# Nuclear Physics

A Level Physics 9702

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# By the end of this topic I can...

- use $E = mc^2$, mass defect and binding energy
- interpret the binding energy per nucleon curve
- describe fission and fusion and calculate the energy released
- define decay constant, activity and half-life
- use $N = N_0 e^{-\lambda t}$ and analyse count-rate data

# Mass-energy equivalence

Mass and energy are equivalent:

$$E = mc^2$$

- $m$ is the **rest mass**
- energy supplied to a system: mass **increases**
- what is conserved is **mass-energy**

# Example: annihilation

# Example: annihilation

(a) Each photon carries the rest energy of one proton:

$$E = m_p c^2 = 1.67 \times 10^{-27} \times (3.0 \times 10^8)^2 = 1.50 \times 10^{-10}\ \text{J}$$

(b) $$f = \frac{E}{h} = \frac{1.50 \times 10^{-10}}{6.63 \times 10^{-34}} = 2.27 \times 10^{23}\ \text{Hz}\ (\gamma\text{-ray})$$

# Mass defect

## Mass defect

The difference between the **mass of the separated nucleons** and the **mass of the nucleus**.

- 1 u = $1.66 \times 10^{-27}$ kg
- 1 MeV = $1.60 \times 10^{-13}$ J

$$\Delta m = Zm_{\mathrm{p}} + (A - Z)m_{\mathrm{n}} - m_X$$

# Binding energy

## Binding energy

The energy needed to **separate the nucleons** of a nucleus to **infinity**.

- equally, the energy **released** when free nucleons combine
- held together by the **strong nuclear force**

$$E_B = \Delta m\, c^2$$

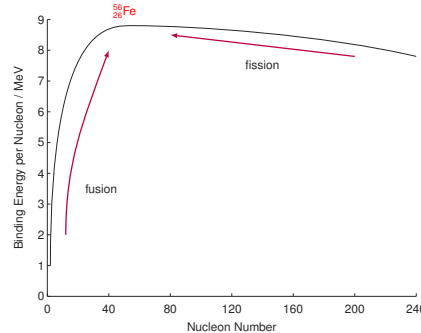
# Example: iron-56

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$\Delta m = 26m_p + 30m_n - m_{\text{Fe}} =$
$26 \times 1.673 \times 10^{-27} + 30 \times 1.675 \times 10^{-27} - 9.288 \times 10^{-26} = 8.68 \times 10^{-28}$ kg
$E_B = \Delta mc^2 = 8.68 \times 10^{-28} \times (3.00 \times 10^8)^2 = 7.81 \times 10^{-11}$ J
per nucleon: $\dfrac{7.81 \times 10^{-11}}{56} = 1.395 \times 10^{-12}$ J $= \dfrac{1.395 \times 10^{-12}}{1.60 \times 10^{-13}} = 8.72$ MeV

# Binding energy per nucleon

- higher means **more stable**
- peaks near **iron-56** ($\approx$ 8.8 MeV)
- moving towards the peak **releases** energy
- fusion of light nuclei, fission of heavy nuclei



# Fusion and fission

- **fusion**: light nuclei join; needs high temperature to beat repulsion
- **fission**: massive nucleus splits into two similar-sized nuclei
- fission may be spontaneous or induced by a neutron
- $\alpha$-decay is **not** fission

# Chain reaction

- each fission releases more than one **neutron**
- **control rods** absorb neutrons: one per fission continues
- **moderator** slows neutrons to thermal energies
- **critical mass** needed to sustain the reaction

# Energy released

Two methods for the energy released $Q$:

$$Q = \Delta m\, c^2 = E_{B,\text{prod}} - E_{B,\text{org}}$$

- **mass**: loss of mass times $c^2$
- **binding energy**: increase in total binding energy

# Example: the Sun

# Example: the Sun

(a) $\Delta m = 4m_p - m_{\text{He}} - 2m_e = 4.40 \times 10^{-29}$ kg

$Q = \Delta mc^2 = 4.40 \times 10^{-29} \times (3.0 \times 10^8)^2 = 3.96 \times 10^{-12}$ J

(b) Reactions per second: $\dfrac{P}{Q} = \dfrac{4.0 \times 10^{26}}{3.96 \times 10^{-12}} = 1.0 \times 10^{38}$ s$^{-1}$

Hydrogen per second: $4 \times 1.0 \times 10^{38} \times 1.67 \times 10^{-27} = 6.7 \times 10^{11}$ kg s$^{-1}$

# Example: fission of uranium-235

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(a) Nucleon number: $1 + 235 = 141 + 92 + x$, so $x = 3$.

(b) $Q = 141 \times 8.326 + 92 \times 8.513 - 235 \times 7.591 = 173.3$ MeV

(c) $\Delta m = \dfrac{Q}{c^2} = \dfrac{173.3 \times 1.60 \times 10^{-13}}{(3.0 \times 10^8)^2} = 3.1 \times 10^{-28}$ kg (a decrease in mass)

# Radioactive decay

## Radioactive decay

The **random** and **spontaneous** emission of radiation from unstable nuclei.

