Momentum

AS Level Physics 9702

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By the end of this topic I can. . .

- define momentum and relate force to the rate of change of momentum
- define impulse and use the area under a force–time graph
- state and use conservation of momentum in one dimension
- distinguish elastic from inelastic collisions

Momentum

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Product of **mass** and **velocity**.

- unit: kg m s^{-1} (= N s)
- a **vector**: direction of the velocity
- choose a positive direction

$$p = mv$$

Force and momentum

Newton's second law

Force is the **rate of change of momentum**.

- longer impact time means smaller force
- gradient of a *p*–*t* graph = force

$$F = \frac{\Delta p}{\Delta t}$$

Example: ball hitting a wall

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change in momentum: $\Delta p = mv - mu = 0.12 \times 10 - 0.12 \times (-10) = 2.4$ kg m s^{-1}
average force: $F = \frac{\Delta p}{\Delta t} = \frac{2.4}{2.5 \times 10^{-3}} = 96$ N

Example: water from a hose

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$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = \frac{\Delta m(v-0)}{\Delta t} = \frac{45 \times 16}{60} \quad \Rightarrow \quad F = 12\ \text{N}$$

Example: wind on a wall

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$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = \frac{\Delta m(v-0)}{\Delta t} \overset{m=\rho V}{=} \frac{\rho \Delta V v}{\Delta t} \overset{V=AL}{=} \frac{\rho A \Delta L v}{\Delta t} \overset{\Delta L = v\Delta t}{=} \rho A v^2$$

$$\Rightarrow \quad F = 1.2 \times 10 \times 30^2 = 10800 \text{ N}$$

Impulse

Impulse

Force × time, equal to the **change in momentum**.

- area under a force–time graph

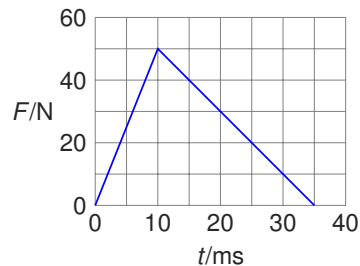
$$J = F\Delta t = \Delta p$$

Example: variable force

An object of mass 70 g is initially at rest. A force that varies with time is exerted on the object. The graph shows the how the force varies during the time of impact. What is the final velocity of the object?

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area under F-t graph gives change in momentum

$$\Delta p = \frac{1}{2} \times 50 \times 35 \times 10^{-3} = 0.875 \text{ kg m s}^{-1}$$

final velocity: $v = \frac{\Delta p}{m} = \frac{0.875}{70 \times 10^{-3}} = 12.5 \text{ m s}^{-1}$

Conservation of momentum

Principle

The total momentum of a **closed system** remains constant.

- no resultant external force
- from Newton's third law

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Example: alpha decay

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(a) No external force, so momentum is conserved. Initial momentum is zero, so the two products carry equal and opposite momenta: $m_\alpha v_\alpha = m_{\mathrm{Th}} v_{\mathrm{Th}} \quad \Rightarrow \quad \dfrac{v_\alpha}{v_{\mathrm{Th}}} = \dfrac{234}{4} = 58.5.$

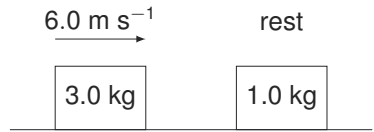
(b) The total momentum must stay zero, so the momenta must be equal in size and opposite in direction.

Example: head-on collision

A 3.0 kg mass moving at 6.0 m s^{-1} has a head-on collision with a stationary 1.0 kg mass. The two masses stick together on impact. What is the final velocity of the two masses?

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$$m_1 u_1 + \cancel{m_2 u_2}^{\,0} = (m_1 + m_2)v \quad \Rightarrow \quad 3.0 \times 6.0 = (3.0 + 1.0) \times v$$

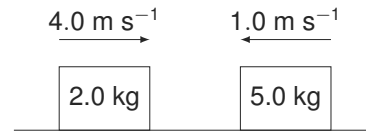
$$\Rightarrow \quad v = 4.5 \text{ m s}^{-1}$$

Example: two masses

A 2.0 kg mass moving at 4.0 m s^{-1} collides head on with a 5.0 kg mass moving at 1.0 m s^{-1}. After the collision, speed of the 5.0 kg mass is unchanged but its direction is reversed. What is the velocity of the 2.0 kg mass after the collision?

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$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow \; 2.0\times 4.0 + 5.0\times(-1.0) = 2.0\times v_1 + 5.0\times 1.0 \;\Rightarrow\; v_1 = -1.0 \text{ m s}^{-1}$$

minus sign means the 2.0 kg mass reverses direction after collision

Elastic and inelastic

Collisions

Elastic: kinetic energy conserved. **Inelastic**: some kinetic energy transferred to other forms.

- momentum conserved in both
- elastic: speed of approach = speed of separation

$$u_1 - u_2 = v_2 - v_1$$

Example: identical spheres

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Momentum: $mv = mv_1 + mv_2 \quad \Rightarrow \quad v = v_1 + v_2$. Elastic: relative speeds, $v = v_2 - v_1$. Adding: $2v = 2v_2$, so $v_2 = v$ and $v_1 = 0$. The first sphere stops and the second moves off with the original velocity.

Example: elastic or not?

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(a) Take right as positive: $4.2 \times 5.40 + 1.5 \times (-1.80) = 4.2v_A + 1.5 \times 4.50 \quad \Rightarrow$
$19.98 = 4.2v_A + 6.75 \quad \Rightarrow \quad v_A = +3.15 \text{ m s}^{-1}$ (to the right).
(b) Before: $E_\text{k} = \frac{1}{2}(4.2)(5.40)^2 + \frac{1}{2}(1.5)(1.80)^2 = 63.7$ J. After:
$\frac{1}{2}(4.2)(3.15)^2 + \frac{1}{2}(1.5)(4.50)^2 = 36.0$ J. Kinetic energy has been lost, so the collision is inelastic.

Key equations

momentum $p = mv$

force $F = \dfrac{\Delta p}{\Delta t}$

impulse $J = F\Delta t = \Delta p$

conservation $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$