Medical Imaging

Name _____ Date _____

By the end of this topic I can...

- explain how X-rays are produced, and use $I = I_0 e^{-\mu x}$
- describe X-ray imaging and CT scanning
- explain ultrasound production, impedance and reflection
- describe nuclear magnetic resonance and MRI

Diagnosis without cutting the patient open needs ___non-invasive___ techniques: ___X-ray/CT___, ___ultrasound___, PET scans and ___MRI___.

X-ray imaging

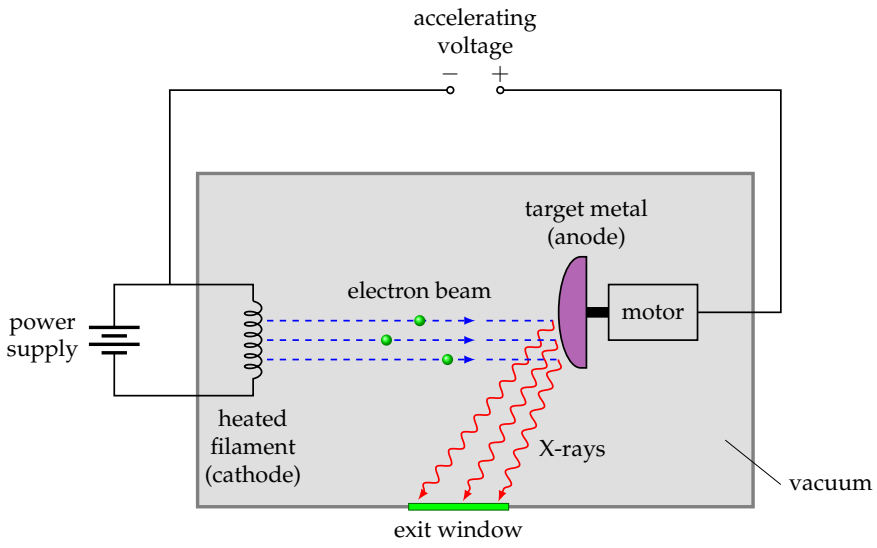
Nature of X-rays

• X-rays are ___electromagnetic radiation___ of short wavelength ($\sim 10^{-12}$ to 10^{-8} m), so high frequency

• ___hardness___ is the penetrating ability: ___hard___ X-rays have higher f, shorter λ, more energy. ___Soft___ X-rays are absorbed easily by the body and do not help form the image, so an ___aluminium___ sheet filters them out.

• radiation emitted by ___electrons___ is called X-rays; radiation emitted by ___nuclei___ is called γ-rays

Production of X-rays



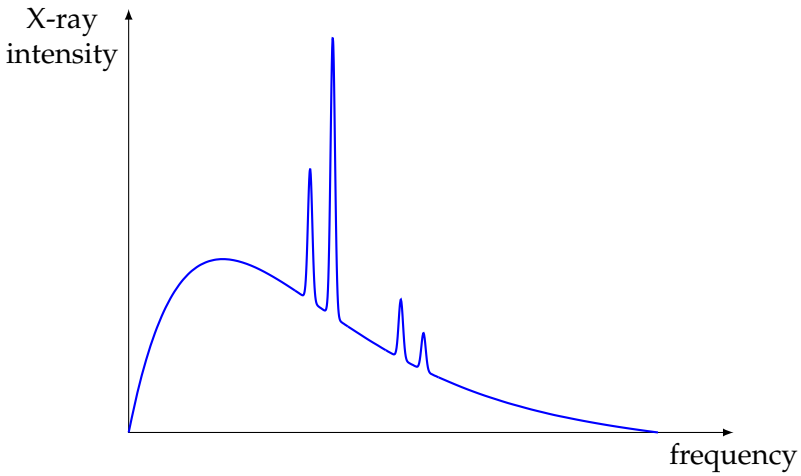
• electrons are pulled from a heated ___cathode___ and accelerated through a high p.d. towards the target ___anode___ (tungsten, etc.)

• the electrons' kinetic energy is converted to ___X-ray photons___, which leave through the exit window

• **accelerating voltage** V_{ac} controls the ___photon energy___: larger V_{ac}, more energetic photons

- **cathode (filament) current** controls the number of electrons, so the intensity of the beam
- only a tiny fraction of the energy becomes X-rays, most is thermal energy in the anode. The anode is rotated by a motor and cooled (e.g. by water).

X-ray spectrum



X-ray spectrum from an X-ray tube.

The spectrum has a continuous background and a few discrete spikes.

