Oscillations

A Level Physics 9702

J. Rowing • Vijay International School Praslin

By the end of this topic I can...

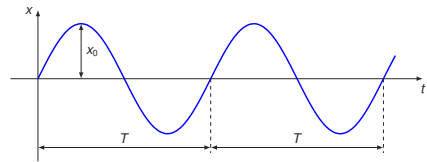
- describe oscillations using amplitude, period, frequency and phase difference
- define SHM and use $a = -\omega^2 x$ and $x = x_0 \sin(\omega t + \phi)$
- use v, a and energy relations, and the periods of a mass–spring system and a pendulum
- describe damping, free and forced oscillations and resonance

Describing oscillations

Oscillation: repetitive back and forth motion about an **equilibrium position**.

- **displacement** x: distance from equilibrium
- **amplitude** x_0: maximum displacement
- **period** T: time for one complete oscillation
- **frequency** f: oscillations per unit time

$$f = \frac{1}{T}$$



Phase difference

The phase angle is the fraction of a cycle reached (in radians):

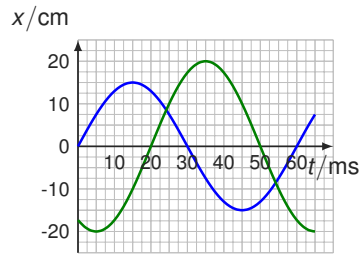
$$\Delta\phi = \frac{\Delta t}{T} \times 2\pi$$

Example: comparing oscillations

Compare the two oscillations in the x–t graph.

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Both have period $T = 60$ ms, so frequency
$f = \dfrac{1}{60 \times 10^{-3}} \approx 16.7$ Hz.
Different amplitudes: $x_0 = 15$ cm and $x_0 = 20$ cm.
Time difference $\Delta t = 20$ ms, so
$\Delta\phi = \dfrac{\Delta t}{T} \times 2\pi = \dfrac{20}{60} \times 2\pi = \dfrac{2\pi}{3}$ rad

Simple harmonic motion

Simple harmonic motion

The acceleration is always **proportional to the displacement** from equilibrium, and in the **opposite direction**.

- ω = angular frequency, ϕ = phase angle

$$a = -\omega^2 x$$

$$x = x_0 \sin(\omega t + \phi)$$

Angular frequency

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- unit: rad s^{-1}
- set by the physical constants of the system (the **natural frequency**)
- ϕ depends on the initial conditions: choose sine or cosine to avoid it

Example: writing x(t) (cosine)

A simple harmonic oscillator is displaced by 6.0 cm and released at $t = 0$. Its period is 0.80 s. State its displacement–time equation.

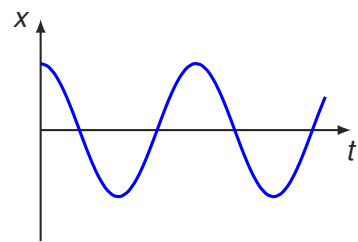
Example: writing x(t) (cosine)

A simple harmonic oscillator is displaced by 6.0 cm and released at $t = 0$. Its period is 0.80 s. State its displacement–time equation.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80} = \frac{5\pi}{2} \text{ rad s}^{-1}$$

At $t = 0$, $x = +x_0$, so use a cosine:

$$x = x_0 \cos \omega t = 6.0 \cos\left(\frac{5\pi}{2} t\right)$$



Example: writing x(t) (sine)

An oscillator at equilibrium is given a speed in the negative direction at $t = 0$. Its frequency is 1.5 Hz and amplitude 5.0 cm. State its displacement–time equation.

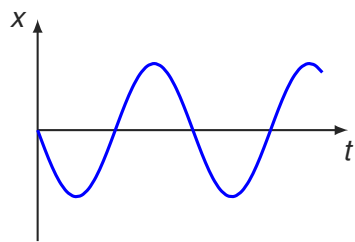
Example: writing x(t) (sine)

An oscillator at equilibrium is given a speed in the negative direction at $t = 0$. Its frequency is 1.5 Hz and amplitude 5.0 cm. State its displacement–time equation.

