Physical Quantities

Name _____ Date _____

By the end of this topic I can...

- recall the SI base quantities and units, and derive units of other quantities
- use metric prefixes and check equations for homogeneity
- distinguish scalars from vectors
- add vectors, and resolve a vector into perpendicular components

Units of measurement

Every physical quantity has a numerical value and a unit. Scientists use the **SI** system (Système International).

SI base units

base quantity	base unit	symbol
mass	kilogram	kg
length	metre	m
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol

All other units are **derived units**, built from the base units.

Example 1.1 Give the SI base units of (a) speed, (b) acceleration, (c) force, (d) work done.

(a) $[v] = \dfrac{[s]}{[t]} = \text{m s}^{-1}$

(b) $[a] = \dfrac{[v]}{[t]} = \text{m s}^{-2}$

(c) $[F] = [m][a] = \text{kg m s}^{-2}$

(d) $[W] = [F][s] = \text{kg m}^2\ \text{s}^{-2}$

Metric prefixes

name	symbol	factor	name	symbol	factor
pico	p	10^{-12}	kilo	k	10^{3}
nano	n	10^{-9}	mega	M	10^{6}
micro	μ	10^{-6}	giga	G	10^{9}
milli	m	10^{-3}	tera	T	10^{12}
centi	c	10^{-2}	deci	d	10^{-1}

Typical sizes: Earth's radius $\approx$ 6370 km; a human hair $\approx$ 60–90 μm; a water molecule $\approx$ 0.3 nm.

Homogeneity and dimensional analysis

An equation is **homogeneous** if the units on both sides are the same. A correct equation must be homogeneous (but a homogeneous equation is not necessarily correct: constants, signs and extra terms are not checked).

Pure numbers, π, the refractive index and the coefficient of friction are dimensionless (they have no units).

Example 1.2 A ball falls in vacuum, all its gravitational potential energy converts into kinetic energy. This is expressed by the equation: $mgh = \frac{1}{2}mv^2$. Show that this equation is homogeneous.

LHS: $[mgh] = [m][g][h] = \text{kg} \times \text{ m s}^{-2} \times \text{m} = \text{kg m}^2\text{s}^{-2}$
RHS: $\left[\frac{1}{2}mv^2\right] = [m][v]^2 = \text{kg} \times (\text{ m s}^{-1})^2 = \text{kg m}^2\text{s}^{-2}$
so we see the equation $mgh = \frac{1}{2}mv^2$ is homogeneous

Example 1.3 The speed of a wave travelling along an elastic string is determined by three things: the tension T in the string, the length L of the string, and the mass m of the string. Let's assume $v = T^a L^b m^c$, where a, b, c are some numerical constants. Find the values of a, b and c.

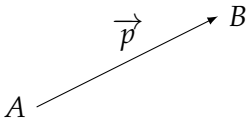
RHS: $[T]^a[L]^b[m]^c = (\text{kg m s}^{-2})^a \text{m}^b \text{kg}^c = \text{kg}^{a+c}\text{m}^{a+b}\text{s}^{-2a}$
for the equation to be homogeneous, we must have:

$$\text{kg}^{a+c}\text{m}^{a+b}\text{s}^{-2a} = \text{m s}^{-1} \quad \Rightarrow \quad \begin{cases} \text{kg}: & a+c=0 \\ \text{m}: & a+b=1 \\ \text{s}: & -2a=-1 \end{cases} \quad \Rightarrow \quad \begin{cases} a=\frac{1}{2} \\ b=\frac{1}{2} \\ c=-\frac{1}{2} \end{cases}$$

so wave speed is given by: $v = T^{1/2}L^{1/2}m^{-1/2}$, or $v = \sqrt{\frac{TL}{m}}$
this happens to be the correct formula for the wave speed on a string

Scalars and vectors

A **scalar** has magnitude only ; a **vector** has magnitude and direction .



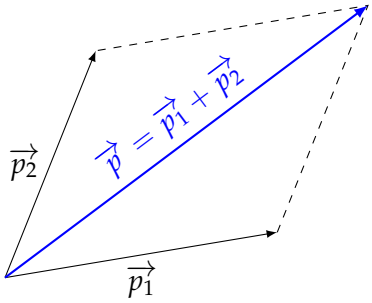
A vector drawn as an arrow from A to B.