- **braking radiation** (continuous): electrons decelerate in the target and lose kinetic energy as photons. There is no constraint on the energy lost, so a range of photon energies.
- there is a maximum frequency: the photon of highest energy comes from an electron losing all its kinetic energy in one collision:

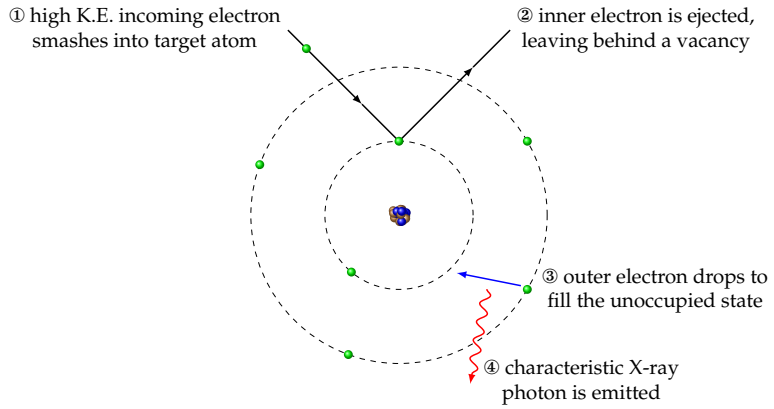
$$eV_{ac} = hf_{max}$$

Example 32.1 An X-ray tube uses an accelerating voltage of 5000 V. What is the highest frequency of X-ray produced?

$eV_{ac} = hf_{max}$, so $f_{max} = \dfrac{eV_{ac}}{h} = \dfrac{1.60 \times 10^{-19} \times 5000}{6.63 \times 10^{-34}} \approx 1.2 \times 10^{18}$ Hz

Question 32.1 Explain why the spectrum of X-rays has a minimum wavelength.

Maximum photon energy eV_{ac} means maximum frequency f_{max}, so $\lambda_{min} = \frac{c}{f_{max}} = \frac{hc}{eV_{ac}}$: no photon can have more energy than one electron's kinetic energy.



Production of characteristic radiation.

- **characteristic radiation** (spikes): an incoming electron knocks an inner-shell electron out of a target atom. An electron from a higher level drops to fill the vacancy and emits an X-ray photon.
- energy levels are discrete, so only certain frequencies are emitted. The lines depend only on the target metal.

Question 32.2 If the accelerating voltage is increased, suggest and explain the changes to the spectrum.

Larger V_{ac} gives electrons more kinetic energy, so f_{max} increases (minimum wavelength decreases) and the intensity at all frequencies rises. The characteristic lines stay at the same frequencies (they depend on the target), though new lines may appear if the electrons can now eject deeper electrons.

Question 32.3 Summarise the principles of production of X-rays.

Electrons are emitted by a heated cathode and accelerated by a high p.d. They strike a metal target, where they decelerate (braking radiation, continuous spectrum) or knock out inner electrons whose places are filled by outer electrons (characteristic lines). Most energy becomes heat.

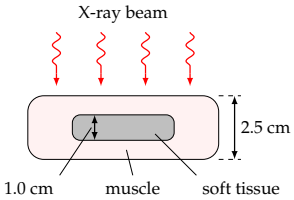
X-ray attenuation

Attenuation is the gradual decrease in intensity of an X-ray beam as it passes through a substance.

For a parallel beam: $I = I_0 e^{-\mu x}$ where μ is the linear attenuation (absorption) constant.

- μ is the probability per unit distance that a photon is absorbed, so the intensity falls exponentially (compare nuclear decay)
- a divergent beam loses intensity even without absorption, so the law applies to parallel beams only
- a high-μ substance is a good absorber: bone absorbs far more than soft tissue ($\mu_{\text{bone}} > \mu_{\text{tissue}}$)
- μ depends on the hardness: higher frequency, smaller μ
- **half-value thickness** $x_{1/2}$: thickness that halves the intensity.

$$x_{1/2} = \frac{\ln 2}{\mu}$$



Example 32.2 A parallel beam of 30 keV X-rays of intensity I_0 passes through a specimen 2.5 cm thick, which has soft tissue ($\mu = 0.40$ cm^{-1}) and, in the middle, a bone 1.0 cm thick ($\mu = 2.55$ cm^{-1}). Find the emergent intensity if the beam passes through (a) soft tissue alone, (b) bone and tissue.

(a) $I = I_0 e^{-0.40\times 2.5} \approx 0.37 I_0$
(b) tissue thickness $= 2.5 - 1.0 = 1.5$ cm:
$I = I_0\, e^{-0.40\times 1.5}\, e^{-2.55\times 1.0} \approx 0.043 I_0$
The very different intensities give a clear difference between bone and tissue on the image.

