Quantum Physics

Name _____ Date _____

By the end of this topic I can...

- use the photon model: $E = hf = hc/\lambda$, and the electronvolt
- explain the photoelectric effect and use $hf = \Phi + E_{k,\max}$
- use photon momentum $p = h/\lambda$ and the de Broglie wavelength $\lambda = h/p$
- explain energy levels and line spectra, and use band theory for conductors, insulators and semiconductors

From classical models to photons

Four ideas take us beyond classical physics: photons and the photoelectric effect, matter waves, atomic energy levels and spectra, and energy bands in solids.

Particle and wave models

• **particle model**: a system of particles obeying _____Newtonian mechanics_____ (electric current, ideal gas pressure, radioactive decay)

• **wave model**: energy transferred by vibrations of a medium or field (water, sound, light). The characteristic wave behaviours come from _____superposition_____: _____interference_____ and _____diffraction_____.

A history of light

• **Corpuscular theory** (Newton, 1671): light is a stream of particles. It explains straight lines and reflection, but not _____interference or diffraction_____.

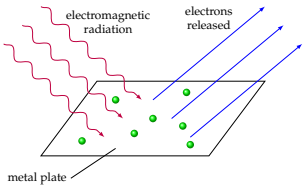
• **Wave theory** (Huygens, 1678): light is a wave in the _____aether_____. It explains interference (Young's double slit) and diffraction, but the aether was disproved.

• **Electromagnetic theory** (Maxwell, 1865): light is an _____electromagnetic wave_____, travelling at $c = \dfrac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m s}^{-1}$ with no medium needed.

By the late 1800s, everything known about light seemed explained by waves...

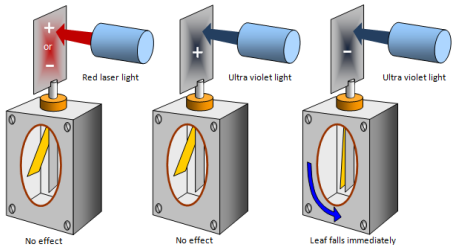
Photons and the photoelectric effect

The photoelectric effect



When electromagnetic radiation of high enough frequency falls on a metal, the photoelectric effect releases electrons, called photoelectrons, from the surface (first seen by Hertz, 1887).

- **observations**:
- there is a minimum threshold frequency f_0: no electrons are emitted if $f < f_0$
- emission is immediate, even for very faint light above f_0
- a greater intensity gives more electrons per second, but no change in their maximum energy
- a higher frequency gives electrons of greater maximum energy
- **wave theory fails**: it predicts that electrons gradually build up energy (so any frequency should work with enough time), that dim light needs a delay, that greater intensity gives greater K.E., and that frequency should not affect K.E.



- **demonstration**: a clean zinc plate on a gold-leaf electroscope. Charged negatively, the leaf falls under UV light (electrons leave). Charged positively, the leaf is unaffected (the electrons are attracted back).

Photon theory

Einstein (1905) proposed that radiation is delivered in packets of energy.

A photon is a packet (quantum) of electromagnetic energy.

- energy of one photon, with the Planck constant $h = 6.63 \times 10^{-34}$ J s:

$$E = hf = \frac{hc}{\lambda}$$

- higher frequency (shorter wavelength) means greater photon energy
- the intensity of a beam is power per unit area. With n photons per second:

$$I = \frac{nhf}{A}$$

- **electronvolt**: 1 eV is the energy gained by an electron moving through a p.d. of 1 V, so 1 eV = 1.60×10^{-19} J

Example 29.1 A laser emits red light of wavelength 650 nm at a power of 2.0 mW. (a) What is the energy of one photon? (b) How many photons are emitted per second?

(a) $E = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{650 \times 10^{-9}} = 3.06 \times 10^{-19}\ \text{J}$

(b) $N = \dfrac{P}{E} = \dfrac{2.0 \times 10^{-3}}{3.06 \times 10^{-19}} = 6.5 \times 10^{15}\ \text{s}^{-1}$

Question 29.1 Calculate the energy, in eV, of a photon of wavelength 440 nm.