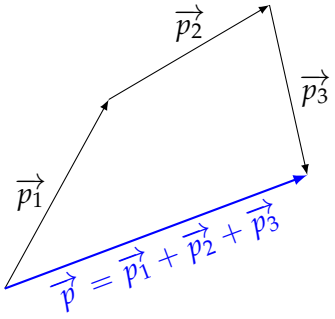
- scalars: time, distance, speed, mass, temperature, energy, density, volume
- vectors: displacement , velocity, acceleration, force, field strength
- the arrow's length shows the magnitude and its direction shows the direction

Adding vectors

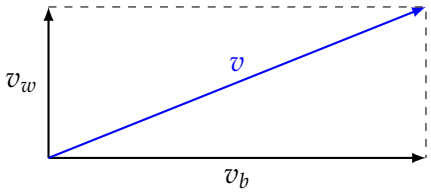
Scalars add like ordinary numbers; vectors need their directions taken into account. The sum is the **resultant**.

- **parallelogram rule**: the resultant lies along the diagonal of the parallelogram formed by the two vectors
- **head-to-tail**: join the vectors one after another; the resultant joins the tail of the first to the head of the last



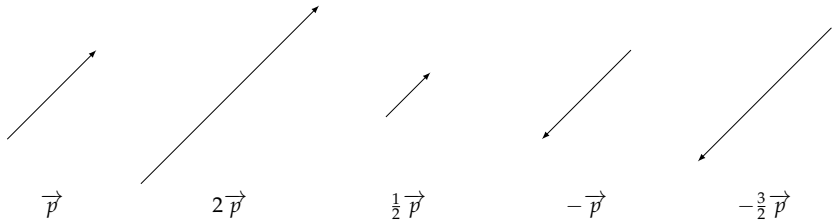


Example 1.4 A river flows from south to north at 2.0 m s^{-1}. A boat has a speed of 5.0 m s^{-1} relative to the water. (a) The boat heads due east from the west bank. What is its resultant velocity? (b) In what direction must it head to reach the point directly opposite?



(a) v_w (north) and v_b (east) are perpendicular: $v = \sqrt{5.0^2 + 2.0^2} \approx 5.4$ m s^{-1}. The crossing takes the shortest time but the boat drifts downstream.
(b) The boat's velocity relative to the water must cancel the river's: $v = \sqrt{5.0^2 - 2.0^2} \approx 4.6$ m s^{-1} straight across, with the boat pointed upstream at $\theta = \sin^{-1}\dfrac{2.0}{5.0} \approx 24°$ to the straight-across line.

Multiplying a vector by a scalar

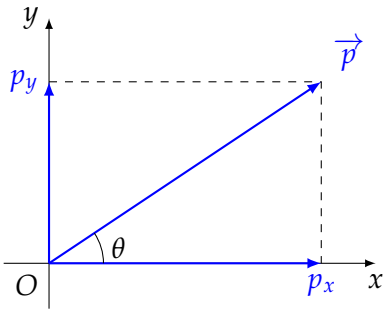


Multiplying by a positive number changes the ___length___ but not the direction. Multiplying by a negative number ___reverses___ the direction.

Resolving vectors

A vector can be split into two perpendicular components. For a vector p at angle θ to the x-axis:

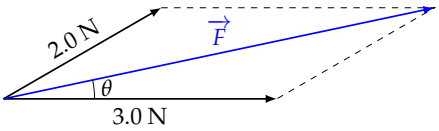
$$p_x = p\cos\theta \qquad p_y = p\sin\theta$$



Resolving $\vec{p}$ into components.

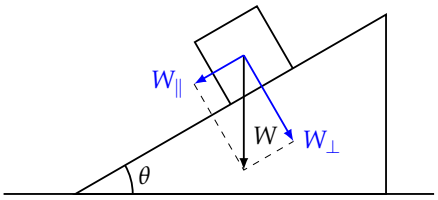
and the magnitude and direction are recovered from

$$p = \sqrt{p_x^2 + p_y^2} \qquad \tan\theta = \frac{p_y}{p_x}$$



Example 1.5 A force of 3.0 N due east and a force of 2.0 N at 30° north of east act on an object. Find the magnitude and direction of the resultant force.

