Nuclear Physics

Name _____ Date _____

By the end of this topic I can...

- use $E = mc^2$, mass defect and binding energy, and the binding energy per nucleon curve
- describe fission and fusion and calculate the energy released
- define decay constant and activity, and use $N = N_0 e^{-\lambda t}$
- use half-life, and analyse count-rate data

Nuclear energy

Mass–energy equivalence

Mass and energy are equivalent : $E = mc^2$

where m is the rest mass and $c = 3.00 \times 10^8$ m s^{-1}.

• for any energy change, $\Delta E =$ Δmc^2 : if energy is supplied to a system, its mass increases

• a moving or hot object has slightly more mass; the effect is tiny in everyday life. What is conserved is mass-energy .

• a stationary object has rest energy $E_0 = m_0 c^2$

Example 30.1 A proton and an antiproton annihilate and produce two photons of equal energy. Find (a) the energy of each photon and (b) its frequency.

(a) Each photon carries the rest energy of one proton: $E = m_p c^2 = 1.67 \times 10^{-27} \times (3.0 \times 10^8)^2 = 1.50 \times 10^{-10}$ J

(b) $f = \dfrac{E}{h} = \dfrac{1.50 \times 10^{-10}}{6.63 \times 10^{-34}} = 2.27 \times 10^{23}$ Hz (γ-ray)

Question 30.1 Burning one mole of carbon releases 394 kJ. Find the change in mass and compare with the mass of carbon (12 g).

$\Delta m = \dfrac{E}{c^2} = \dfrac{394 \times 10^3}{(3.0 \times 10^8)^2} = 4.4 \times 10^{-12}$ kg

Fraction $= \dfrac{4.4 \times 10^{-12}}{0.012} = 3.6 \times 10^{-10}$: negligible.

Question 30.2 The specific heat capacity of copper is 380 J kg^{-1} K^{-1}. A copper block is heated from 300 K to 1000 K. Find the additional mass as a fraction of its rest mass.

Energy per kg: $c\Delta T = 380 \times 700 = 2.66 \times 10^5$ J kg^{-1}

$$\frac{\Delta m}{m} = \frac{c\Delta T}{c^2} = \frac{2.66 \times 10^5}{(3.0 \times 10^8)^2} = 3.0 \times 10^{-12}$$

Binding energy

Nucleons are held together by the strong nuclear force. Work must be done to pull them apart, so separated nucleons have more energy, and more mass.

The mass defect is the difference between the mass of the separated nucleons and the mass of the nucleus.

The binding energy E_B is the energy needed to separate the nucleons of a nucleus to infinity (equivalently, the energy released when free nucleons combine).

- for nuclide ${}^A_Z X$: $\Delta m = Z m_\text{p} + (A - Z) m_\text{n} - m_X$ and

$$E_B = \Delta m\, c^2$$

- units: 1 u = 1.66×10^{-27} kg (a nuclide has mass $\approx A$ u);
1 MeV = 1.60×10^{-13} J

Example 30.2 An iron-56 nucleus has mass 9.288×10^{-26} kg. Find the binding energy per nucleon, in MeV ($m_p = 1.673 \times 10^{-27}$ kg, $m_n = 1.675 \times 10^{-27}$ kg).

$\Delta m = 26m_p + 30m_n - m_{\text{Fe}} = 26 \times 1.673 \times 10^{-27} + 30 \times 1.675 \times 10^{-27} - 9.288 \times 10^{-26} = 8.68 \times 10^{-28}$ kg

$E_B = \Delta mc^2 = 8.68 \times 10^{-28} \times (3.00 \times 10^8)^2 = 7.81 \times 10^{-11}$ J

per nucleon: $\dfrac{7.81 \times 10^{-11}}{56} = 1.395 \times 10^{-12}$ J $= \dfrac{1.395 \times 10^{-12}}{1.60 \times 10^{-13}} = 8.72$ MeV

Question 30.3 Show that the energy equivalent of 1.0 u is 934 MeV.