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{440 \times 10^{-9}} = 4.52 \times 10^{-19}\ \text{J} = \frac{4.52 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.8\ \text{eV}$$

Question 29.2 The red laser is replaced by a green laser of the same power. How does the number of photons emitted per second change?

Green light has a shorter wavelength, so each photon has **more** energy. $N = P/E$, so **fewer** photons are emitted per second.

The photoelectric effect explained

- the work function Φ is the minimum energy needed for an electron to escape from the metal surface
- one photon is absorbed by one electron (all or nothing). If $hf > \Phi$ the electron escapes and the excess becomes kinetic energy:

$$hf = \Phi + E_{k,\max}$$

This is Einstein's photoelectric equation; $E_{k,\max}$ is the maximum K.E. (electrons from deeper below the surface lose extra energy).

- at the threshold frequency the photon has just enough energy:

$$hf_0 = \Phi$$

- **observations explained**: $f < f_0$ gives photon energy $< \Phi$, so no emission; one-to-one interaction means no delay; more intensity means more photons so more electrons; higher f means more photon energy so greater K.E.

Example 29.2 The work function of gold is 4.9 eV. Find the longest wavelength of radiation that releases electrons from gold.

$$f_0 = \frac{\Phi}{h} = \frac{4.9 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.18 \times 10^{15} \text{ Hz}$$
$$\lambda_0 = \frac{c}{f_0} = \frac{3.00 \times 10^8}{1.18 \times 10^{15}} = 2.54 \times 10^{-7} \text{ m (ultraviolet)}$$

Question 29.3 Sodium has a work function of 3.8×10^{-19} J. (a) Find its threshold frequency. (b) Can light of wavelength 500 nm release electrons?

(a) $f_0 = \dfrac{3.8 \times 10^{-19}}{6.63 \times 10^{-34}} = 5.7 \times 10^{14}$ Hz

(b) $f = \dfrac{3.0 \times 10^8}{500 \times 10^{-9}} = 6.0 \times 10^{14}$ Hz $> f_0$, so **yes**.

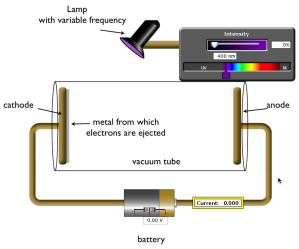
Question 29.4 Radiation of wavelength 1200 nm releases electrons of maximum K.E. 5.4×10^{-20} J from a metal. What is the work function?

$$hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^8}{1200 \times 10^{-9}} = 1.66 \times 10^{-19} \text{ J}$$
$$\Phi = hf - E_k = 1.66 \times 10^{-19} - 0.54 \times 10^{-19} = 1.1 \times 10^{-19} \text{ J}$$

Question 29.5 Light of one frequency and intensity releases electrons from a metal. Light of the same intensity but higher frequency is used. What happens to the rate of emission?

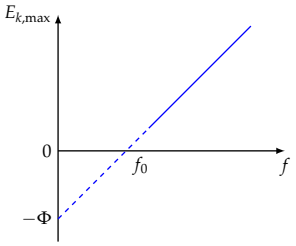
Each photon has more energy, so for the same intensity there are **fewer photons per second**, and the rate of emission of electrons **decreases**. (Each electron has greater K.E.)

