Momentum

Name _____ Date _____

By the end of this topic I can…

- define momentum and relate force to the rate of change of momentum
- define impulse and use the area under a force–time graph
- state and use the principle of conservation of momentum in one dimension
- distinguish elastic from inelastic collisions

Momentum and force

Momentum

Momentum is the product of mass and velocity:

$$p = mv \qquad [p] = \text{kg m s}^{-1} = \text{N s}$$

- momentum is a vector, in the same direction as the velocity
- to add momenta or find a change in momentum, keep track of directions (take one direction as positive)

Newton's second law and momentum

For a constant force: $F = ma = \dfrac{m\Delta v}{\Delta t}$, so

$$F = \frac{\Delta p}{\Delta t}$$

Force is the rate of change of momentum; the resultant force is in the direction of the change in momentum.

- this form allows for a changing mass (water from a hose, wind on a wall)
- a longer time of impact means a smaller force for the same change in momentum (crumple zones, bending your knees)
- the gradient of a momentum–time graph is the force

Example 5.1 A ball of 120 g strikes a wall at right angle with a speed of 10 m s^{-1}. It rebounds with the same speed. If the time of impact is 25 ms, find the average force exerted on the ball.

change in momentum: $\Delta p = mv - mu = 0.12 \times 10 - 0.12 \times (-10) = 2.4$ kg m s^{-1}
average force: $F = \frac{\Delta p}{\Delta t} = \frac{2.4}{2.5 \times 10^{-3}} = 96$ N

Example 5.2 Water is pumped through a hose-pipe. A man is holding the hose-pipe horizontally and water emerges from the hose-pipe with a speed of 16 m s^{-1} at a rate of 45 kg per minute. Find the force required from this man to hold steady the hose-pipe.

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = \frac{\Delta m(v-0)}{\Delta t} = \frac{45 \times 16}{60} \quad \Rightarrow \quad F = 12 \text{ N}$$

Example 5.3 A strong wind of speed 30 m s^{-1} blows against a wall of area 10 m^2 at right angles. The density of the air is 1.2 kg m^{-3}. Assume air speed reduces to zero when it hits the wall. What is the approximate force exerted by the air on the wall?

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = \frac{\Delta m(v-0)}{\Delta t} \overset{m=\rho V}{=} \frac{\rho \Delta V v}{\Delta t} \overset{V=AL}{=} \frac{\rho A \Delta L v}{\Delta t} \overset{\Delta L = v\Delta t}{=} \rho A v^2$$

$$\Rightarrow \quad F = 1.2 \times 10 \times 30^2 = 10800 \text{ N}$$

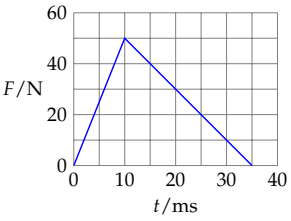
Impulse

Impulse is the force multiplied by the time for which it acts, and equals the change in momentum:

$$J = F\Delta t = \Delta p$$

• the area under a force–time graph is the impulse (the change in momentum)

Example 5.4 An object of mass 70 g is initially at rest. A force that varies with time is exerted on the object. The graph shows the how the force varies during the time of impact. What is the final velocity of the object?



area under F-t graph gives change in momentum

$$\Delta p = \tfrac{1}{2} \times 50 \times 35 \times 10^{-3} = 0.875 \text{ kg m s}^{-1}$$

final velocity: $v = \frac{\Delta p}{m} = \frac{0.875}{70 \times 10^{-3}} = 12.5 \text{ m s}^{-1}$

Conservation of momentum

The **principle of conservation of momentum**: the total momentum of a closed system (no resultant external force) remains constant.

Why: by Newton's third law the forces two bodies exert on each other are equal and opposite and act for the same time, so the changes in their momenta cancel.

Example 5.5 A uranium-238 nucleus (initially at rest) disintegrates, emitting an α-particle of mass 4u and leaving a thorium-234 nucleus of mass 234u. (a) Find the ratio v_α / v_{Th}. (b) Explain why they move in opposite directions.

