Force and Motion

Name _____ Date _____

By the end of this topic I can...

- state and apply Newton's three laws of motion
- draw free-body diagrams and find the resultant force on a body
- describe weight, drag, upthrust, normal contact and friction, and explain terminal velocity
- solve problems on slopes and for connected bodies

Newton's laws of motion

A force appears when two bodies interact. The **resultant** (net) force is the vector sum of all the forces on an object. Forces you will meet: weight, tension, normal contact, friction, drag, upthrust, electric and magnetic forces.

- a resultant force can change the speed, the direction of motion or the shape of an object, or make it rotate
- a **point mass** is an idealisation: the object has mass but takes up no space

Newton's first law

An object stays at rest or continues to move at constant velocity unless acted on by a resultant force.

- the tendency of a body to resist changes in its motion is its **inertia**
- with no resultant force the object is in **equilibrium**

Newton's second law

The acceleration of an object is directly proportional to the resultant force on it and inversely proportional to its mass; it is in the same direction as the resultant force:

$$F_{\text{net}} = ma$$

- the **newton** (N) is the force that gives a mass of 1 kg an acceleration of $1\ \text{m s}^{-2}$, so $1\ \text{N} = 1\ \text{kg m s}^{-2}$

- **mass** is the property of a body that resists changes in its motion (a measure of its inertia)

Example 3.1 A box of 6.0 kg is being pushed along a horizontal surface with a force of 30 N. The resistive force acting is 21 N. What is the acceleration of the box?

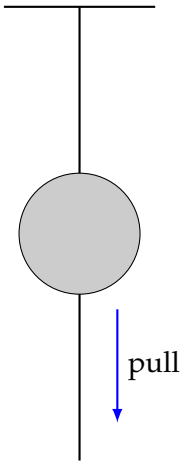
$$F_{\text{net}} = F - f = ma \quad \Rightarrow \quad a = \frac{F-f}{m} = \frac{30-21}{6.0} = 1.5 \text{ m s}^{-2}$$

Example 3.2 A car of mass 800 kg is travelling at a speed of 20 m s^{-1}. The driver then operates the brake pedal so a braking force of 2000 N gradually brings the car to stop. (a) What is the deceleration for the car? (b) What is the stopping distance?

Using Newton's second law and noticing braking force acts opposite to direction of motion:

$$F_{\text{net}} = ma \quad \Rightarrow \quad -2000 = 800 \times a \quad \Rightarrow \quad a = -2.5 \text{ m s}^{-2}$$
$$2as = v^2 - u^2 \quad \Rightarrow \quad s = \frac{v^2-u^2}{2a} = \frac{0^2-20^2}{2\times(-2.5)} = 80 \text{ m}$$

Example 3.3 A massive ball is suspended on a string. A second string is attached to the bottom of the ball. If one pulls the bottom string with a gradually increasing force, does the top string or the bottom string break first? What if the bottom string is jerked, which string breaks?



When tension gradually increases, system is always in equilibrium, the tensions in strings must have $T_{\text{top}} = T_{\text{bottom}} + mg$. The top string suffers a greater force, so it breaks first, however, when bottom string is jerked, the ball tends to remain at rest due to its large mass, preventing sudden change to the tension in top string so in this case bottom string is more likely to snap.

Newton's third law

When body A exerts a force on body B, body B exerts a force on A that is equal in magnitude, opposite in direction and of the same type. The two forces act on different bodies.

Example 3.4 Suggest the action and reaction force in the following cases: (a) A person stands on a bathroom scale. (b) A helicopter hovers in air. (c) The earth orbits around the sun.

(a) The person exerts downward force on scale, scale exerts an upward reaction on the person.
(b) The rotors of helicopter push air downwards, air exerts an upward force on helicopter.
(c) The Sun pulls the earth through gravitational attraction, the Earth also attracts the sun in return.

Free-body diagrams

A **free-body diagram** shows one object with arrows for every force acting *on it*. Routine:

• (1) draw the FBD; (2) ___resolve___ forces and find the resultant; (3) apply ___Newton's second law___; (4) solve for the acceleration; (5) use the ___equations of motion___ to find speeds, distances and times

Types of forces

Weight

The **weight** of an object is the ___gravitational force___ on it from the Earth:

$$W = mg$$

$g \approx$ ___9.81___ N kg^{-1} is the ___gravitational field strength___.