$E = mc^2 = 1.66 \times 10^{-27} \times (3.00 \times 10^8)^2 = 1.494 \times 10^{-10}$ J $= \dfrac{1.494 \times 10^{-10}}{1.60 \times 10^{-13}} =$ 934 MeV

Question 30.4 Proton mass is 1.007 u, neutron 1.009 u and uranium-235 is 234.992 u. Find the binding energy per nucleon of $^{235}_{92}$U.

$\Delta m = 92 \times 1.007 + 143 \times 1.009 - 234.992 = 1.939$ u

$E_B = 1.939 \times 934 = 1811$ MeV, so per nucleon $= \dfrac{1811}{235} \approx 7.7$ MeV (the masses are only given to 3 d.p., so this is only approximate; the data book value is 7.59 MeV).

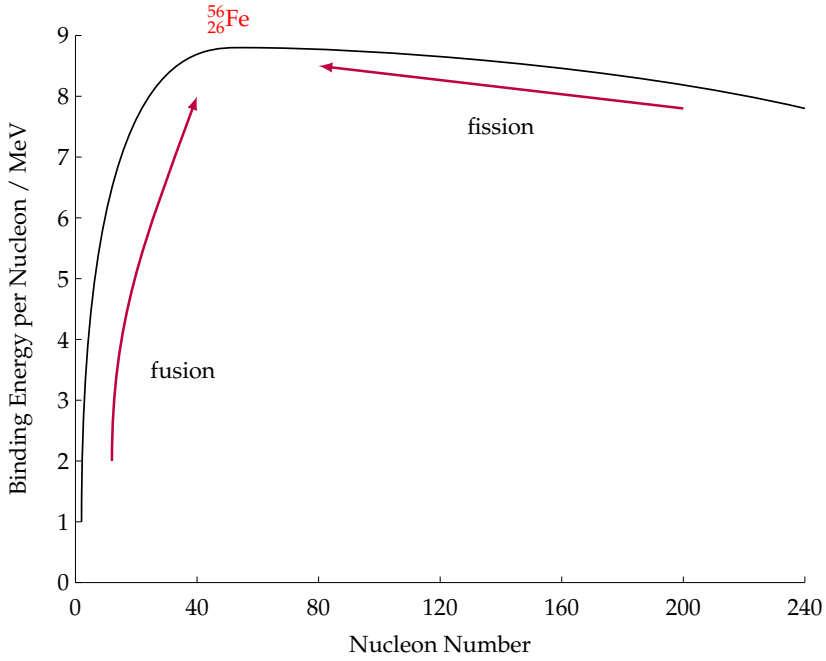
Question 30.5 What is the binding energy of the hydrogen nucleus $^1_1$H?

It is a single proton, so there is nothing to bind: $E_B = 0$.

Question 30.6 Why is the rest mass of a proton slightly larger than 1 u?

1 u is $\frac{1}{12}$ of the mass of a carbon-12 atom, where the nucleons are bound and have lost mass (mass defect). A free proton has no such loss, so its mass is slightly more than the average carbon nucleon (1 u).

Nuclear stability



- the binding energy per nucleon is the average energy to remove one nucleon: higher means more stable
- the curve rises steeply, peaks at about iron-56 ($\approx$ 8.8 MeV), then falls slowly. (Strictly nickel-62 is the most tightly bound, but iron is far more abundant.)
- reactions that move nuclei towards the peak release energy

Fusion and fission

Fusion: two light nuclei join to form a heavier nucleus.
Fission: a massive nucleus splits into two nuclei of similar size (usually with several neutrons).

