Nuclear Physics

Name _____ Date _____

By the end of this topic I can…

- use $E = mc^2$, mass defect and binding energy, and the binding energy per nucleon curve
- describe fission and fusion and calculate the energy released
- define decay constant and activity, and use $N = N_0 e^{-\lambda t}$
- use half-life, and analyse count-rate data

Nuclear energy

Mass–energy equivalence

_____:

$$\square$$

where m is the _____ and $c = 3.00 \times 10^8$ m s^{-1}.

• for any energy change, $\Delta E =$ _____: if energy is supplied to a system, its mass _____

• a moving or hot object has slightly more mass; the effect is tiny in everyday life. What is conserved is _____.

• a stationary object has _____ $E_0 = m_0 c^2$

Example 30.1 A proton and an antiproton annihilate and produce two photons of equal energy. Find (a) the energy of each photon and (b) its frequency.

Question 30.1 Burning one mole of carbon releases 394 kJ. Find the change in mass and compare with the mass of carbon (12 g).

Question 30.2 The specific heat capacity of copper is 380 J kg^{-1} K^{-1}. A copper block is heated from 300 K to 1000 K. Find the additional mass as a fraction of its rest mass.

Binding energy

Nucleons are held together by the _____. Work must be done to pull them apart, so separated nucleons have more energy, and more mass.

The _____ is the difference between the _____ and the _____.

The _____ E_B is the energy needed to separate the nucleons of a nucleus to _____ (equivalently, the energy released when free nucleons combine).

- for nuclide ${}^{A}_{Z}X$:

and

- units: 1 u = _____ (a nuclide has mass $\approx A$ u); 1 MeV = _____

Example 30.2 An iron-56 nucleus has mass 9.288×10^{-26} kg. Find the binding energy per nucleon, in MeV ($m_p = 1.673 \times 10^{-27}$ kg, $m_n = 1.675 \times 10^{-27}$ kg).

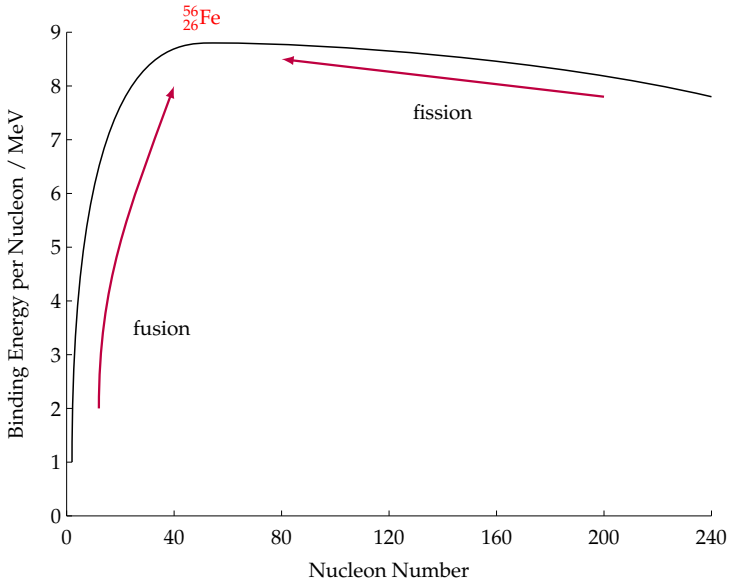
Question 30.3 Show that the energy equivalent of 1.0 u is 934 MeV.

Question 30.4 Proton mass is 1.007 u, neutron 1.009 u and uranium-235 is 234.992 u. Find the binding energy per nucleon of $^{235}_{92}\mathrm{U}$.

Question 30.5 What is the binding energy of the hydrogen nucleus $^{1}_{1}\mathrm{H}$?

Question 30.6 Why is the rest mass of a proton slightly larger than 1 u?

Nuclear stability



• the _____ is the average energy to remove one nucleon: higher means _____

• the curve rises steeply, peaks at about _____ ($\approx$ 8.8 MeV), then falls slowly. (Strictly nickel-62 is the most tightly bound, but iron is far more abundant.)

• reactions that move nuclei towards the peak _____ energy

Fusion and fission

_____: two _____ nuclei join to form a heavier nucleus.
_____: a _____ nucleus splits into two nuclei of similar size (usually with several neutrons).