• weight is a ___vector___ measured in newtons; mass is a ___scalar___ measured in kilograms

• mass is the same everywhere; weight changes with the ___gravitational field strength___

Example 3.5 An astronaut finds that he weighs 300 N on the surface of Mars, where the gravitational field strength is known to be 3.7 N kg^{-1}. Find his mass and hence estimate his weight if he returns to his home on the Earth.

Mass of astronaut: $m = \frac{W_M}{g_M} = \frac{300}{3.7} \approx 81.1$ kg
Weight on earth: $W_E = mg_E = 81.1 \times 9.81 \approx 795$ N

Free fall

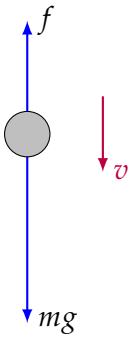
With gravity as the only force, $F_{\text{net}} = W$, so $ma = mg$:

$$a = g$$

Free-fall acceleration equals the field strength and is
independent of mass.

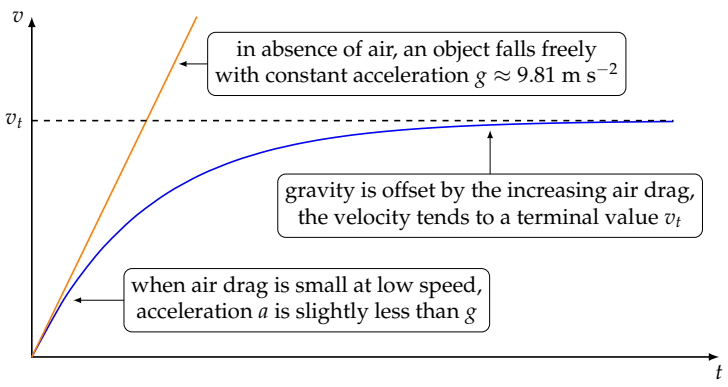
Drag and terminal velocity

Drag is the force on an object moving through a fluid. It acts opposite to the motion, and increases with the speed, the cross-sectional area and the density of the fluid (and depends on shape).

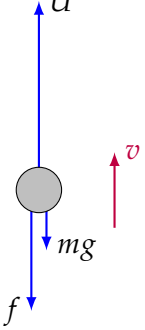


Falling through air: $F_{\text{net}} = mg - f = ma$.

- at the start $v = 0$, $f = 0$ so $a =$ g
- as v increases, f increases, F_{net} decreases and the acceleration decreases
- when $f = mg$ the resultant force is zero, $a = 0$, and the object falls at constant **terminal velocity**



Velocity–time graph of an object falling through air.



A bubble rising in a liquid has weight mg down, drag f down and ___upthrust___ U up: $F_{\text{net}} = U - mg - f$. At terminal speed $U =$ ___f+mg___.

Example 3.6 An object of 5.0 kg falls through the atmosphere from a very high altitude. After some time, it falls at a constant speed of 70 m s^{-1}. Assume there is no significant change in gravitational field during the fall and the air resistance is proportional to speed: $f = bv$. (a) Find the value of the coefficient k. (b) Find the acceleration of the object when it is falling at 30 m s^{-1}.

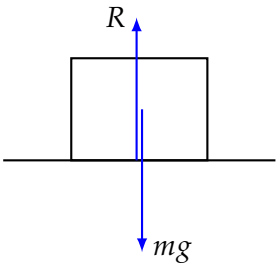
equilibrium between weight and air drag when falling at terminal speed, so

$$mg = bv_t \quad \Rightarrow \quad b = \frac{mg}{v_t} = \frac{5.0 \times 9.81}{70} \approx 0.70 \text{ kg s}^{-1}$$

at any instant, equation of motion is: $F_{\text{net}} = ma = mg - bv$
at 30 m s^{-1}, acceleration is: $a = \frac{mg-bv}{m} = \frac{5.0\times9.81-0.70\times30}{5.0} \approx 5.6 \text{ m s}^{-2}$

Normal contact force

The **normal contact force** R is the component of the contact force ___perpendicular___ to the surfaces. It comes from the ___electrostatic repulsion___ between atoms.

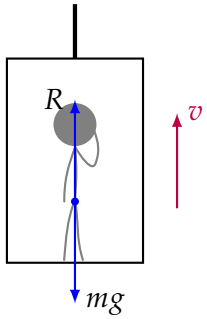


Example 3.7 A box of mass $m = 4.0$ kg is resting on a horizontal ground. What is the normal contact force acting?