$\omega = 2\pi f = 2\pi \times 1.5 = 3\pi$ rad s^{-1}

At $t = 0$, $x = 0$ and $v < 0$, so use a negative sine:

$x = -x_0 \sin \omega t = -5.0 \sin(3\pi t)$

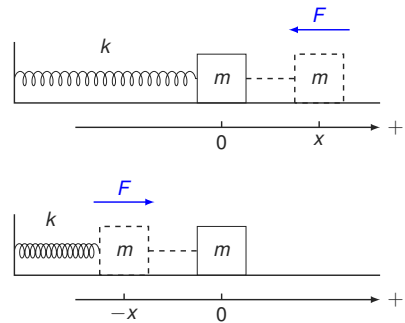


Mass–spring oscillator

Hooke's law: $F_{\text{net}} = -kx$, so $a = -\dfrac{k}{m}x$: SHM.

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$



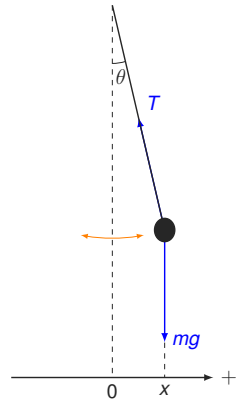
- larger m: longer period; larger k: shorter period
- period does not depend on amplitude

Simple pendulum

For small angles $a \approx -\frac{g}{L}x$: SHM.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- depends on L and g only
- independent of amplitude (small angles)



Question: the floating tube

Question: the floating tube

(a) The resultant force acts to restore the tube, so it is opposite to y:
$ma = -\rho g A y \quad \Rightarrow \quad a = -\dfrac{\rho g A}{m}\, y$. The acceleration is proportional to y and opposite to it: SHM.
(b) $\omega^2 = \dfrac{\rho g A}{m}$, so $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{\rho g A}{m}}$

Velocity in SHM

- velocity = rate of change of displacement

$$v = \omega x_0 \cos(\omega t + \phi)$$

$$v = \pm\omega\sqrt{x_0^2 - x^2}$$

$$v_{max} = \omega x_0$$

Acceleration in SHM

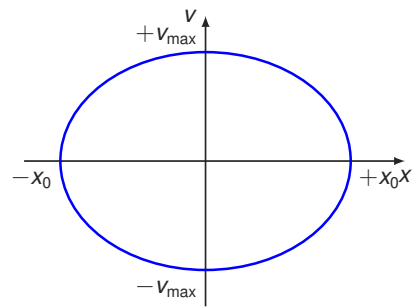
- acceleration = rate of change of velocity
- zero at equilibrium; maximum at $x = \pm x_0$

$$a = -\omega^2 x_0 \sin(\omega t + \phi)$$

$$a_{\text{max}} = \omega^2 x_0$$

Velocity and acceleration against displacement

- v–x: an **ellipse**
- a–x: a straight line through the origin with **negative gradient**



Example: a swinging pendulum

Example: a swinging pendulum

(a) $T = 2 \times 1.0 = 2.0$ s and $x_0 = \frac{1}{2} \times 6.0 = 3.0$ cm

(b) $\omega = \dfrac{2\pi}{T} = \pi$ rad s^{-1}, $v_{\max} = \omega x_0 = \pi \times 3.0 \approx 9.4$ cm s^{-1}

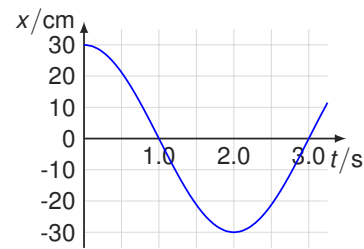
(c) $v = \omega\sqrt{x_0^2 - x^2} = \pi\sqrt{3.0^2 - 1.2^2} \approx 8.6$ cm s^{-1}

Example: reading an x–t graph

For the SHM in the graph, find (a) its speed at $t = 0$,
(b) its greatest speed, (c) its acceleration at $t = 1.0$ s.

