# Oscillations

Name _____ Date _____

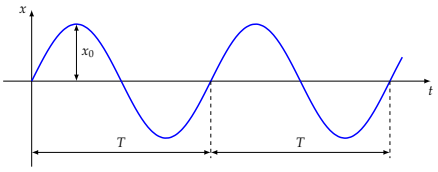
**By the end of this topic I can…**

- describe oscillations using amplitude, period, frequency and phase difference
- define simple harmonic motion and use $a = -\omega^2 x$ and $x = x_0 \sin(\omega t + \phi)$
- use $v$, $a$ and energy relations for SHM, and the periods of a mass–spring system and a pendulum
- describe damping, free and forced oscillations and resonance

## Describing oscillations

_____ is a repetitive back and forth motion about an _____.

At the equilibrium position all the forces balance: released from rest there, the object stays at rest. Examples: a pendulum, a vibrating string, a swing.




*Displacement–time graph of a typical oscillator.*

### Amplitude, period, frequency

• **Displacement** $x$: the distance from the _____

• **Amplitude** $x_0$: the _____ from equilibrium

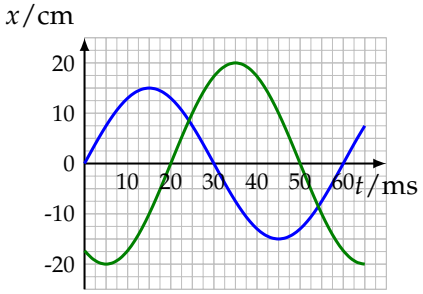
• **Period** $T$: the time for _____

• **Frequency** $f$: the number of oscillations _____:

### Phase

The **phase angle** $\phi$ tells how far through its cycle an oscillator is. Unit: _____. Think of it as the fraction of a complete cycle, not as a geometric angle.

The **phase difference** compares two oscillators:




*Two oscillations (time in ms, displacement in*

**Example 20.1** Compare the two oscillations in the $x$–$t$ graph.

## Simple harmonic motion

### Restoring force

In any oscillation the acceleration always points towards the _____. The resultant force that does this is the **restoring force**; at equilibrium it is _____.

_____ (SHM): the acceleration is always _____ from the equilibrium position, and _____ to it.

SHM approximates a pendulum, molecular vibrations and more. Complicated motions can be built from a set of simple harmonics.[1]

[1] This is Fourier analysis. For example, uniform circular motion is two SHMs, in the $x$- and $y$-directions.

### Equation of motion

The defining equation of SHM:

$\omega$ is a constant, and the _____ shows $a$ and $x$ are in opposite directions.

The solution[2] is

where $x_0$ is the _____, $\omega$ the _____ and $\phi$ the phase angle.

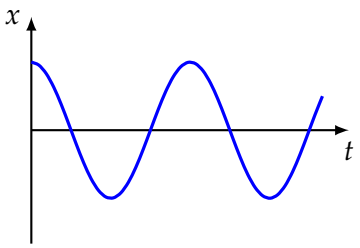
[2] Since $a = \mathrm{d}^2x/\mathrm{d}t^2$, the defining equation is the second-order differential equation $\mathrm{d}^2x/\mathrm{d}t^2 + \omega^2 x = 0$.

- **angular frequency**:

$[\omega] =$ _____

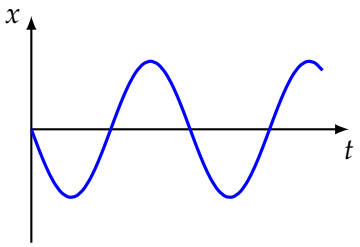
• $\omega$ depends only on the _____ of the system; a freely oscillating system has a fixed **natural frequency**

• $\phi$ depends on the _____; choose sine or cosine so that $\phi$ is not needed




*Released from $x = +x_0$: a cosine.*

**Example 20.2** A simple harmonic oscillator is displaced by 6.0 cm and released at $t = 0$. Its period is 0.80 s. State its displacement–time equation.

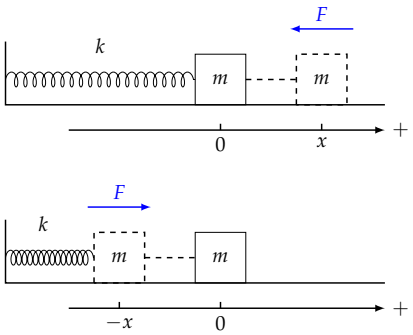



*At $t = 0$ at equilibrium, moving in the negative direction.*

**Example 20.3** An oscillator at equilibrium is given a speed in the negative direction at $t = 0$. Its frequency is 1.5 Hz and amplitude 5.0 cm. State its displacement–time equation.