Equilibrium between weight and normal contact, so

$$R - W = 0 \quad \Rightarrow \quad R = mg = 4.0 \times 9.81 \approx 39.2 \text{ N}$$

Example 3.8 A man of 80 kg stands in a lift. Find his apparent weight, i.e., the contact force, when the lift is (a) moving upwards at steady speed of 2.0 m s^{-1}, (b) accelerating upwards at 2.0 m s^{-2}, (c) moving upwards but slowing down at a deceleration of 1.5 m s^{-2}.



The forces acting on man are weight and normal contact, for either case, equation of motion for the man reads:

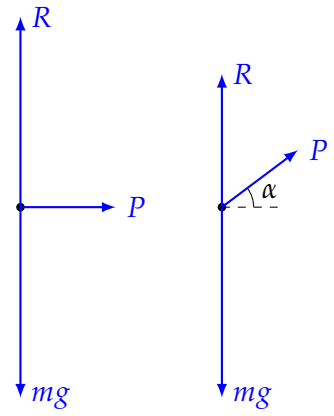
$$F_{\text{net}} = ma = R - mg$$

so normal contact force: $R = mg + ma$

when rising at steady speed, man is in equilibrium ($a = 0$), so: $R = mg = 80 \times 9.81 \approx 785$ N

when accelerating upwards ($a = +2.0$ m s^{-2}): $R = 80 \times 9.81 + 80 \times 2.0 \approx 945$ N

when decelerating upwards ($a = -1.5$ m s^{-2}): $R = 80 \times 9.81 + 80 \times (-1.5) \approx 665$ N



(a) (b)

Example 3.9 A sleigh of mass 15 kg lies at rest on an icy ground. The surface is frictionless. A force P of 75 N is applied to the sleigh. Find the normal contact force and the acceleration of the sleigh if P is acting (a) horizontally, (b) at an angle α to the horizontal where $\tan\alpha = \frac{3}{4}$.

No vertical acceleration, so the vertical forces balance; the horizontal resultant gives the acceleration.

(a) $R = mg = 15 \times 9.81 \approx 147$ N, $a = \dfrac{P}{m} = \dfrac{75}{15} = 5.0$ m s^{-2}

(b) $\tan\alpha = \frac{3}{4}$ gives $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$.

Vertical: $R + P\sin\alpha = mg \quad \Rightarrow \quad R = 147 - 75 \times \frac{3}{5} \approx 102$ N. Horizontal: $P\cos\alpha = ma \quad \Rightarrow \quad a = \dfrac{75 \times \frac{4}{5}}{15} = 4.0$ m s^{-2}

Friction

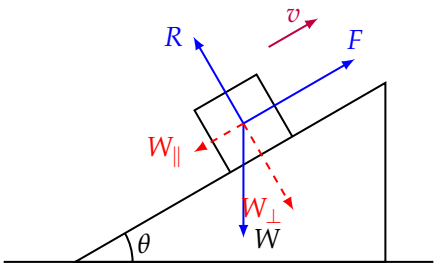
Friction is the component of the contact force ___parallel___ to the surfaces; it opposes (relative) sliding.

- **static friction**: acts when surfaces ___tend___ to slide; it adjusts to equal the applied force, up to a ___maximum___
- **kinetic (dynamic) friction**: acts when surfaces are ___already sliding___

Slopes and connected bodies

Inclined slopes

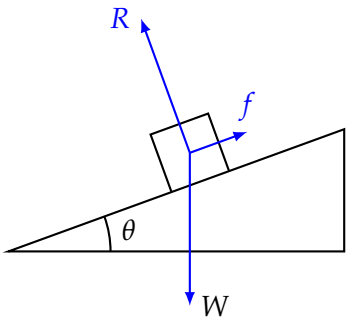
Resolve all forces ___parallel___ and ___perpendicular___ to the slope:



Resolving the weight on a slope.

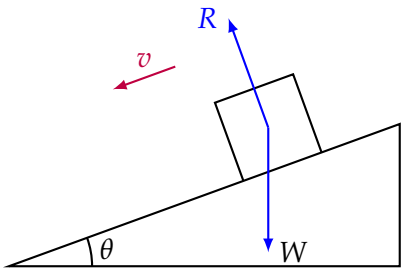
$$W_\parallel = mg\sin\theta \qquad W_\perp = mg\cos\theta$$

Example 3.10 A block of mass m stays at rest on an inclined plane. The plane makes an angle θ with the horizontal. Find the normal contact force R and the frictional force f acting on the block.