• fusion needs very high _____ and density to overcome the _____; it powers the Sun and stars (hydrogen fusion needs $\sim 1.5 \times 10^7$ K)

• α-decay is _____ fission (it does not give two similar-sized fragments)

• fission may be _____ or _____ by a neutron

Nuclear reactors (extension)

• chain reaction: each fission releases more than one _____, which can cause more fission. Example: $^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{141}_{56}\mathrm{Ba} + ^{92}_{36}\mathrm{Kr} + 3\,^{1}_{0}\mathrm{n}$

• _____ (boron) absorb neutrons: $^{10}_{5}\mathrm{B} + ^{1}_{0}\mathrm{n} \rightarrow ^{7}_{3}\mathrm{Li} + ^{4}_{2}\mathrm{He}$. Adjusted so exactly _____ neutron per fission goes on to cause another.

• a _____ of fuel is needed to sustain the chain reaction

• a _____ (e.g. water) slows the fast neutrons to _____ energies so $^{235}\mathrm{U}$ can capture them

• a _____ removes heat to drive generators; a concrete _____ stops radiation

Energy released in nuclear reactions

For original $\rightarrow$ products $+ Q$, two methods give Q:

• **mass**: $Q = (m_{\text{org}} - m_{\text{prod}})c^2$, the _____ times c^2

• **binding energy**: $Q = E_{B,\text{prod}} - E_{B,\text{org}}$, the _____

Example 30.3 The Sun fuses hydrogen: $4\,^{1}_{1}\mathrm{H} \rightarrow ^{4}_{2}\mathrm{He} + 2\,^{0}_{+1}\mathrm{e}$. (a) Find the energy released per reaction. (b) The Sun's power is 4.0×10^{26} W. Estimate the hydrogen burned per second. ($m_p = 1.672622 \times 10^{-27}$ kg, $m_{\text{He}} = 6.644657 \times 10^{-27}$ kg, $m_e = 9.11 \times 10^{-31}$ kg)

Example 30.4 ${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \to {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + x\,{}^{1}_{0}\mathrm{n}$. Binding energy per nucleon: U 7.591 MeV, Ba 8.326 MeV, Kr 8.513 MeV. (a) Find x. (b) Find the energy released. (c) Find the change in mass.

Question 30.7 ${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \to {}^{144}_{55}\mathrm{Cs} + {}^{90}_{37}\mathrm{Rb} + 2\,{}^{1}_{0}\mathrm{n}$. Binding energy per nucleon: U 7.591, Cs 8.212, Rb 8.631 MeV. Find the energy released.

Question 30.8 ${}^{1}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{D} \to {}^{3}_{2}\mathrm{He} + \text{energy}$. Masses: ${}^{1}_{1}\mathrm{H}$ 1.00728 u, D 2.01356 u, ${}^{3}_{2}\mathrm{He}$ 3.01605 u. Find the energy released.

Question 30.9 ${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2\,{}^1_1\text{p} + 12.86$ MeV. The binding energy per nucleon of ${}^4_2\text{He}$ is 7.074 MeV. Find that of ${}^3_2\text{He}$.

Question 30.10 In a nuclear reaction the products have more total mass than the original particles. What does this tell you about the original particles?

Radioactive decay

_____ is the _____ and _____ emission of α, β or γ radiation from unstable nuclei.

- _____: we cannot say when a given nucleus will decay, only the _____
- _____: not affected by external conditions such as temperature or pressure

Decay constant and activity

The _____ λ is the _____ of a nucleus per unit time:

_____ is the

_____,

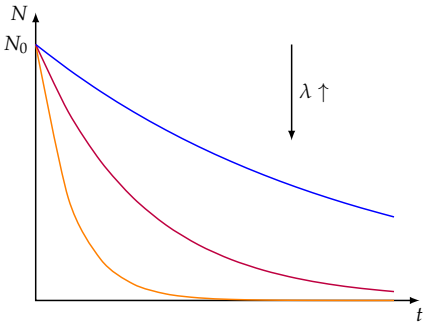
$A = \dfrac{\Delta N}{\Delta t} = -\dfrac{dN}{dt}$, so

- units: $[\lambda] = [A] = \text{s}^{-1}$; one decay per second is one _____: $1\ \text{Bq} = 1\ \text{s}^{-1}$

The decay equations

From $-\dfrac{dN}{dt} = \lambda N$, separating variables and integrating gives $\ln \dfrac{N}{N_0} = -\lambda t$, so

- N_0 is the number of undecayed nuclei at $t =$ _____; a larger λ means a _____ decay



Exponential decay; larger λ decays faster.

Example 30.5 A sample of 6.0×10^7 iodine-131 nuclei has activity 60 Bq. Find (a) the decay constant and (b) the number undecayed after 20 days.

Example 30.6 Sodium-24 has $\lambda = 1.28 \times 10^{-5}\ \text{s}^{-1}$. A sample initially has mass 9.0 μg. Find (a) the initial number of nuclei, (b) the initial activity, (c) the activity after 48 hours.