- fusion needs very high temperature and density to overcome the electrostatic repulsion; it powers the Sun and stars (hydrogen fusion needs $\sim 1.5 \times 10^7$ K)
- α-decay is not fission (it does not give two similar-sized fragments)
- fission may be spontaneous or induced by a neutron

Nuclear reactors (extension)

• chain reaction: each fission releases more than one ___neutron___, which can cause more fission. Example: $^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\,^{1}_{0}\text{n}$

• ___control rods___ (boron) absorb neutrons: $^{10}_{5}\text{B} + ^{1}_{0}\text{n} \rightarrow ^{7}_{3}\text{Li} + ^{4}_{2}\text{He}$. Adjusted so exactly ___one___ neutron per fission goes on to cause another.

• a ___critical mass___ of fuel is needed to sustain the chain reaction

• a ___moderator___ (e.g. water) slows the fast neutrons to ___thermal___ energies so ^{235}U can capture them

• a ___coolant___ removes heat to drive generators; a concrete ___shield___ stops radiation

Energy released in nuclear reactions

For original $\rightarrow$ products $+ Q$, two methods give Q:

• **mass**: $Q = (m_{\text{org}} - m_{\text{prod}})c^2$, the ___loss of mass___ times c^2

• **binding energy**: $Q = E_{B,\text{prod}} - E_{B,\text{org}}$, the ___increase in total binding energy___

Example 30.3 The Sun fuses hydrogen: $4\,^{1}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + 2\,^{0}_{+1}\text{e}$. (a) Find the energy released per reaction. (b) The Sun's power is 4.0×10^{26} W. Estimate the hydrogen burned per second. ($m_p = 1.672622 \times 10^{-27}$ kg, $m_{\text{He}} = 6.644657 \times 10^{-27}$ kg, $m_e = 9.11 \times 10^{-31}$ kg)

(a) $\Delta m = 4m_p - m_{\text{He}} - 2m_e = 4.40 \times 10^{-29}$ kg
$Q = \Delta m c^2 = 4.40 \times 10^{-29} \times (3.0 \times 10^8)^2 = 3.96 \times 10^{-12}$ J
(b) Reactions per second: $\dfrac{P}{Q} = \dfrac{4.0 \times 10^{26}}{3.96 \times 10^{-12}} = 1.0 \times 10^{38}\ \text{s}^{-1}$
Hydrogen per second: $4 \times 1.0 \times 10^{38} \times 1.67 \times 10^{-27} = 6.7 \times 10^{11}\ \text{kg s}^{-1}$

Example 30.4 ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + x\,{}^{1}_{0}\text{n}$. Binding energy per nucleon: U 7.591 MeV, Ba 8.326 MeV, Kr 8.513 MeV. (a) Find x. (b) Find the energy released. (c) Find the change in mass.

(a) Nucleon number: $1 + 235 = 141 + 92 + x$, so $x = 3$.
(b) $Q = 141 \times 8.326 + 92 \times 8.513 - 235 \times 7.591 = 173.3$ MeV
(c) $\Delta m = \dfrac{Q}{c^2} = \dfrac{173.3 \times 1.60 \times 10^{-13}}{(3.0 \times 10^8)^2} = 3.1 \times 10^{-28}$ kg (a ___decrease___ in mass)

Question 30.7 ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{144}_{55}\text{Cs} + {}^{90}_{37}\text{Rb} + 2\,{}^{1}_{0}\text{n}$. Binding energy per nucleon: U 7.591, Cs 8.212, Rb 8.631 MeV. Find the energy released.

$Q = 144 \times 8.212 + 90 \times 8.631 - 235 \times 7.591 = 1182.5 + 776.8 - 1783.9 = 175$ MeV

Question 30.8 ${}^{1}_{1}\text{H} + {}^{2}_{1}\text{D} \rightarrow {}^{3}_{2}\text{He} + \text{energy}$. Masses: ${}^{1}_{1}\text{H}$ 1.00728 u, D 2.01356 u, ${}^{3}_{2}\text{He}$ 3.01605 u. Find the energy released.