The block is in equilibrium, so $F_{\text{net}} = 0$ in any direction.
Parallel to slope: $f = W_\parallel \quad \Rightarrow \quad f = mg\sin\theta$
Normal to slope: $R = W_\perp \quad \Rightarrow \quad R = mg\cos\theta$

Example 3.11 A block of mass m slides down a *smooth* slope. The angle of the slope to the horizontal is θ. Find the acceleration of the block.

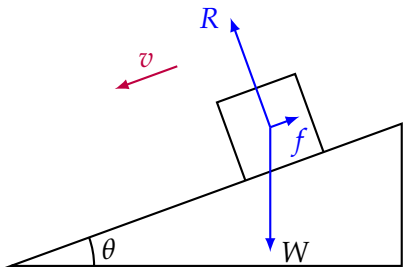


only force acting along the slope is component of weight down the slope, so:

$$F_{\text{net}} = ma = mg\sin\theta \quad \Rightarrow \quad a = g\sin\theta$$

as $\theta \to 0$, $a \to 0$, this shows if plane becomes horizontal, the block simply stays put
as $\theta \to 90°$, slope becomes vertical, block would undergo free fall, so naturally $a \to g$

Example 3.12 A block of mass 2.0 kg slides down a rough slope from rest. The slope is inclined at angle $\theta = 20^\circ$ to the horizontal, and the block experiences a constant friction of 5.0 N. (a) What is the block's acceleration? (b) What is the distance travelled in 2.5 seconds?



Resolving along slope:

$$F_{\text{net}} = mg\sin\theta - f = ma$$

$$a = \frac{2.0\times 9.81\times\sin 20^\circ - 5.0}{2.0} \approx 0.855 \text{ m s}^{-2}$$

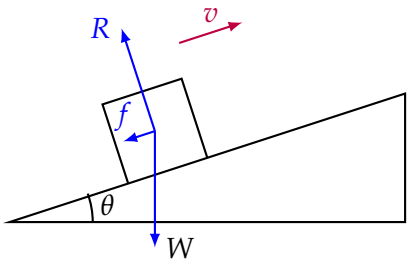
Distance travelled: $s = \cancel{ut}^{0} + \frac{1}{2}at^2 = \frac{1}{2}\times 0.855\times 2.5^2 \approx 2.67$ m

Example 3.13 A block of mass 3.0 kg is travelling up an inclined slope at an initial speed of 2.8 m s^{-1}. The slope makes an angle of 18° with the horizontal. A constant friction of 7.5 N acts on the block.

(a) What is the block's deceleration?

(b) How far does the block travel along the slope before its speed decreases to zero?

(c) Suggest whether the block could stay on the slope.



Resolving along slope (take direction of initial velocity as positive direction):

$$F_{\text{net}} = -mg\sin\theta - f = ma \quad \Rightarrow \quad a = \frac{-mg\sin\theta - f}{m} = \frac{-3.0\times 9.81\times\sin 18^\circ - 7.5}{3.0} \approx -5.53 \text{ m s}^{-2}$$

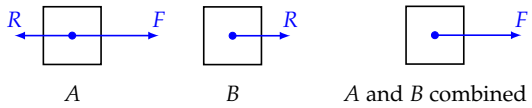
$$v^2 - u^2 = 2as \quad \Rightarrow \quad s = \frac{v^2-u^2}{2a} = \frac{0^2-2.8^2}{2\times(-5.5.3)} \approx 0.709 \text{ m}$$

Note that component of weight down the slope is: $W_{\parallel} = mg\sin\theta = 3.0\times 9.81\times 18^\circ \approx 9.1$ N

$W_{\parallel} > f$, so friction is not enough to prevent block from sliding back down the slope

Connected bodies

Treat each body separately (internal forces such as tension come in equal and opposite pairs), or the whole system together (only external forces matter). Two of the three equations are independent.