Measuring h and the work function



• photocell: illuminate the metal with (nearly) monochromatic light of known frequency and raise the reverse p.d. until the current is zero: this is the stopping voltage V_{stop}:

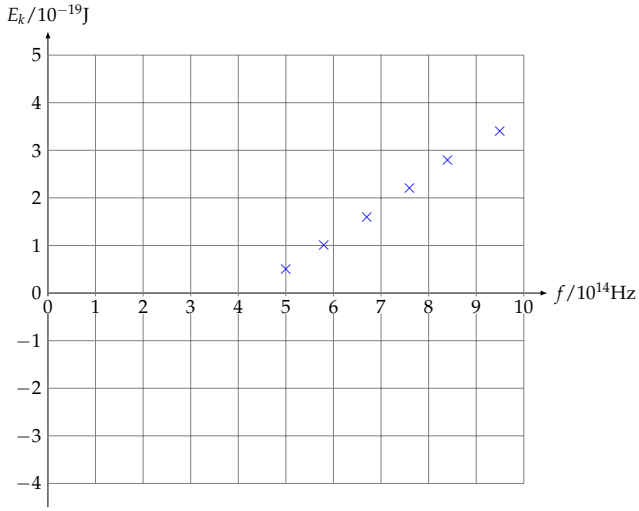
$$E_{k,\max} = eV_{\text{stop}}$$



• repeat for several frequencies, plot $E_{k,\max}$ against f. Comparing $E_{k,\max} = hf - \Phi$ with $y = mx + c$: gradient = h, y-intercept = $-\Phi$, x-intercept = f_0

Question 29.6 A metal with a greater work function is used. Describe the change to the $E_{k,\max}$–f line.

The gradient is still h (same slope), but the line shifts to the **right**: the intercept $-\Phi$ is more negative and the threshold frequency $f_0 = \Phi/h$ is greater.



Question 29.7 The graph shows the maximum K.E. of photoelectrons against the frequency of the radiation. Use it to find (a) the threshold frequency and (b) a

value of the Planck constant.

(a) The best-fit line cuts the f-axis at about 4.2×10^{14} Hz.
(b) Gradient $= \dfrac{(3.4 - 0.5) \times 10^{-19}}{(9.5 - 5.0) \times 10^{14}} \approx 6.4 \times 10^{-34}$ J s, so $h \approx 6.4 \times 10^{-34}$ J s (accepted value 6.63×10^{-34}).

Photon momentum

Photons have ___momentum___ even though they have no mass (Compton, 1923):

$$p = \frac{h}{\lambda}$$

This follows from $E = pc$ for photons and $E = hf$. Radiation exerts ___radiation pressure___, small in everyday life but noticeable on spacecraft and comet tails.

Example 29.3 A beam of wavelength 600 nm, power 5.0 mW and cross-sectional area 0.16 cm^2 falls normally on a surface and is completely absorbed. For 1.0 s, find (a) the number of photons, (b) the change of momentum, (c) the pressure.

$E = \dfrac{hc}{\lambda} = 3.32 \times 10^{-19}$ J
(a) $N = \dfrac{5.0 \times 10^{-3} \times 1.0}{3.32 \times 10^{-19}} = 1.5 \times 10^{16}$
(b) $p = \dfrac{h}{\lambda} = 1.11 \times 10^{-27}$ kg m s^{-1} each, so $\Delta P = Np = 1.67 \times 10^{-11}$ kg m s^{-1}
(c) $F = \dfrac{\Delta P}{\Delta t} = 1.67 \times 10^{-11}$ N, $p = \dfrac{F}{A} = \dfrac{1.67 \times 10^{-11}}{0.16 \times 10^{-4}} = 1.0 \times 10^{-6}$ Pa

Question 29.8 Show that a laser of power P on a spot of area A (fully absorbed) causes pressure $p = \dfrac{P}{cA}$.

Photons per second $= \dfrac{P}{E}$, each with momentum $\dfrac{E}{c}$, so force $= \dfrac{P}{E} \times \dfrac{E}{c} = \dfrac{P}{c}$.
Pressure $= \dfrac{F}{A} = \dfrac{P}{cA}$.

Question 29.9 An electron and a positron with negligible K.E. annihilate to produce two γ-photons. Explain why the photons move off in opposite directions with equal energies.

