Medical Imaging

A Level Physics 9702

J. Rowing • Vijay International School Praslin

By the end of this topic I can...

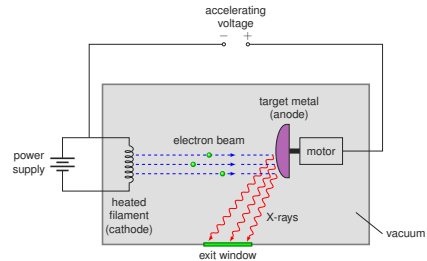
- explain the production and spectrum of X-rays
- use $I = I_0 e^{-\mu x}$ and describe X-ray and CT imaging
- explain ultrasound production, impedance and reflection
- describe nuclear magnetic resonance and MRI

X-rays

- short-wavelength electromagnetic radiation
- **hard** X-rays: more penetrating, form the image
- **soft** X-rays: absorbed by the body, filtered out (aluminium)
- X-rays from **electrons**, γ-rays from **nuclei**

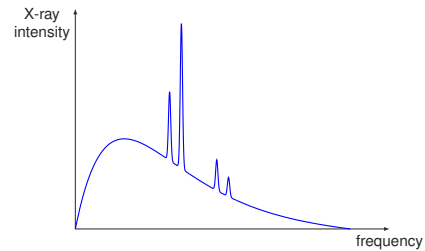
The X-ray tube

- heated cathode emits electrons; accelerated by high p.d.
- electrons strike the **target** anode
- V_{ac} sets photon energy
- **filament current** sets intensity
- most energy becomes heat: rotating anode



X-ray spectrum

- **braking radiation**: continuous background
- **characteristic radiation**: discrete lines, set by the target
- maximum frequency: $eV_{ac} = hf_{max}$



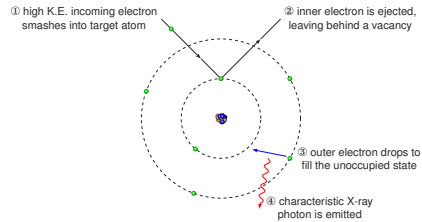
Example: maximum frequency

Example: maximum frequency

$$eV_{\text{ac}} = hf_{\text{max}}, \text{ so } f_{\text{max}} = \frac{eV_{\text{ac}}}{h} = \frac{1.60 \times 10^{-19} \times 5000}{6.63 \times 10^{-34}} \approx 1.2 \times 10^{18} \text{ Hz}$$

Characteristic radiation

- electron knocks out an **inner** electron
- outer electron drops to fill the vacancy
- photon emitted: discrete energy levels give discrete lines



X-ray attenuation

For a **parallel** beam:

$$I = I_0 \mathrm{e}^{-\mu x}$$

$$x_{1/2} = \frac{\ln 2}{\mu}$$

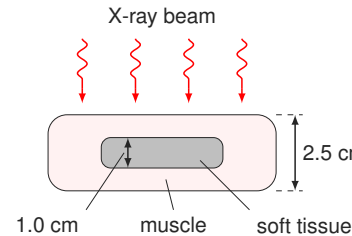
- μ: linear attenuation constant
- bone has a much larger μ than tissue
- μ is smaller for harder X-rays

Example: bone and tissue

A parallel beam of 30 keV X-rays of intensity I_0 passes through a specimen 2.5 cm thick, which has soft tissue ($\mu = 0.40$ cm^{-1}) and, in the middle, a bone 1.0 cm thick ($\mu = 2.55$ cm^{-1}). Find the emergent intensity if the beam passes through (a) soft tissue alone, (b) bone and tissue.

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(a) $I = I_0 \mathrm{e}^{-0.40\times2.5} \approx 0.37 I_0$

(b) tissue thickness $= 2.5 - 1.0 = 1.5$ cm:

$I = I_0 \, \mathrm{e}^{-0.40\times1.5} \, \mathrm{e}^{-2.55\times1.0} \approx 0.043 I_0$

The very different intensities give a clear difference between bone and tissue on the image.

Example: copper filter

Example: copper filter

$I/I_0 = 0.10 = \mathrm{e}^{-\mu x}$, so $x = \dfrac{\ln 10}{0.41} = 5.6$ mm

Image quality

- **sharpness**: narrow parallel beam, collimation, anti-scatter screen
- **contrast**: large difference in μ; contrast media (iodine, barium)
- **dose**: ionising, so image intensifier reduces exposure

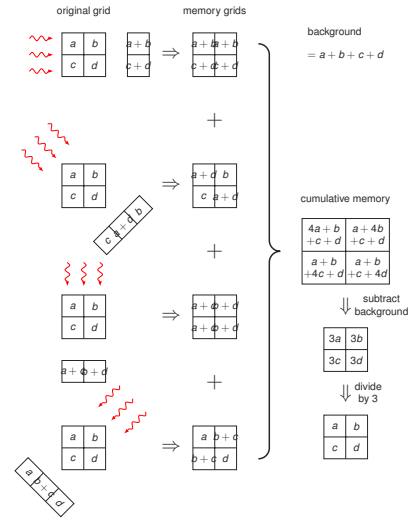


CT scans

- body treated as 2D **slices** of **voxels**
- exposed from all directions; detectors opposite
- computer builds slices, combined to a 3D image
- expensive, higher dose

CT toy model

- four directions, readings added
- cumulative $= 3 \times$ voxel $+$ background
- subtract background, divide by 3