$\Delta m = 1.00728 + 2.01356 - 3.01605 = 0.00479$ u $= 0.00479 \times 1.66 \times 10^{-27} = 7.95 \times 10^{-30}$ kg
$Q = \Delta mc^2 = 7.16 \times 10^{-13}$ J $= 4.5$ MeV

Question 30.9 ${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2\,{}^1_1\text{p} + 12.86$ MeV. The binding energy per nucleon of ${}^4_2\text{He}$ is 7.074 MeV. Find that of ${}^3_2\text{He}$.

$Q = E_B(\text{He-4}) - 2E_B(\text{He-3})$ (protons have none): $12.86 = 4 \times 7.074 - 2E_B$
$E_B(\text{He-3}) = 7.72$ MeV, so per nucleon $= \dfrac{7.72}{3} = 2.57$ MeV

Question 30.10 In a nuclear reaction the products have more total mass than the original particles. What does this tell you about the original particles?

Energy is needed to create the extra mass, so the original particles must have enough **kinetic energy** ($\geq \Delta mc^2$) for the reaction to occur.

Radioactive decay

Radioactive decay is the random and spontaneous emission of α, β or γ radiation from unstable nuclei.

- random: we cannot say when a given nucleus will decay, only the probability
- spontaneous: not affected by external conditions such as temperature or pressure

Decay constant and activity

The decay constant λ is the probability of decay of a nucleus per unit time: $\lambda = \dfrac{\Delta N}{N\,\Delta t}$

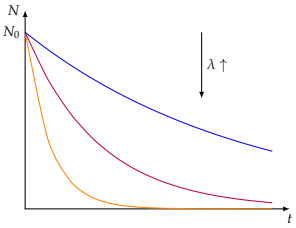
Activity is the number of nuclei decaying per unit time,
$A = \dfrac{\Delta N}{\Delta t} = -\dfrac{\mathrm{d}N}{\mathrm{d}t}$, so $A = \lambda N$

- units: $[\lambda] = [A] = \text{s}^{-1}$; one decay per second is one becquerel: $1\text{ Bq} = 1\text{ s}^{-1}$

The decay equations

From $-\dfrac{dN}{dt} = \lambda N$, separating variables and integrating gives $\ln \dfrac{N}{N_0} = -\lambda t$, so

$$N = N_0 e^{-\lambda t} \qquad A = A_0 e^{-\lambda t}$$



Exponential decay; larger λ decays faster.

- N_0 is the number of undecayed nuclei at $t =$ 0 ; a larger λ means a faster decay

Example 30.5 A sample of 6.0×10^7 iodine-131 nuclei has activity 60 Bq. Find (a) the decay constant and (b) the number undecayed after 20 days.

(a) $\lambda = \dfrac{A}{N} = \dfrac{60}{6.0 \times 10^7} = 1.0 \times 10^{-6}\ \text{s}^{-1}$
(b) $N = 6.0 \times 10^7 \times e^{-1.0\times10^{-6}\times20\times24\times3600} = 1.1 \times 10^7$

Example 30.6 Sodium-24 has $\lambda = 1.28 \times 10^{-5}\ \text{s}^{-1}$. A sample initially has mass 9.0 μg. Find (a) the initial number of nuclei, (b) the initial activity, (c) the activity after 48 hours.

(a) $N_0 = \dfrac{9.0 \times 10^{-9}}{24 \times 1.66 \times 10^{-27}} = 2.26 \times 10^{17}$
(b) $A_0 = \lambda N_0 = 1.28 \times 10^{-5} \times 2.26 \times 10^{17} = 2.89 \times 10^{12}$ Bq
(c) $A = A_0 e^{-\lambda t} = 2.89 \times 10^{12} \times e^{-1.28\times10^{-5}\times48\times3600} = 3.2 \times 10^{11}$ Bq

Question 30.11 A sample of polonium-205 has activity 6.7×10^{15} Bq and $\lambda = 1.16 \times 10^{-4}$ s^{-1}. Find (a) the number of nuclei, (b) the mass, (c) the time for the activity to fall to 1% of its initial value.

