Physical Quantities

AS Level Physics 9702

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By the end of this topic I can. . .

- recall the SI base quantities and units, and derive units of other quantities
- use metric prefixes and check equations for homogeneity
- distinguish scalars from vectors
- add vectors, and resolve a vector into perpendicular components

SI base units

Every quantity = **numerical value** × **unit**. The six base quantities in 9702:

- mass: kilogram (kg)
- length: metre (m)
- time: second (s)
- electric current: ampere (A)
- temperature: kelvin (K)
- amount of substance: mole (mol)

Example: derived units

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(a) $[v] = \dfrac{[s]}{[t]} = \mathrm{m\ s^{-1}}$

(b) $[a] = \dfrac{[v]}{[t]} = \mathrm{m\ s^{-2}}$

(c) $[F] = [m][a] = \mathrm{kg\ m\ s^{-2}}$

(d) $[W] = [F][s] = \mathrm{kg\ m^2\ s^{-2}}$

Metric prefixes

- pico p 10^{-12}, nano n 10^{-9}, micro μ 10^{-6}
- milli m 10^{-3}, centi c 10^{-2}, deci d 10^{-1}
- kilo k 10^{3}, mega M 10^{6}, giga G 10^{9}, tera T 10^{12}

Homogeneous equations

Homogeneous

The **units on both sides** of the equation are the same.

- a correct equation must be homogeneous
- a homogeneous equation is not necessarily correct
- dimensionless: numbers, π, refractive index

Example: checking homogeneity

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LHS: $[mgh] = [m][g][h] = \text{kg} \times \text{ m s}^{-2} \times \text{m} = \text{kg m}^2\text{s}^{-2}$
RHS: $\left[\frac{1}{2}mv^2\right] = [m][v]^2 = \text{kg} \times (\text{ m s}^{-1})^2 = \text{kg m}^2\text{s}^{-2}$
so we see the equation $mgh = \frac{1}{2}mv^2$ is homogeneous

Example: finding a formula from units

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RHS: $[T]^a[L]^b[m]^c = (\text{kg m s}^{-2})^a \text{m}^b \text{kg}^c = \text{kg}^{a+c}\text{m}^{a+b}\text{s}^{-2a}$
for the equation to be homogeneous, we must have:

$$\text{kg}^{a+c}\text{m}^{a+b}\text{s}^{-2a} = \text{m s}^{-1} \quad \Rightarrow \quad \begin{cases} \text{kg}: & a+c=0 \\ \text{m}: & a+b=1 \\ \text{s}: & -2a=-1 \end{cases} \quad \Rightarrow \quad \begin{cases} a=\frac{1}{2} \\ b=\frac{1}{2} \\ c=-\frac{1}{2} \end{cases}$$

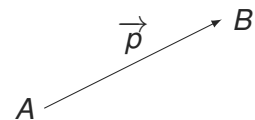
so wave speed is given by: $v = T^{1/2}L^{1/2}m^{-1/2}$, or $v = \sqrt{\frac{TL}{m}}$
this happens to be the correct formula for the wave speed on a string

Scalars and vectors

Scalar and vector

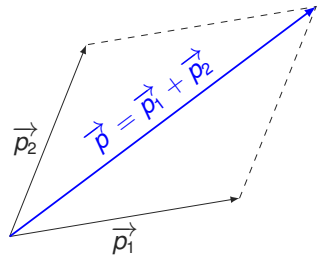
A **scalar** has magnitude only. A **vector** has magnitude and direction.

- scalars: time, distance, speed, mass, energy
- vectors: displacement, velocity, acceleration, force



Adding vectors

- **parallelogram rule**: resultant is the diagonal
- **head-to-tail**: tail of first to head of last



Example: a boat crossing a river

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(a) v_w (north) and v_b (east) are perpendicular: $v = \sqrt{5.0^2 + 2.0^2} \approx 5.4$ m s^{-1}. The crossing takes the shortest time but the boat drifts downstream.

(b) The boat's velocity relative to the water must cancel the river's: $v = \sqrt{5.0^2 - 2.0^2} \approx 4.6$ m s^{-1} straight across, with the boat pointed upstream at $\theta = \sin^{-1}\dfrac{2.0}{5.0} \approx 24^\circ$ to the straight-across line.

Resolving vectors

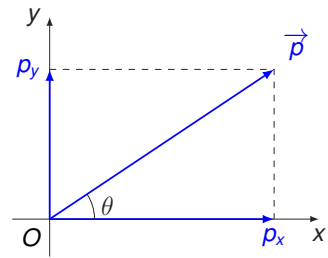
A vector p at angle θ to the x-axis:

$$p_x = p\cos\theta$$

$$p_y = p\sin\theta$$

$$p = \sqrt{p_x^2 + p_y^2}$$

$$\tan\theta = \frac{p_y}{p_x}$$

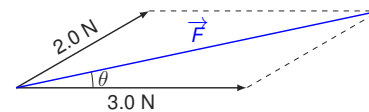


Example: adding two forces

A force of 3.0 N due east and a force of 2.0 N at 30° north of east act on an object. Find the magnitude and direction of the resultant force.

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Add components: $F_x = 3.0 + 2.0\cos 30^\circ \approx 4.73$ N, $F_y = 2.0 \sin 30^\circ = 1.0$ N.

$F = \sqrt{4.73^2 + 1.0^2} \approx 4.8$ N, $\theta = \tan^{-1} \dfrac{1.0}{4.73} \approx 12^\circ$ north of east.

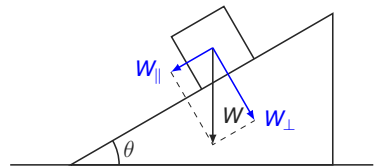
(A scale diagram, 1 cm for 1 N, gives the same answer to lower precision.)

Example: weight on a slope

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the vector diagram is shown
component of weight parallel to slope:
$W_\parallel = W \sin\theta = 20.0 \times \sin 30^\circ = 10.0$ N
component of weight normal to slope:
$W_\perp = W \cos\theta = 20.0 \times \cos 30^\circ \approx 17.3$ N

Question: powers of a wind turbine

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$[P] = \text{kg m}^2\text{s}^{-3}$. $[\rho A v^n] = \text{kg m}^{-3} \times \text{m}^2 \times (\text{m s}^{-1})^n = \text{kg m}^{n-1}\text{s}^{-n}$.
Matching the powers of s gives $n = 3$ (and then $\text{m}^{n-1} = \text{m}^2$ agrees).