- **random**: only the probability of decay is known
- **spontaneous**: unaffected by temperature or pressure

# Decay constant

## Decay constant

The **probability of decay** of a nucleus per unit time.

- unit: $s^{-1}$

$$\lambda = \frac{\Delta N}{N \Delta t}$$

# Activity

## Activity

The **number of nuclei decaying per unit time**.

- unit: becquerel, 1 Bq = 1 $s^{-1}$

$$A = \lambda N$$

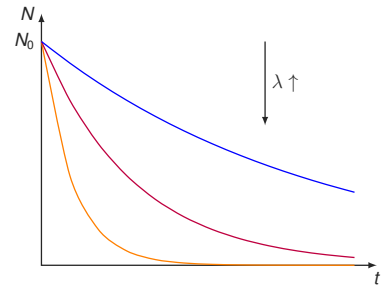
# Exponential decay

Integrating $-\mathrm{d}N/\mathrm{d}t = \lambda N$:

$$N = N_0 \mathrm{e}^{-\lambda t}$$

$$A = A_0 \mathrm{e}^{-\lambda t}$$

- larger $\lambda$: faster decay



# Example: iodine-131

# Example: iodine-131

(a) $\lambda = \dfrac{A}{N} = \dfrac{60}{6.0 \times 10^7} = 1.0 \times 10^{-6}\ \mathrm{s}^{-1}$

(b) $N = 6.0 \times 10^7 \times \mathrm{e}^{-1.0 \times 10^{-6} \times 20 \times 24 \times 3600} = 1.1 \times 10^7$

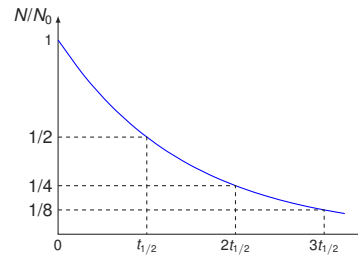
# Half-life

## Half-life

The time for the number of undecayed nuclei (or the activity) to fall to **half**.

- a **constant**: independent of the initial number

$$\lambda t_{1/2} = \ln 2$$



# Example: radium-224

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$$\lambda = \frac{\ln 2}{3.63 \times 24 \times 3600} = 2.21 \times 10^{-6}\ \mathrm{s}^{-1}$$

(a) $N_0 = \dfrac{6.0 \times 10^{-6}}{224 \times 1.66 \times 10^{-27}} = 1.61 \times 10^{19}$, so $A_0 = \lambda N_0 = 3.6 \times 10^{13}$ Bq

(b) $N = N_0 \mathrm{e}^{-\lambda t} = 1.61 \times 10^{19} \times \mathrm{e}^{-2.21 \times 10^{-6} \times 10 \times 24 \times 3600} = 2.39 \times 10^{18}$

Decayed: $\Delta N = 1.61 \times 10^{19} - 2.39 \times 10^{18} = 1.37 \times 10^{19}$

# Example: carbon dating

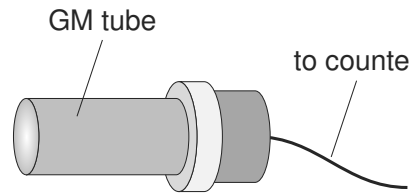
# Example: carbon dating

$0.42 = 1.60\,\mathrm{e}^{-\frac{\ln 2}{5570}t}$, so $-\dfrac{\ln 2}{5570}t = \ln\dfrac{0.42}{1.60}$

$t \approx 10\,700$ years

# Measuring decay

- GM tube: each particle gives a **pulse**
- count rate $R$ is a **fraction** of the activity
- plot ln $R$ against $t$: straight line, gradient $-\lambda$
- errors: **background**, dead time, absorption

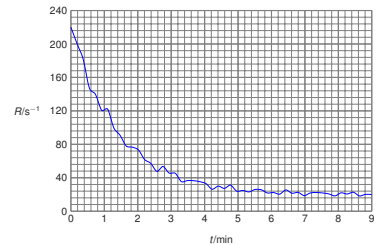


# Example: GM tube half-life

A GM tube measures the count rate $R$ of an isotope as shown. Find the half-life.

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The count rate levels off at about 20 $s^{-1}$: this is **background**, so subtract it to get the true rate $R_T$. The readings fluctuate (random decay), so take several half-life values and average.

$R_T$ : 200 $\rightarrow$ 100 takes about 1.0 min; 160 $\rightarrow$ 80 about 0.9 min; 120 $\rightarrow$ 60 about 0.9 min; 80 $\rightarrow$ 40 about 1.0 min.

Average: $t_{1/2} \approx 0.95$ min

# Key equations

| | | | |
|---|---|---|---|
| E = mc2 | $E = mc^2$ | activity | $A = \lambda N$ |
| mass defect | $\Delta m = Z m_{\mathrm{p}} + (A - Z) m_{\mathrm{n}} - m_X$ | decay | $N = N_0 \mathrm{e}^{-\lambda t}$ |
| binding | $E_B = \Delta m\, c^2$ | half-life | $\lambda\, t_{1/2} = \ln 2$ |