## Examples of simple harmonic oscillators

### Mass–spring oscillator




*Restoring force on a mass–spring oscillator.*

A block of mass $m$ on an ideal spring. The spring's restoring force obeys _____, $F = kx$, directed _____ to the displacement. So

$$F_{\text{net}} = ma \quad \Rightarrow \quad -kx = ma \quad \Rightarrow \quad a = -\frac{k}{m}x$$

$k$ and $m$ are constant, so $a \propto x$ with a minus sign: it is _____. Comparing with $a = -\omega^2 x$:

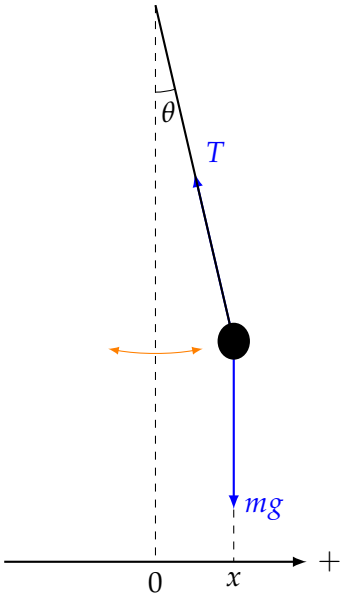
so

The period depends only on $m$ and $k$, _____.

• larger $m$: _____ period (more inertia, slower)

• larger $k$: _____ period (stiffer spring, larger restoring force)

## Simple pendulum



*Simple pendulum.*

A bob on a light string swings in SHM for _____. Then the bob has almost no vertical displacement:

vertically: $T\cos\theta \approx mg$, so the tension $\approx mg$

horizontally: $-T\sin\theta = ma$ with $\sin\theta = x/L$, so $a \approx -\dfrac{g}{L}x$

Comparing with $a = -\omega^2 x$ gives $\omega = \sqrt{g/L}$:

The period depends on $L$ and $g$ only, not on the _____ (for small angles).

- longer pendulum: _____ period
- smaller $g$: _____ period (with $g = 0$ the bob would not move: $T \to \infty$)

**Question 20.1** A cylindrical tube of mass $m$ and cross-sectional area $A$ floats upright in a liquid of density $\rho$. For a small vertical displacement $y$, the resultant force on it has magnitude $F_{\text{net}} = \rho g A y$. (a) Show that it performs SHM. (b) Find an expression for the frequency.

**Question 20.2** A glider on a frictionless horizontal air track bounces elastically off buffers at the ends. Does it describe SHM? Explain.

# Velocity, acceleration and energy

## Velocity and acceleration

From $x = x_0 \sin(\omega t + \phi)$, velocity is the _____:

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} \quad \Rightarrow$$

Eliminating the sine and cosine: $v^2 + \omega^2 x^2 = \omega^2 x_0^2$, so

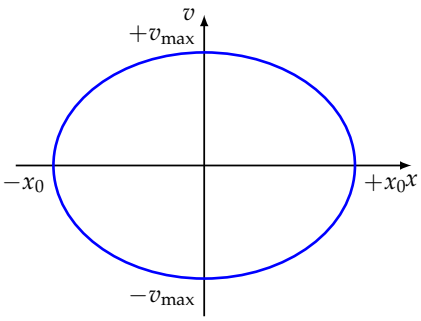
- at $x = 0$ the speed is a _____:
- at $x = \pm x_0$ the oscillator is momentarily _____: $v = 0$

Acceleration is the rate of change of velocity:

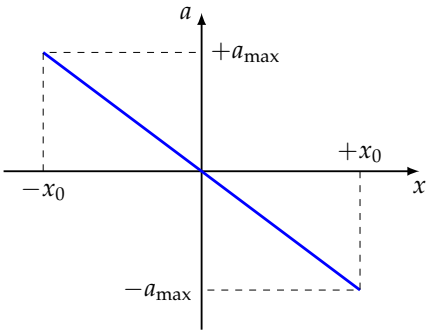
which is just $a = -\omega^2 x$ again.