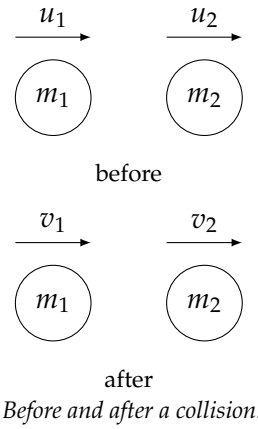
(a) No external force, so momentum is conserved. Initial momentum is zero, so the two products carry equal and opposite momenta: $m_\alpha v_\alpha = m_{\text{Th}} v_{\text{Th}} \Rightarrow \frac{v_\alpha}{v_{\text{Th}}} = \frac{234}{4} = 58.5.$

(b) The total momentum must stay zero, so the momenta must be equal in size and opposite in direction.

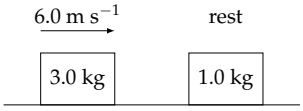
Collisions in one dimension

During a collision external forces are negligible, so total momentum is conserved. Take right as positive:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

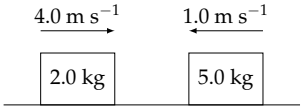


Before and after a collision.



Example 5.6 A 3.0 kg mass moving at 6.0 m s^{-1} has a head-on collision with a stationary 1.0 kg mass. The two masses stick together on impact. What is the final velocity of the two masses?

$$m_1u_1 + \cancel{m_2u_2}^{0} = (m_1 + m_2)v \quad \Rightarrow \quad 3.0 \times 6.0 = (3.0 + 1.0) \times v$$
$$\Rightarrow \quad v = 4.5 \text{ m s}^{-1}$$



Example 5.7 A 2.0 kg mass moving at 4.0 m s^{-1} collides head on with a 5.0 kg mass moving at 1.0 m s^{-1}. After the collision, speed of the 5.0 kg mass is unchanged but its direction is reversed. What is the velocity of the 2.0 kg mass after the collision?

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$\Rightarrow \; 2.0 \times 4.0 + 5.0 \times (-1.0) = 2.0 \times v_1 + 5.0 \times 1.0 \; \Rightarrow \; v_1 = -1.0 \text{ m s}^{-1}$$

minus sign means the 2.0 kg mass reverses direction after collision

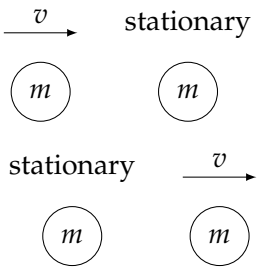
Elastic and inelastic collisions

In an **elastic collision** the total kinetic energy is conserved. In an **inelastic collision** some kinetic energy is transferred to other forms (such as thermal energy); total momentum is conserved in both.

- for an elastic collision the relative speed of approach equals the relative speed of separation:

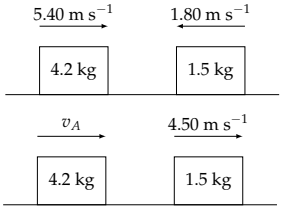
$$u_1 - u_2 = v_2 - v_1$$

- kinetic energy: $E_k = \frac{1}{2}mv^2 = \dfrac{p^2}{2m}$
- in a perfectly inelastic collision the bodies stick together



Example 5.8 A sphere of mass m moves on a smooth horizontal surface at speed v and collides elastically and head-on with an identical sphere at rest. Find the final velocities.

Momentum: $mv = mv_1 + mv_2 \quad \Rightarrow \quad v = v_1 + v_2$. Elastic: relative speeds, $v = v_2 - v_1$.
Adding: $2v = 2v_2$, so $v_2 = v$ and $v_1 = 0$. The first sphere stops and the second moves off with the original velocity.



Example 5.9 A 4.2 kg mass A and a 1.5 kg mass B move towards each other on a frictionless plane at 5.40 m s^{-1} and 1.80 m s^{-1}. After the collision B moves to the right at 4.50 m s^{-1}. (a) Find the velocity of A after the collision. (b) Is the collision elastic?

(a) Take right as positive: $4.2 \times 5.40 + 1.5 \times (-1.80) = 4.2v_A + 1.5 \times 4.50 \quad \Rightarrow$
$19.98 = 4.2v_A + 6.75 \quad \Rightarrow \quad v_A = +3.15$ m s^{-1} (to the right).
(b) Before: $E_k = \frac{1}{2}(4.2)(5.40)^2 + \frac{1}{2}(1.5)(1.80)^2 = 63.7$ J. After: $\frac{1}{2}(4.2)(3.15)^2 + \frac{1}{2}(1.5)(4.50)^2 = 36.0$ J. Kinetic energy has been lost, so the collision is inelastic.