(a) $N = \dfrac{A}{\lambda} = \dfrac{6.7 \times 10^{15}}{1.16 \times 10^{-4}} = 5.8 \times 10^{19}$
(b) $m = 5.8 \times 10^{19} \times 205 \times 1.66 \times 10^{-27} = 2.0 \times 10^{-5}$ kg (20 mg)
(c) $0.01 = \mathrm{e}^{-\lambda t}$: $t = \dfrac{\ln 100}{\lambda} = \dfrac{4.61}{1.16 \times 10^{-4}} = 3.97 \times 10^{4}$ s $= 11$ hours

Question 30.12 A technetium-99m tracer of activity 900 MBq is injected. After 24 hours the activity is 56.4 MBq. Find (a) the decay constant and (b) the number of nuclei still in the body.

(a) $\lambda = \dfrac{1}{t} \ln \dfrac{A_0}{A} = \dfrac{\ln(900/56.4)}{24 \times 3600} = 3.2 \times 10^{-5}$ s^{-1}
(b) $N = \dfrac{A}{\lambda} = \dfrac{56.4 \times 10^{6}}{3.2 \times 10^{-5}} = 1.8 \times 10^{12}$

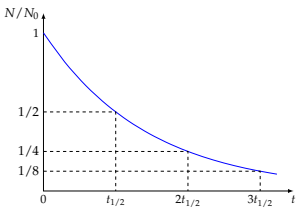
Half-life

The ___half-life___ $t_{1/2}$ is the time taken for the number of undecaying nuclei (or the activity) to fall to ___half___ its initial value.

At $t = t_{1/2}$: $N = \frac{1}{2}N_0$, so $\mathrm{e}^{-\lambda t_{1/2}} = \frac{1}{2}$, giving

$$\lambda\, t_{1/2} = \ln 2$$

- $t_{1/2}$ is a ___constant___: it does not depend on the initial number of nuclei



Each half-life halves N.

Example 30.7 Radium-224 has half-life 3.63 days. (a) What is the activity of 6.0 mg of pure radium-224? (b) How many nuclei have decayed after 10 days?

$\lambda = \dfrac{\ln 2}{3.63 \times 24 \times 3600} = 2.21 \times 10^{-6}\ \text{s}^{-1}$

(a) $N_0 = \dfrac{6.0 \times 10^{-6}}{224 \times 1.66 \times 10^{-27}} = 1.61 \times 10^{19}$, so $A_0 = \lambda N_0 = 3.6 \times 10^{13}$ Bq

(b) $N = N_0 e^{-\lambda t} = 1.61 \times 10^{19} \times e^{-2.21 \times 10^{-6} \times 10 \times 24 \times 3600} = 2.39 \times 10^{18}$

Decayed: $\Delta N = 1.61 \times 10^{19} - 2.39 \times 10^{18} = 1.37 \times 10^{19}$

Example 30.8 Carbon-14 has half-life 5570 years. A sample of dead wood has activity 0.42 Bq; an equal mass of living wood has 1.60 Bq. Find the age of the dead wood.

$0.42 = 1.60\, e^{-\frac{\ln 2}{5570} t}$, so $-\dfrac{\ln 2}{5570} t = \ln \dfrac{0.42}{1.60}$

$t \approx 10\,700$ years

Question 30.13 The number of uranium-238 nuclei in a rock fell from 3.50×10^{17} to 3.27×10^{17} in 480 million years. Estimate the half-life.