- at $x = 0$ the acceleration is _____
- at $x = \pm x_0$ it is a _____:

| time $t$ | 0 | $\frac{1}{4}T$ | $\frac{1}{2}T$ | $\frac{3}{4}T$ | $T$ |
|---|---|---|---|---|---|
| $x = x_0 \sin \omega t$ | ____ | _____ | ____ | _____ | ____ |
| $v = \omega x_0 \cos \omega t$ | _____ | ____ | _____ | ____ | _____ |
| $a = -\omega^2 x$ | ____ | _____ | ____ | _____ | ____ |



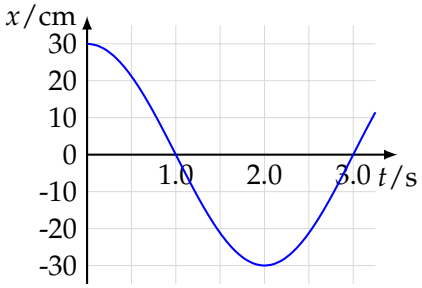

*v–x graph: an ellipse.*




*a–x graph: a straight line with negative gradient.*

**Example 20.4** A pendulum swings from one end to the other, a distance of 6.0 cm, in 1.0 s. (a) State the period and amplitude. (b) Find the greatest speed. (c) Find the speed when $x = 1.2$ cm.

**Example 20.5** For the SHM in the graph, find (a) its speed at $t = 0$, (b) its greatest speed, (c) its acceleration at $t = 1.0$ s.




*x–t graph (cm, s).*

**Question 20.3** A car-engine piston moves in SHM, completing 3000 oscillations per minute with amplitude 4.0 cm. Find (a) the greatest speed, (b) the greatest acceleration.

## Vibrational energy

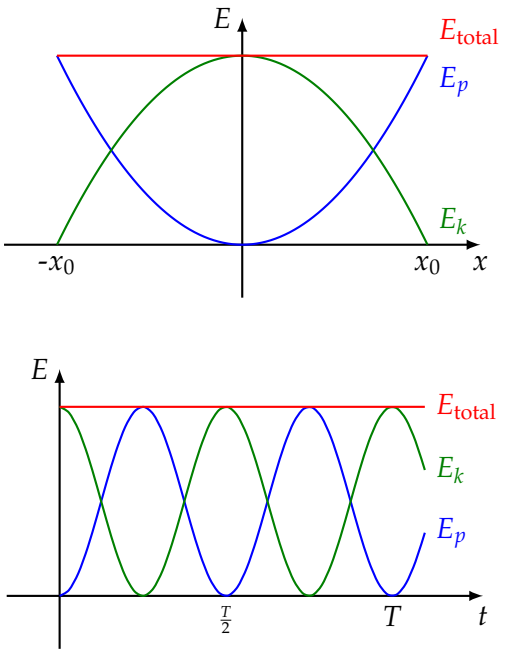
For an ideal mass–spring oscillator the energy has two parts:

- kinetic: $E_k = \frac{1}{2}mv^2 =$ _____
- elastic potential: $E_p = \frac{1}{2}kx^2 =$ _____

Adding them:

This holds for _____ simple harmonic oscillators, and the total is _____ for an ideal system.

- at $x = 0$: $E_k$ is _____, $E_p = 0$ (all kinetic)
- at $x = \pm x_0$: $E_k = 0$, $E_p$ is _____ (all potential)




**Example 20.6** A 150 g block on a spring oscillates with period 0.80 s and maximum displacement 12 cm. Find the vibrational energy.

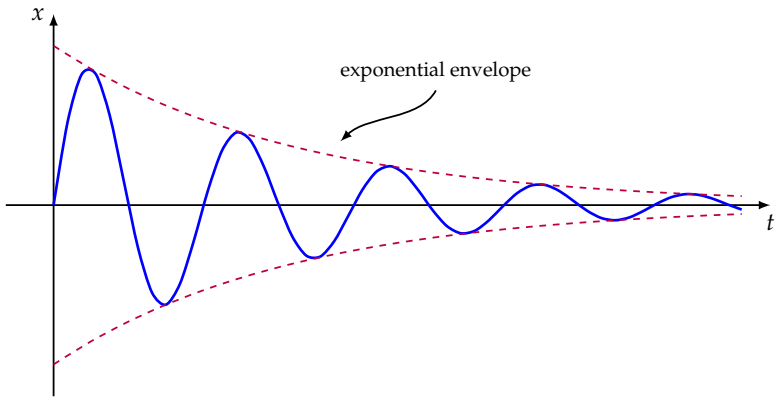
**Question 20.4** An oscillator given 20 mJ reaches an amplitude of 8.0 cm. What energy is needed to double the amplitude?

# Damping and resonance

## Damped oscillations

In reality friction and resistive forces remove energy.