Add components: $F_x = 3.0 + 2.0\cos 30° \approx 4.73$ N, $F_y = 2.0\sin 30° = 1.0$ N.
$F = \sqrt{4.73^2 + 1.0^2} \approx 4.8$ N, $\theta = \tan^{-1}\dfrac{1.0}{4.73} \approx 12°$ north of east.
(A scale diagram, 1 cm for 1 N, gives the same answer to lower precision.)



Example 1.6 A box of weight $W = 20.0$ N is resting on an inclined slope at 30° to the horizontal. Find the components of weight parallel to the slope and normal to the slope.

the vector diagram is shown
component of weight parallel to slope: $W_\parallel = W\sin\theta = 20.0 \times \sin 30° = 10.0$ N
component of weight normal to slope: $W_\perp = W\cos\theta = 20.0 \times \cos 30° \approx 17.3$ N

Practice questions

Question 1.1 What are the SI base units of (a) density, (b) pressure, (c) energy, (d) electric charge?

(a) density $= \frac{m}{V}$: $\mathrm{kg\,m^{-3}}$
(b) pressure $= \frac{F}{A}$: $\frac{\mathrm{kg\,m\,s^{-2}}}{\mathrm{m^2}} = \mathrm{kg\,m^{-1}s^{-2}}$
(c) energy $= Fs$: $\mathrm{kg\,m^2s^{-2}}$
(d) charge $= It$: A s

Question 1.2 For a substance of mass m, the heat energy Q needed to change its temperature by ΔT is given by: $Q = cm\Delta T$. Find the SI base units of the constant c.

$c = \frac{Q}{m\Delta T}$: $\frac{\mathrm{kg\,m^2s^{-2}}}{\mathrm{kg\,K}} = \mathrm{m^2s^{-2}K^{-1}}$

Question 1.3 The resistive force F on a metal ball falling at low speeds in water is given by the equation $F = krv$, where r is the radius of the metal ball, v is its speed and k is a constant. Find the base units of k in the SI system.

$k = \frac{F}{rv} = \frac{\mathrm{kg\,m\,s^{-2}}}{\mathrm{m} \times \mathrm{m\,s^{-1}}} = \mathrm{kg\,s^{-1}}$

Question 1.4 The speed of sound in air can be given by $c = \sqrt{\frac{\gamma p}{\rho}}$, where p is the pressure of the air and ρ is the air density. Show that γ is unit free.

$\gamma = \frac{c^2\rho}{p}$: $\frac{(\mathrm{m\,s^{-1}})^2\,\mathrm{kg\,m^{-3}}}{\mathrm{kg\,m^{-1}s^{-2}}} = \frac{\mathrm{kg\,m^{-1}s^{-2}}}{\mathrm{kg\,m^{-1}s^{-2}}} = 1$, so γ has no units.

Question 1.5 The effective power output from a wind turbine is given by the equation $P = \frac{1}{2}\eta\rho A v^n$, where ρ is the air density, A is the area of the turbine blades, and v is the wind speed. Given that η is a constant with no units, what is the value of n?

$[P] = \text{kg m}^2\text{s}^{-3}$. $[\rho A v^n] = \text{kg m}^{-3} \times \text{m}^2 \times (\text{m s}^{-1})^n = \text{kg m}^{n-1}\text{s}^{-n}$. Matching the powers of s gives $n = 3$ (and then $\text{m}^{n-1} = \text{m}^2$ agrees).

Question 1.6 Three forces of 5.0 N, 12 N and 13 N act at one point on an object. The angles at which the forces act can vary. What is the maximum and the minimum resultant force?

Maximum: all three in the same direction, $5.0 + 12 + 13 = 30$ N.
Minimum: $5^2 + 12^2 = 13^2$, so the forces can be arranged head-to-tail as a closed (right-angled) triangle: the minimum resultant is 0.