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For the SHM in the graph, find (a) its speed at $t = 0$, (b) its greatest speed, (c) its acceleration at $t = 1.0$ s.



(a) At $t = 0$, $x = +x_0$: gradient zero, so $v = 0$.

(b) $x_0 = 30$ cm, $T = 4.0$ s, $\omega = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$ rad s^{-1};

$v_{max} = \omega x_0 = \dfrac{\pi}{2} \times 30 \approx 47$ cm s^{-1}

(c) At $t = 1.0$ s, $x = 0$ (equilibrium), so $a = 0$.

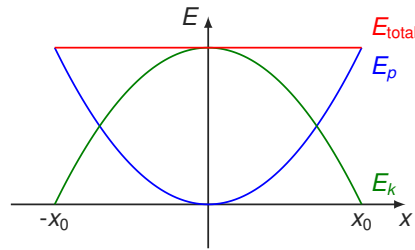
Vibrational energy

Total energy of SHM

Kinetic plus potential energy is **constant** for an ideal system; energy swaps between the two.

- at $x = 0$: all kinetic
- at $x = \pm x_0$: all potential

$$E = \tfrac{1}{2} m\omega^2 x_0^2$$



Example: energy of a mass on a spring

Example: energy of a mass on a spring

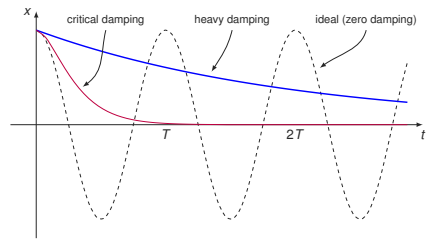
$$E = \tfrac{1}{2} m\omega^2 x_0^2 = \tfrac{1}{2} \times 0.15 \times \frac{4\pi^2}{0.80^2} \times 0.12^2 \approx 6.7 \times 10^{-2}\ \text{J}$$

Damping

Damping

The decrease in amplitude of an oscillator because of **energy loss** to resistive forces.

- **light**: amplitude falls gradually, period almost unchanged
- **heavy**: no oscillation, very slow return
- **critical**: shortest return to equilibrium, no oscillation



Example: damped oscillator

Example: damped oscillator

(a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{160}{0.25}} \approx 25.3 \text{ rad s}^{-1}$

(b) $E = \frac{1}{2} k x_0^2 = \frac{1}{2} \times 160 \times 0.050^2 = 0.20 \text{ J}$

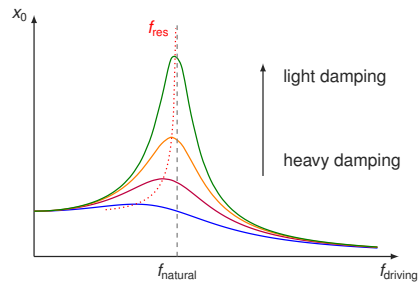
(c) $E \propto x_0^2$ and 60% remains: $x_0' = \sqrt{0.6}\, x_0 = \sqrt{0.6} \times 5.0 \approx 3.9 \text{ cm}$

Resonance

Resonance

When the driving frequency equals the **natural frequency**, the amplitude is a **maximum**.

- useful: microwave ovens, MRI, radio tuning
- harmful: buildings in earthquakes, bridges in wind
- more damping: lower, flatter peak, slightly lower frequency



Key equations

SHM	$a = -\omega^2 x$	pendulum	$T = 2\pi\sqrt{\dfrac{L}{g}}$
solution	$x = x_0 \sin(\omega t + \phi)$	max speed	$v_{\max} = \omega x_0$
angular freq.	$\omega = \dfrac{2\pi}{T} = 2\pi f$	max accel.	$a_{\max} = \omega^2 x_0$
spring	$T = 2\pi\sqrt{\dfrac{m}{k}}$	energy	$E = \frac{1}{2} m\omega^2 x_0^2$