Example: CT pixels

Example: CT pixels

(a) Background $= 19$. Subtract it and divide by 3: top-left $(34 - 19)/3 = 5$, top-right $(28 - 19)/3 = 3$, bottom-left $(40 - 19)/3 = 7$, bottom-right $(31 - 19)/3 = 4$. Check: $5 + 3 + 7 + 4 = 19$.
(b) Pixels: $\begin{smallmatrix} 5 & 3 \\ 7 & 4 \end{smallmatrix}$.
from the left (horizontal beam): rows $5 + 3 = 8$ and $7 + 4 = 11$
from above (vertical beam): columns $5 + 7 = 12$ and $3 + 4 = 7$
from the upper left (diagonal): 3, $5 + 4 = 9$, 7
from the upper right (diagonal): 5, $3 + 7 = 10$, 4

Ultrasound

Sound with frequency above 20 kHz:

$$v = f\lambda$$

$$I = I_0 \mathrm{e}^{-\alpha x}$$

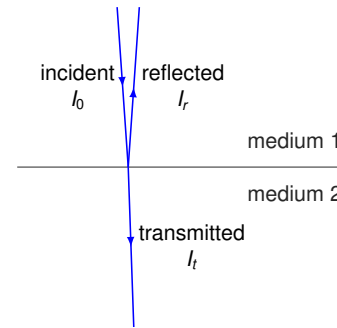
- higher f: shorter λ, better resolution
- piezo-electric crystal, thickness $d = \lambda/2$
- damping block stops vibration before echo returns

Reflection at a boundary

Normal incidence, acoustic impedance $Z = \rho c$:

$$\frac{I_r}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

$$Z = \rho c$$



- large Z difference: strong reflection
- gel matches impedance of skin
- air and bone: poor imaging

Example: air to tissue

Example: air to tissue

$Z_{\text{air}} = 1.29 \times 330 = 4.26 \times 10^2, \quad Z_{\text{tissue}} = 1060 \times 1540 = 1.63 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}$

$$\frac{I_r}{I_0} = \left(\frac{4.26 \times 10^2 - 1.63 \times 10^6}{4.26 \times 10^2 + 1.63 \times 10^6} \right)^2 \approx 99.9\%$$

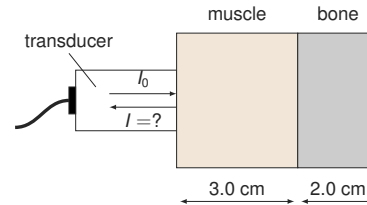
Almost all is reflected. A gel with impedance close to that of tissue is put between transducer and skin (impedance matching).

Example: muscle and bone

An ultrasonic pulse of intensity I_0 enters a layer of muscle 3.0 cm thick, then reaches a muscle-bone boundary (bone 2.0 cm thick). Muscle: $c = 1600$ m s^{-1}, $Z = 1.7 \times 10^6$, $\mu = 23$ m^{-1}. Bone: $c = 4000$ m s^{-1}, $Z = 6.4 \times 10^6$, $\mu = 130$ m^{-1} (Z in kg m^{-2} s^{-1}). (a) Find the intensity of the echo back at the muscle surface. (b) Find its arrival time.

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(a) The pulse crosses the muscle twice (2×0.030 m), and is reflected once:

$$I = I_0 \times \mathrm{e}^{-23 \times 2 \times 0.030} \times \left(\frac{6.4 - 1.7}{6.4 + 1.7}\right)^2 \approx$$

$$I_0 \times 0.252 \times 0.337 \approx 0.085 I_0$$

(b) $$\Delta t = \frac{2x_\mathrm{m}}{c_\mathrm{m}} = \frac{2 \times 0.030}{1600} \approx 3.75 \times 10^{-5} \text{ s}$$

Ultrasound imaging

- pulses sent in, **echoes** detected by the same transducer
- **arrival time**: depth of structures
- **intensity**: nature of structures
- safe, real-time, cheap, portable
- poor behind bone or air

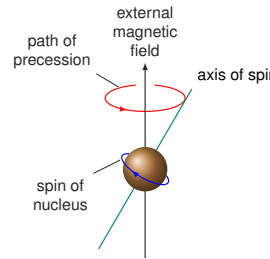
Larmor frequency

Larmor frequency

Nuclei in a field B_0 **precess** about the field direction.

- γ: gyromagnetic ratio
- resonance: RF pulse at the Larmor frequency
- low-energy nuclei absorb photons and flip

$$\omega_0 = \gamma B_0$$



Example: resonance of hydrogen

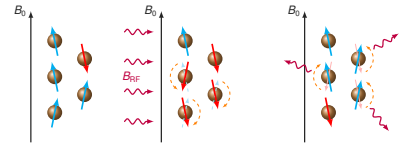
Example: resonance of hydrogen

$$\omega_0 = \gamma B_0 = 2.68 \times 10^8 \times 1.8 = 4.82 \times 10^8 \text{ rad s}^{-1}$$

$$f = \frac{\omega_0}{2\pi} = \frac{4.82 \times 10^8}{2\pi} \approx 7.7 \times 10^7 \text{ Hz: a radio frequency}$$

Relaxation

- pulse off: nuclei flip back, emit **RF photons**
- spin-lattice (T_1) and spin-spin (T_2) relaxation
- relaxation times identify the **tissue**



MRI scanner

- **superconducting magnet**: strong uniform B_0
- **gradient coils**: frequency depends on position
- **RF coils**: transmit pulses, detect signals
- computer builds the image
- no known hazard; no metal implants; expensive

Key equations

max frequency	$eV_{\text{ac}} = hf_{\text{max}}$	impedance	$Z = \rho c$
attenuation	$I = I_0 \mathrm{e}^{-\mu x}$	reflection	$\frac{I_r}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$
half-value	$x_{1/2} = \frac{\ln 2}{\mu}$	Larmor	$\omega_0 = \gamma B_0$