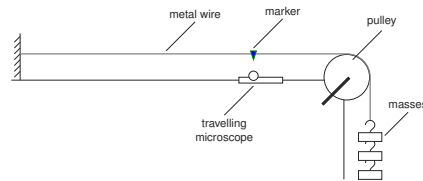
Example: comparing wires

$$E = \frac{FL}{Ax} \quad \Rightarrow \quad A = \frac{1}{4}\pi d^2 = \frac{FL}{Ex} \quad \Rightarrow$$

$$d^2 \propto \frac{1}{E} \quad \Rightarrow \quad \frac{d_A}{d_B} = \sqrt{\frac{E_B}{E_A}} = \frac{1}{\sqrt{2}}$$

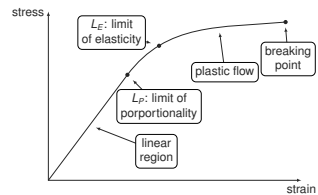
Measuring the Young modulus

- length: metre rule; diameter: micrometer
- add masses, record extension
- plot stress against strain
- **gradient** = Young modulus



Stress–strain graphs

- linear up to the limit of proportionality
- ductile: plastic deformation before breaking
- brittle: breaks with little plastic deformation
- area under graph = energy per unit volume



Metals, polymers and rubber

- metals: dislocations allow plastic deformation
- alloying and work hardening pin dislocations
- polymers: chains untangle then straighten
- rubber: hysteresis, area between curves = energy lost

Key equations

Hooke	$F = kx$	strain	$\epsilon = \dfrac{x}{L}$
energy	$E_\mathrm{p} = \frac{1}{2}Fx = \frac{1}{2}kx^2$	Young	$E = \dfrac{\sigma}{\epsilon} = \dfrac{FL}{Ax}$
stress	$\sigma = \dfrac{F}{A}$		