Initial momentum is zero, so by conservation of momentum the photons' momenta must cancel: equal magnitudes ($p = E/c$, so equal energies) in **opposite** directions.

Wave–particle duality

Matter waves

Inspired by photons, de Broglie (1924) suggested that all matter has a wave nature.

The de Broglie wavelength of a particle of momentum $p = mv$:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Example 29.4 What is the wavelength of a 70 kg human walking at 2.0 m s^{-1}?

$\lambda = \dfrac{h}{mv} = \dfrac{6.63 \times 10^{-34}}{70 \times 2.0} = 4.7 \times 10^{-36}$ m, far smaller than anything we meet, so no observable wave behaviour.

Example 29.5 An electron is accelerated from rest through 50 V. What is its de Broglie wavelength?

$\frac{1}{2}mv^2 = eV$: $v = \sqrt{\dfrac{2 \times 1.60 \times 10^{-19} \times 50}{9.11 \times 10^{-31}}} = 4.19 \times 10^6 \text{ m s}^{-1}$

$\lambda = \dfrac{h}{mv} = \dfrac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 4.19 \times 10^6} = 1.74 \times 10^{-10} \text{ m}$

This is about an atomic spacing, so crystals can **diffract** the electrons.

Question 29.10 An α-particle has K.E. 2.4×10^{-15} J. Find its speed and de Broglie wavelength.

$m = 6.64 \times 10^{-27}$ kg: $v = \sqrt{\dfrac{2E_k}{m}} = \sqrt{\dfrac{4.8 \times 10^{-15}}{6.64 \times 10^{-27}}} = 8.5 \times 10^5 \text{ m s}^{-1}$

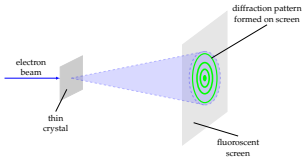
$\lambda = \dfrac{h}{mv} = \dfrac{6.63 \times 10^{-34}}{6.64 \times 10^{-27} \times 8.5 \times 10^5} = 1.2 \times 10^{-13} \text{ m}$

Question 29.11 A proton and an electron are accelerated through the same voltage. Compare (a) their energies and (b) their wavelengths.

(a) Both gain $E_k = eV$: the **same** energy.

(b) $\lambda = \dfrac{h}{\sqrt{2mE_k}}$: the proton is heavier, so its wavelength is shorter, by a factor $\sqrt{m_e/m_p} \approx \dfrac{1}{43}$.

Electron diffraction



Davisson and Thomson (1927) fired electrons at a thin crystal and saw concentric rings on a fluorescent screen: a diffraction pattern, which proves the wave nature of electrons. Electron wavelengths are much shorter than light, so an electron microscope has greater resolving power.

Question 29.12 A higher accelerating p.d. is used in the electron diffraction experiment. How does the pattern change?

Greater speed means greater momentum and a **smaller** wavelength, so the diffraction angles decrease: the rings become **smaller** (closer to the centre).

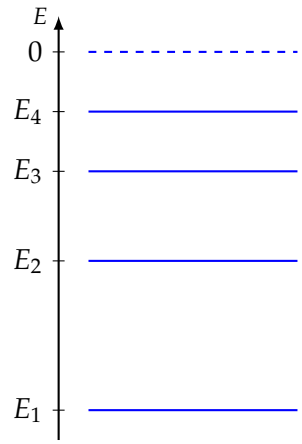
Wave–particle duality

	radiation	electron
particle-like	photoelectric effect, radiation pressure	deflection in fields, collisions
wave-like	interference, diffraction, refraction	electron diffraction

Moving through space, light and electrons behave like waves; when they interact, they behave like particles. This is wave–particle duality, and it applies to all matter.

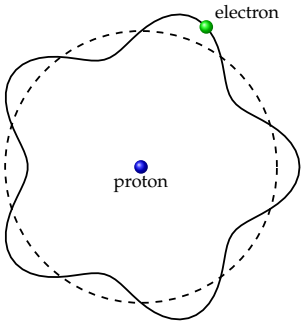
Energy levels and spectra

Quantised energy levels



An electron in an atom behaves as a wave. Only orbits where it can superpose constructively with itself exist, so the electron can only have certain energies: energy levels.