$\lambda = \dfrac{1}{t} \ln \dfrac{N_0}{N} = \dfrac{\ln(3.50/3.27)}{4.8 \times 10^8\ \text{y}} = 1.4 \times 10^{-10}\ \text{y}^{-1}$

$t_{1/2} = \dfrac{\ln 2}{\lambda} = 4.9 \times 10^9$ years

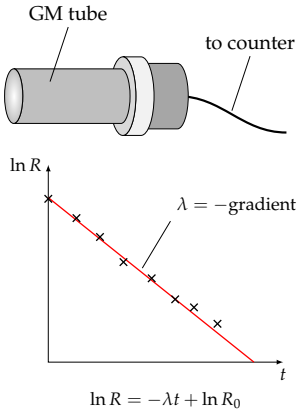
Question 30.14 Plutonium-238 (half-life 87.7 years) releases 9.0×10^{-13} J per decay. A pacemaker battery has initial activity 6.0×10^{10} Bq. (a) Find the initial power. (b) Find the mass of plutonium needed. (c) The power must stay above 60% of its initial value. For how many years?

(a) $P = AE = 6.0 \times 10^{10} \times 9.0 \times 10^{-13} = 0.054$ W
(b) $\lambda = \dfrac{\ln 2}{87.7 \times 3.16 \times 10^7} = 2.5 \times 10^{-10}\ \text{s}^{-1}$, $N = \dfrac{A}{\lambda} = 2.4 \times 10^{20}$, so $m = 2.4 \times 10^{20} \times 238 \times 1.66 \times 10^{-27} = 9.5 \times 10^{-5}$ kg
(c) $0.60 = \text{e}^{-\lambda t}$: $t = \dfrac{\ln(1/0.6)}{\lambda} \times \dfrac{1}{3.16 \times 10^7} \approx 65$ years

Question 30.15 Show that $N(t) = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$.

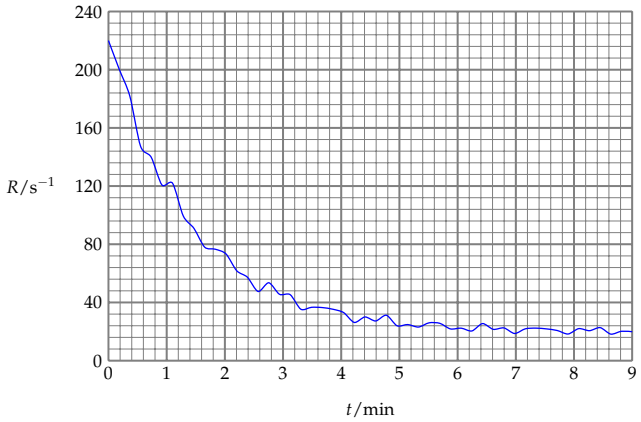
$\lambda = \dfrac{\ln 2}{t_{1/2}}$, so $N = N_0 \text{e}^{-(\ln 2)t/t_{1/2}} = N_0 \left(\text{e}^{\ln 2}\right)^{-t/t_{1/2}} = N_0\, 2^{-t/t_{1/2}} = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$

Measuring radioactive decay



A Geiger–Müller (GM) tube detects ionising radiation: each particle gives a pulse that is counted. The count rate R is only a fraction of the activity (the tube only detects part of the radiation), but it follows the same law: $R = R_0 \text{e}^{-\lambda t}$

- **to verify the law**: $\ln R = -\lambda t + \ln R_0$. Plot $\ln R$ against t: a straight line confirms exponential decay and $\lambda =$ −gradient.
- **sources of error**: background radiation (must be subtracted); absorption of α, β within the sample; radioactive products; GM tube dead time (it cannot count while resetting)



Example 30.9 A GM tube measures the count rate R of an isotope as shown. Find the half-life.

The count rate levels off at about 20 s^{-1}: this is **background**, so subtract it to get the true rate R_T. The readings fluctuate (random decay), so take several half-life values and average.
R_T : 200 $\rightarrow$ 100 takes about 1.0 min; 160 $\rightarrow$ 80 about 0.9 min; 120 $\rightarrow$ 60 about 0.9 min; 80 $\rightarrow$ 40 about 1.0 min.
Average: $t_{1/2} \approx 0.95$ min