Question 30.11 A sample of polonium-205 has activity 6.7×10^{15} Bq and $\lambda = 1.16 \times 10^{-4}\ \mathrm{s}^{-1}$. Find (a) the number of nuclei, (b) the mass, (c) the time for the activity to fall to 1% of its initial value.

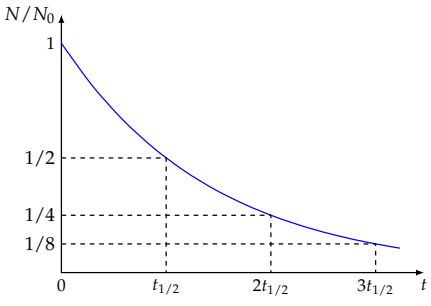
Question 30.12 A technetium-99m tracer of activity 900 MBq is injected. After 24 hours the activity is 56.4 MBq. Find (a) the decay constant and (b) the number of nuclei still in the body.

Half-life

The _____ $t_{1/2}$ is the time taken for the number of undecaying nuclei (or the activity) to fall to _____ its initial value.

At $t = t_{1/2}$: $N = \frac{1}{2}N_0$, so $\mathrm{e}^{-\lambda t_{1/2}} = \frac{1}{2}$, giving

- $t_{1/2}$ is a _____: it does not depend on the initial number of nuclei



Each half-life halves N.

Example 30.7 Radium-224 has half-life 3.63 days. (a) What is the activity of 6.0 mg of pure radium-224? (b) How many nuclei have decayed after 10 days?

Example 30.8 Carbon-14 has half-life 5570 years. A sample of dead wood has activity 0.42 Bq; an equal mass of living wood has 1.60 Bq. Find the age of the dead wood.

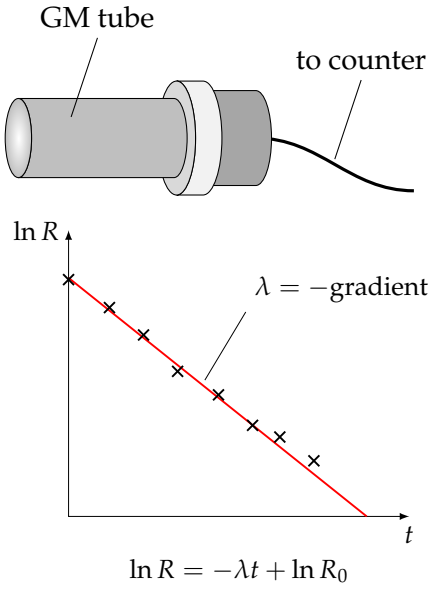
Question 30.13 The number of uranium-238 nuclei in a rock fell from 3.50×10^{17} to 3.27×10^{17} in 480 million years. Estimate the half-life.

Question 30.14 Plutonium-238 (half-life 87.7 years) releases 9.0×10^{-13} J per decay. A pacemaker battery has initial activity 6.0×10^{10} Bq. (a) Find the initial power. (b) Find the mass of plutonium needed. (c) The power must stay above 60% of its initial value. For how many years?

Question 30.15 Show that $N(t) = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$.

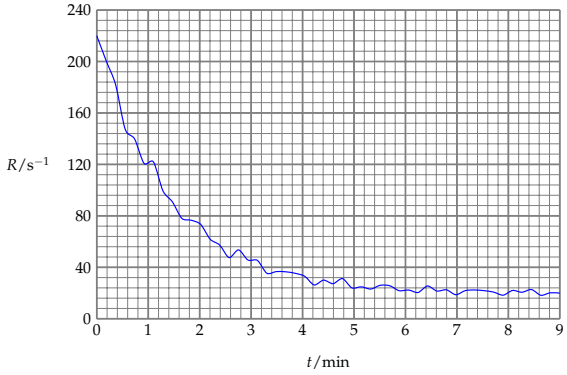
Measuring radioactive decay

A _____ detects ionising radiation: each particle gives a pulse that is _____. The _____ R is only a _____ of the activity (the tube only detects part of the radiation), but it follows the same law:



$\ln R = -\lambda t + \ln R_0$

- **to verify the law**: $\ln R = -\lambda t + \ln R_0$. Plot $\ln R$ against t: a _____ confirms exponential decay and $\lambda =$ _____.
- **sources of error**: _____ (must be subtracted); absorption of α, β within the sample; _____; GM tube _____ (it cannot count while resetting)



Example 30.9 A GM tube measures the count rate R of an isotope as shown. Find the half-life.