- energy is quantised: no values between the levels. The lowest level is the ground state; higher levels are excited states.
- levels are negative: energy must be supplied to pull the electron away. At infinity $E = 0$ (a free electron).
- electrons transit between levels by emitting or absorbing a photon



The hydrogen atom (Bohr model, extension)

Electrostatic attraction provides the centripetal force: $\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{m_e v^2}{r}$.

The orbit must fit a whole number of de Broglie wavelengths: $2\pi r = n\lambda = \frac{nh}{m_e v}$.

Eliminating v: $r_n = n^2 a_0$ with $a_0 = \frac{h^2\epsilon_0}{\pi m_e e^2} = 5.29 \times 10^{-11}$ m

Total energy $E = E_k + E_p = -\dfrac{e^2}{8\pi\epsilon_0 r}$ gives

$$E_n = -\frac{R_E}{n^2} \qquad R_E = 2.18 \times 10^{-18}\,\text{J} = 13.6\,\text{eV}$$

Emission and absorption

Photon energy equals the difference between two levels:

$$hf = E_{\text{high}} - E_{\text{low}}$$

• **emission spectrum**: hot gas; electrons fall to lower levels emitting photons; bright lines on a dark background. Each element has its own unique set of lines.

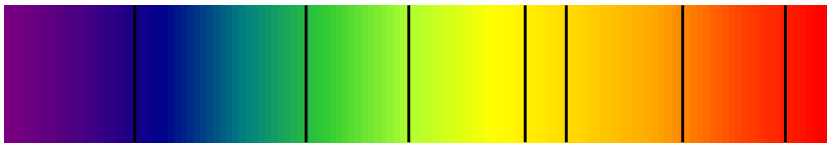
• **absorption spectrum**: white light through cool gas; photons of the right energy are absorbed (electrons move up); dark lines on a continuous spectrum. Re-emitted photons go in random directions.



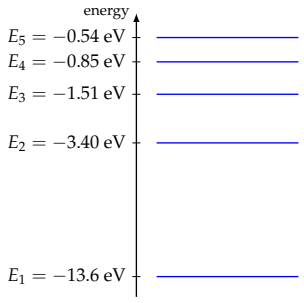
Continuous spectrum (white light).



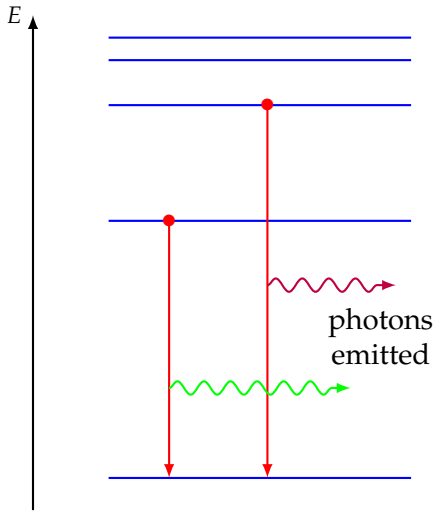
Emission line spectrum (hydrogen).



Absorption line spectrum.



• uses: flame colours, identifying elements, gas-discharge lamps, composition of stars



Example 29.6 Some energy levels of hydrogen are shown ($E_1 = -13.6$, $E_2 = -3.40$, $E_3 = -1.51$, $E_4 = -0.85$, $E_5 = -0.54$ eV). Find the longest and shortest wavelengths produced by transitions between them.