_____ is the decrease in amplitude of an oscillator because of _____ to resistive forces.



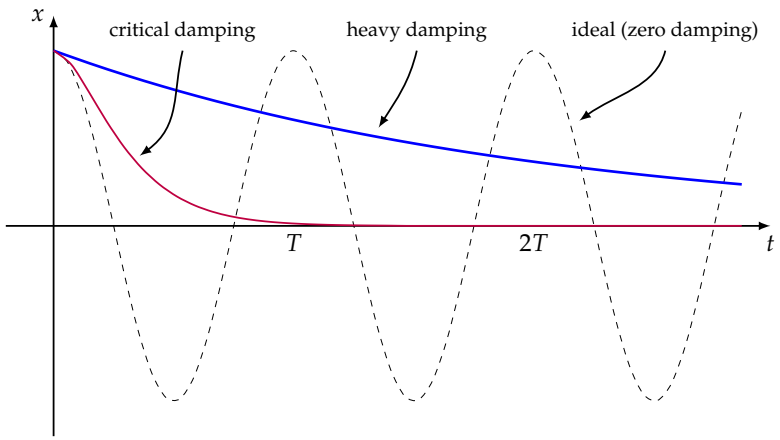

*Light damping: the amplitude decays exponentially.*

• **light damping**: the amplitude falls _____; the oscillator makes many oscillations; the period is almost _____

• **heavy damping**: so much resistance that there is _____ and the system returns to equilibrium _____

• **critical damping**: the border between them: the system returns to equilibrium in the _____ without oscillating[3]

[3] Some boards define critical damping only by 'shortest time'; A-level exam boards insist on 'no oscillation'.




*Displacement for different amounts of damping.*

Critical damping is wanted in door closers, vehicle shock absorbers and artillery.

**Example 20.7** A 250 g block on a spring ($k = 1.6$ N cm$^{-1}$) is displaced 5.0 cm and released. (a) Find $\omega$. (b) Find the initial vibrational energy. (c) After some oscillations 40% of the energy has been lost to damping. Find the new amplitude.

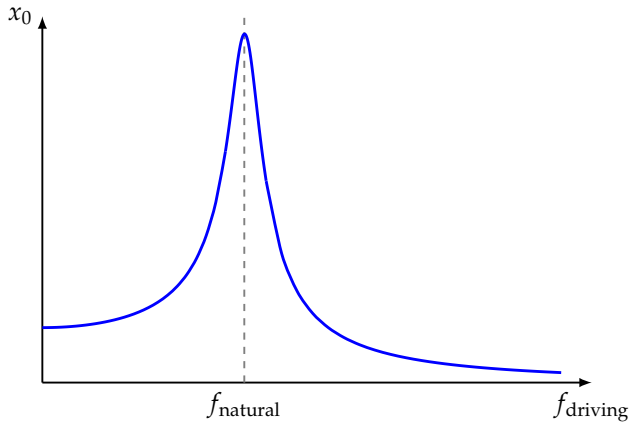
**Question 20.5** A 360 g toy boat floats on water. It is pushed down and released. In the first four cycles its amplitude falls from 5.0 cm to 2.0 cm in 6.0 s. Find the energy loss.

## Free and forced oscillations

- **free oscillation**: no gain or loss of energy; constant amplitude; frequency is the _____
- **forced oscillation**: moves under an _____; after a long time its frequency tends to the _____

## Resonance

_____: when the driving frequency equals the system's _____, the amplitude of the oscillation is a _____.



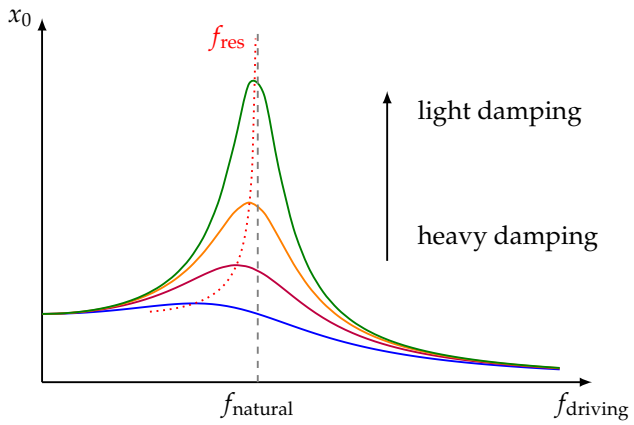

*Amplitude against driving frequency (undamped: the peak tends to infinity).*

• useful: microwave ovens (water molecules), MRI (nuclear precession), radio tuning circuits

• harmful: buildings in earthquakes, car suspension on bumps, bridges and skyscrapers in wind

## Damping and resonance

Damping lowers the amplitude at _____ frequencies; heavier damping makes the resonance peak _____ and shifts it slightly to a _____ frequency. Engineering systems are often deliberately damped to reduce resonance.




*Resonance curves for different amounts of damping.*