A B A and B combined

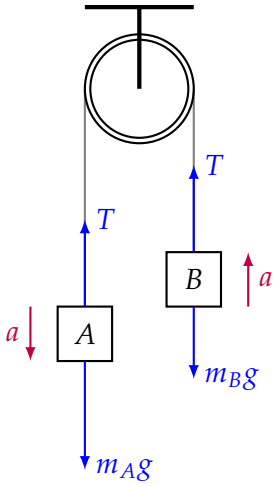
Example 3.14 Two boxes A and B are placed on a smooth surface. They are accelerated together by a horizontal force F as shown. Find the acceleration and the contact force between them.

For A: $F - R = M_A a$. For B: $R = M_B a$. For the system: $F = (M_A + M_B)a$.
$a = \dfrac{F}{M_A + M_B}$ and the contact force $R = M_B a = \dfrac{M_B}{M_A + M_B} F$.

Example 3.15 A vehicle of mass 1500 kg is towing a trailer of mass 500 kg by a light inextensible tow-bar. The engine of the vehicle exerts a driving force of 9600 N, and the tractor and the trailer experience resistances of 3600 N and 1800 N respectively. Find the acceleration of the vehicle and the tension in the tow-bar.

Whole system: $F_{\text{net}} = 9600 - 3600 - 1800 = 4200$ N, $m = 2000$ kg,
so $a = \dfrac{4200}{2000} = 2.1$ m s^{-2}.
Trailer: $T - 1800 = 500 \times 2.1 \quad \Rightarrow \quad T = 2850$ N.

Example 3.16 Two blocks of mass m_A and m_B ($m_A > m_B$) are joined together by a light inextensible string. The string passes over a smooth pulley as shown. The two blocks are suddenly released from rest. Find the acceleration of each block and the tension in the string.



Apply Newton's second law to each block:

$$\begin{cases} \text{for } A: & m_Ag - T &= m_Aa \\ \text{for } B: & T - m_Bg &= m_Ba \end{cases}$$

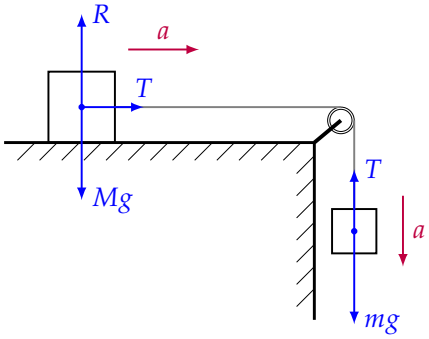
adding the two, one obtains equation of motion for whole system:

$$m_Ag - m_Bg = (m_A + m_B)a$$

solving these equations, we find

$$a = \frac{m_A - m_B}{m_A + m_B}g \qquad T = \frac{2m_Am_Bg}{m_A + m_B}$$

Example 3.17 A mass $M = 4.0$ kg is attached to a block of mass $m = 2.0$ kg through a light string which passes over a frictionless pulley as shown. When both masses are released, find the acceleration and the tension in the string.



(Smooth table.) M is pulled along the table by T; m hangs: $T = Ma$ and $mg - T = ma$. Adding: $mg = (M + m)a$.
$a = \dfrac{mg}{M+m} = \dfrac{2.0 \times 9.81}{6.0} \approx 3.3$ m s^{-2} and $T = Ma = 4.0 \times 3.27 \approx 13$ N.

Practice questions

Question 3.1 A little girl tries to lift a luggage bag of mass 25 kg. She pulls upwards with a force of 150 N. The bag does not move. What is the normal reaction from the floor?

The bag is in equilibrium: $R + 150 = mg = 25 \times 9.81 = 245$ N, so $R \approx 95$ N.

Question 3.2 A worker is pulling a wagon of mass of 40 kg across a lawn at a constant velocity. He applies a force of 200 N at an angle of 15° above the horizontal. (a) Draw a free-body diagram for the wagon. (b) Find the frictional force. (c) Find the normal contact force.

(a) Forces on the wagon: weight mg down, normal contact R up, the pull 200 N at 15° above horizontal (forwards) and friction f backwards.
Constant velocity, so the forces balance. (b) $f = 200\cos 15° \approx 193$ N.
(c) $R + 200\sin 15° = mg \quad \Rightarrow \quad R = 40 \times 9.81 - 52 \approx 340$ N.

Question 3.3 (a) Forces of 3.0 N and 4.0 N act at right angles upon a mass of 160 g. What is the acceleration produced? (b) If the angle between the two forces are allowed to vary, what is the maximum possible acceleration they produce on the same mass? (c) What about the minimum possible acceleration?