Smallest ΔE gives the longest wavelength: $E_5 \to E_4$: $\lambda = \dfrac{hc}{\Delta E} = \dfrac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{(0.85 - 0.54) \times 1.60 \times 10^{-19}} = 4.0 \times 10^{-6}$ m (infrared)

Largest ΔE: $E_5 \to E_1$: $\lambda = \dfrac{hc}{(13.6 - 0.54) \times 1.60 \times 10^{-19}} = 9.5 \times 10^{-8}$ m (ultraviolet)

Example 29.7 Visible light has wavelengths 380 nm to 740 nm. Find the energies of visible photons produced by transitions between the levels shown.

$E_{\max} = \dfrac{hc}{380\text{ nm}} = 5.23 \times 10^{-19}\text{ J} = 3.27\text{ eV}$; $E_{\min} = \dfrac{hc}{740\text{ nm}} = 2.69 \times 10^{-19}\text{ J} = 1.68\text{ eV}$

Look for level differences between 1.68 and 3.27 eV:

$E_3 \to E_2$: $3.40 - 1.51 = 1.89$ eV (656 nm, H_α)

$E_4 \to E_2$: $3.40 - 0.85 = 2.55$ eV (486 nm, H_β)

$E_5 \to E_2$: $3.40 - 0.54 = 2.86$ eV (435 nm, H_γ)

Example 29.8 White light passes through cool hydrogen gas. A dark line appears at 435 nm. Which transition causes it?

$E = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{435 \times 10^{-9}} = 4.57 \times 10^{-19}\text{ J} = 2.86\text{ eV}$

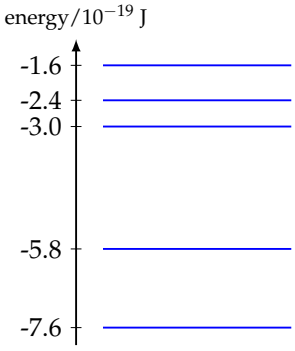
$3.40 - 0.54 = 2.86$ eV, so the electron moves $E_2 \to E_5$ (absorption).

Question 29.13 With only the five levels in the example, how many different wavelengths can appear in the emission spectrum?

Each pair of levels gives one line: $\dfrac{5 \times 4}{2} =$ **10** wavelengths.

Question 29.14 Hydrogen lines are seen at 486 nm, 656 nm and 1880 nm. (a) Calculate the photon energies. (b) Draw an energy level diagram with three labelled levels showing these transitions.

(a) $E = \dfrac{hc}{\lambda}$: 486 nm: 4.09×10^{-19} J (2.55 eV); 656 nm: 3.03×10^{-19} J (1.89 eV); 1880 nm: 1.06×10^{-19} J (0.66 eV).
(b) Levels A (top), B (middle), C (bottom): $A \to C$ gives 2.55 eV, $B \to C$ gives 1.89 eV, $A \to B$ gives $2.55 - 1.89 = 0.66$ eV.

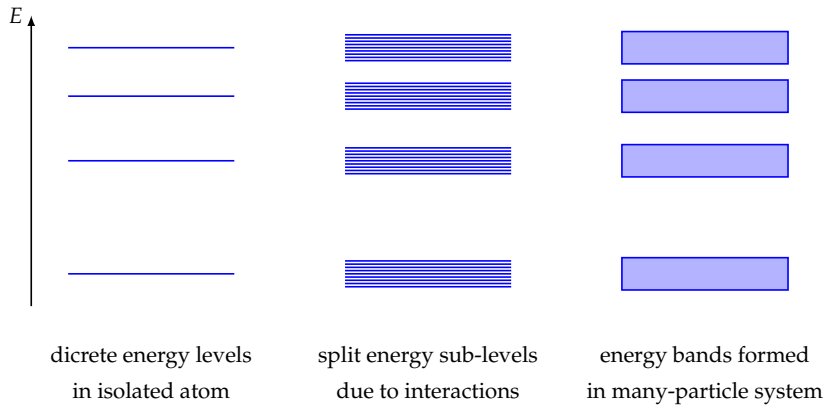


Question 29.15 Sunlight shows a dark line at 590 nm. Using the helium energy levels (in 10^{-19} J: -7.6, -5.8, -3.0, -2.4, -1.6), suggest how this shows helium in the Sun's atmosphere.