Example 5.10 A ball of mass $m_1 = 1.0$ kg travelling with a speed of $u = 6.0$ m s^{-1} in the x-direction strikes a stationary ball of mass $m_2 = 2.0$ kg. The direction of the balls' velocities v_1 and v_2 after the collision are shown in the diagram. Find v_1 and v_2.

Start with momentum conservation equations:

$$\begin{cases} m_1 u = m_1 v_1 \cos 60^\circ + m_2 v_2 \cos 30^\circ \\ 0 = m_1 v_1 \sin 60^\circ - m_2 v_2 \sin 30^\circ \end{cases}$$

$$\begin{cases} 1.0 \times 6.0 = 1.0 \times v_1 \times \frac{1}{2} + 2.0 \times v_2 \times \frac{\sqrt{3}}{2} \\ 0 = 1.0 \times v_1 \times \frac{\sqrt{3}}{2} - 2.0 \times v_2 \times \frac{1}{2} \end{cases}$$

Simplify and solve the equations:

$$\begin{cases} \frac{1}{2} v_1 + \sqrt{3} v_2 = 6 \\ v_2 = \frac{\sqrt{3}}{2} v_1 \end{cases} \Rightarrow \begin{cases} v_1 = 3.0 \text{ m s}^{-1} \\ v_2 \approx 2.6 \text{ m s}^{-1} \end{cases}$$

One can also draw and use the vector triangle for this question, this happens to be a right-angled triangle, so things become much easier.

$$\begin{cases} m_1 v_1 = m_1 u \cos 60^\circ \\ m_2 v_2 = m_1 u \cos 30^\circ \end{cases} \Rightarrow \begin{cases} 1.0 \times v_1 = 1.0 \times 6.0 \times \frac{1}{2} \\ 2.0 \times v_2 = 1.0 \times 6.0 \times \frac{\sqrt{3}}{2} \end{cases} \Rightarrow$$

$$\begin{cases} v_1 = 3.0 \text{ m s}^{-1} \\ v_2 \approx 2.6 \text{ m s}^{-1} \end{cases}$$

Practice questions

Question 5.1 A tennis ball of mass 57 g arrives horizontally at 30 m s^{-1} and rebounds in the opposite direction at 20 m s^{-1}. The contact lasts 8.0 ms. Find the average force on the ball.

Take the rebound direction as positive: $\Delta p = 0.057 \times 20 - 0.057 \times (-30) = 2.85$ N s.

$$F = \frac{\Delta p}{\Delta t} = \frac{2.85}{8.0 \times 10^{-3}} \approx 3.6 \times 10^2 \text{ N}.$$

Question 5.2 A 0.50 kg trolley moving at 2.0 m s^{-1} collides with a stationary 1.5 kg trolley and they stick together. Find (a) their common velocity, (b) the kinetic energy lost.

(a) $0.50 \times 2.0 = (0.50 + 1.5)v \quad \Rightarrow \quad v = 0.50$ m s^{-1}
(b) Before: $\frac{1}{2}(0.50)(2.0)^2 = 1.0$ J. After: $\frac{1}{2}(2.0)(0.50)^2 = 0.25$ J. Lost: 0.75 J.

Question 5.3 When speed cars run out of control in a racing game, why are they stopped by haystacks instead of concrete walls? When you jump from an elevated position and land on the ground, you naturally bend your knees instead of keeping your legs stiff. How does that reduce the chance of causing harmful injuries?

A haystack takes longer to stop the car than a concrete wall does. The change in momentum is the same, so a longer collision time gives a smaller average force: $F = \Delta p / \Delta t$.

Question 5.4 Automobiles were manufactured to be as rigid as possible, but nowadays many cars are designed to crumple upon impact. Can you explain why?

Crumpling lengthens the time over which the car stops. The change in momentum of the occupants is the same, so the average force on them is smaller. The crumple zone also absorbs kinetic energy by deforming.

Question 5.5 When a piece of putty falls and hits the floor without bouncing, what becomes of its momentum before impact? What becomes of its kinetic energy?

Its momentum is transferred to the Earth: the putty and the Earth exert equal and opposite impulses on each other. The Earth's velocity change is immeasurably small because its mass is huge, so momentum of the putty–Earth system is conserved. The kinetic energy becomes thermal energy.