(a) $F = \sqrt{3.0^2 + 4.0^2} = 5.0$ N, $a = \dfrac{5.0}{0.160} \approx 31$ m s^{-2}
(b) Maximum when the forces are in the same direction: $a = \dfrac{7.0}{0.160} \approx 44$ m s^{-2}
(c) Minimum when opposite: $a = \dfrac{1.0}{0.160} \approx 6.3$ m s^{-2}

Question 3.4 A rocket of mass 30,000 kg is launched vertically upwards at uniform acceleration of 1.6 m s^{-2}. What is the minimum thrust force required?

$T - mg = ma \quad \Rightarrow \quad T = m(g + a) = 30000 \times (9.81 + 1.6) \approx 3.4 \times 10^5$ N.

Question 3.5 A fire-fighter of mass 85 kg slides down a vertical pole. He descends through a distance of 6.0 m in 2.0 seconds. (a) Find the average acceleration. (b) Find the average frictional force acting on the fire-fighter.

(a) $s = \frac{1}{2}at^2 \quad \Rightarrow \quad a = \dfrac{2 \times 6.0}{2.0^2} = 3.0 \text{ m s}^{-2}$
(b) $mg - f = ma \quad \Rightarrow \quad f = 85 \times (9.81 - 3.0) \approx 5.8 \times 10^2 \text{ N}.$

Question 3.6 A girl stands onto a bathroom scale and finds the reading is 35.0 kg. She then takes the scale into a lift, what mass reading would she observe if the lift (a) is going down at a constant speed, (b) is accelerating downwards at 2.1 m s^{-2}?

The scale reads R/g.
(a) Constant speed: $R = mg$, so the reading is 35.0 kg.
(b) $mg - R = ma \quad \Rightarrow \quad R = m(g - a)$, so the reading is $35.0 \times \dfrac{9.81 - 2.1}{9.81} \approx 27.5$ kg.

Question 3.7 A small mass slides down a frictionless slope with an acceleration of 2.8 m s^{-2}. Determine the angle that the slope makes with the horizontal.

$a = g \sin\theta \quad \Rightarrow \quad \sin\theta = \dfrac{2.8}{9.81} = 0.285$, so $\theta \approx 17°$.

Question 3.8 A car of mass 1400 kg is moving up a slope at a constant velocity of 13.5 m s^{-1}. The slope makes an angle of 6.0° to the horizontal. Total resistive force of 650 N acts on the car. What is the driving force required to push the car up the slope?

Constant velocity, so the driving force balances the weight component down the slope and the resistance: $F = mg\sin\theta + f = 1400 \times 9.81 \times \sin 6.0° + 650 \approx 1436 + 650 \approx 2.1 \times 10^3$ N.

Question 3.9 A shopping trolley somehow loses control and runs down a straight slope from rest. The slope makes an angle of 3.0° to the horizontal. The resistive force acting on the trolley is a constant 15 N. The trolley and its contents have a total mass of 40 kg. (a) Find the acceleration of the trolley. (b) Determine the time for the trolley to travel a distance of 4.0 m along the the slope. (c) Suggest why the slope in shopping malls are not made any steeper.

(a) $mg\sin\theta - f = ma \quad \Rightarrow \quad a = \dfrac{40 \times 9.81 \times \sin 3.0^\circ - 15}{40} \approx 0.14 \text{ m s}^{-2}$

(b) $s = \frac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\dfrac{2 \times 4.0}{0.139}} \approx 7.6 \text{ s}$

(c) A steeper slope would give a larger acceleration, so a runaway trolley would reach a dangerous speed.

Question 3.10 Block A of mass 5.0 kg is connected by means of a light string to block B of mass 3.0 kg. The two blocks are placed on a horizontal table. A force of 30 N is applied to pull on block A. Given that the friction on each block is 30% of its own weight. (a) Find the acceleration of the blocks. (b) Find the tension in the string.

Total friction $= 0.30 \times (5.0 + 3.0) \times 9.81 \approx 23.5$ N.

(a) Whole system: $30 - 23.5 = (5.0 + 3.0)a \quad \Rightarrow \quad a \approx 0.81 \text{ m s}^{-2}$

(b) Block B: $T - 0.30 \times 3.0 \times 9.81 = 3.0a \quad \Rightarrow \quad T \approx 8.83 + 2.42 \approx 11$ N.