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{590 \times 10^{-9}} = 3.4 \times 10^{-19}\ \text{J}$$
This equals the gap between the -5.8 and -2.4 levels ($5.8 - 2.4 = 3.4$), so helium atoms in the cool outer atmosphere absorb this wavelength by an electron transition from the -5.8 level to the -2.4 level.

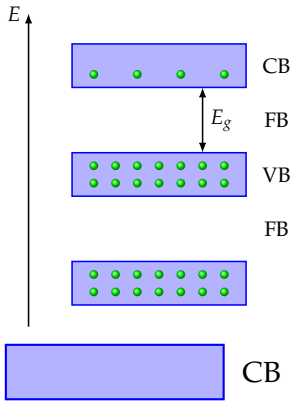
Band theory (extension)

Energy bands



In a solid, interaction between atoms splits each level into many closely spaced sub-levels (one per atom), forming energy bands, separated by forbidden bands.

• the highest filled band is the valence band (VB); the next band is the conduction band (CB); the gap between them is the band gap E_g



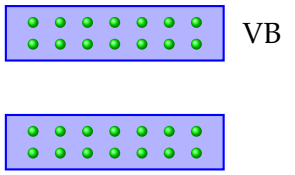
Conductivity

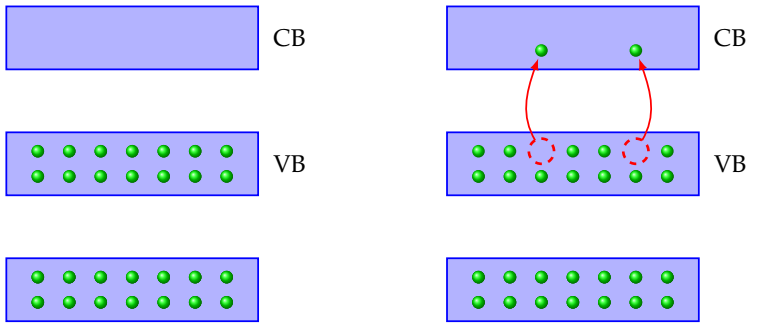
• **metals**: the CB overlaps the VB (no gap), so there are many free electrons. Resistance increases with temperature (more lattice vibrations). Metals are opaque to visible light, which is absorbed by conduction electrons, but transparent to X-rays.

• **insulators**: VB full, CB empty, and a large band gap ($\gtrsim$ 5 eV). Thermal energy cannot promote electrons, so very few charge carriers.

Question 29.16 Diamond has $E_g \approx 6.0$ eV. Explain why it is transparent to visible light.

Visible photons have energies $\sim$ 1.7 to 3.3 eV, less than E_g. They cannot lift an electron from the VB to the CB, so they are not absorbed and pass through.





band structure of an intrinsic semiconductor at low temperature

at higher temperature VB electrons cross band gap and enter CB

• **intrinsic semiconductors** (Si, Ge): similar to insulators but with a narrow band gap. As temperature rises, VB electrons cross into the CB, leaving holes in the VB (positive charge carriers). More carriers means lower resistance.

Example 29.9 Silicon has $E_g \approx 1.1$ eV. Does it become conducting in 600 nm red light?

$E = \dfrac{hc}{\lambda} = 3.32 \times 10^{-19}\ \text{J} = 2.1\ \text{eV} > E_g$
Photons can lift valence electrons into the conduction band, creating electron–hole pairs, so silicon **becomes conducting**.

Question 29.17 Use band theory to explain why an LDR has high resistance in the dark but low resistance in the light.

In the dark few electrons are in the CB, so there are few charge carriers: high resistance. Light photons with $hf \geq E_g$ raise electrons into the CB, creating electron–hole pairs. More carriers means a **lower** resistance.