Question 32.4 Copper has $\mu = 0.41$ mm^{-1} for 100 keV X-rays. What thickness reduces the intensity of a parallel beam by 90%?

$I/I_0 = 0.10 = e^{-\mu x}$, so $x = \dfrac{\ln 10}{0.41} = 5.6$ mm

Image quality

A film or detector records the attenuated X-rays as a shadow image. Good quality needs good sharpness, good contrast and minimal dose.

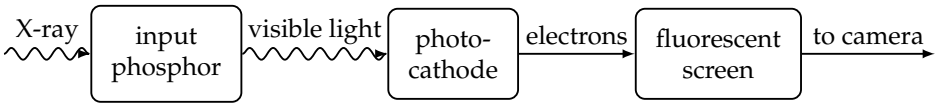
- **sharpness**: depends mainly on the width of the beam, so a narrow parallel beam is wanted. Achieve this by a small target, a small exit window, lead slits (collimation), an anti-scatter screen, and a patient who keeps still.
- **contrast**: different structures must be blackened by different amounts, so a large difference in μ. Soft tissues are all poor absorbers, so a contrast medium of high atomic number (iodine injection, barium meal) is used.

Question 32.5 The table gives μ for bone and muscle. Why is it better to use low-energy X-rays to distinguish bone from muscle?

energy/keV	bone μ/cm^{-1}	muscle μ/cm^{-1}
30	2.55	0.40
80	0.43	0.19
200	0.25	0.14
800	0.14	0.082

At 30 keV, $\mu_{\text{bone}}/\mu_{\text{muscle}} \approx 6.4$; at 800 keV it is only about 1.7. A larger difference in μ gives a larger difference in transmitted intensity, so better contrast.

• **dosage**: X-rays are ionising and harmful, so keep exposure short. An image intensifier converts X-rays to light, then to electrons, and amplifies the image so a lower dose is needed.



CT scans

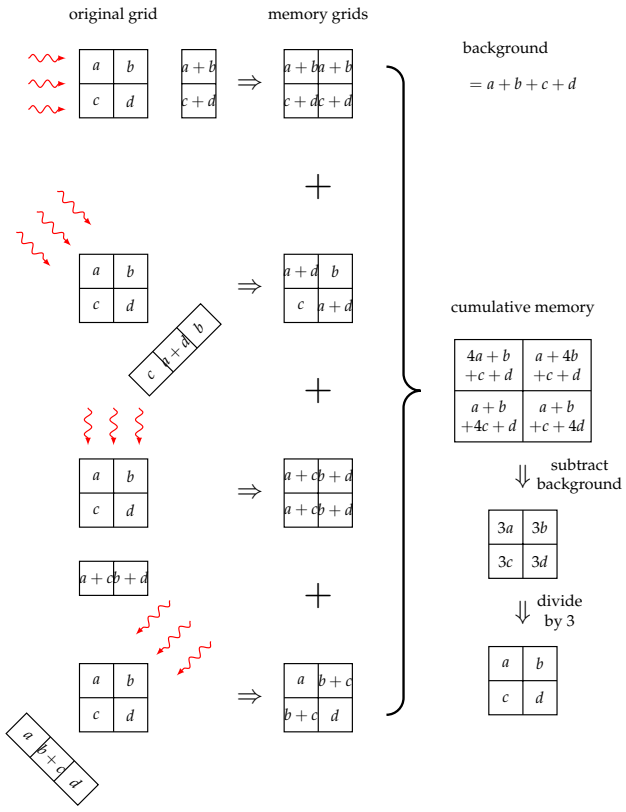
• a conventional X-ray gives a 2D shadow: no information on depth. A CT scan (computerised axial tomography) builds a 3D image.

• the body is treated as many 2D slices, each divided into a grid of voxels. The X-ray tube rotates and exposes the slice from all directions; detectors opposite measure the emergent intensities; a computer deduces each voxel's attenuation. The slice images are combined into a 3D image.

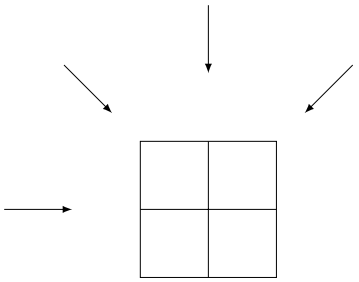
• **advantages**: shows precise position, shape and size; can distinguish tissues with similar μ

• **disadvantages**: more expensive; higher dose of radiation

Toy model: a 2×2 grid of voxels a, b, c, d is scanned from four directions. Each reading is stored in a memory grid, and the four grids are added. Each voxel then appears four times and each of the others once, so cumulative $= 3 \times (\text{voxel}) + (a + b + c + d)$. So: subtract the background $(a + b + c + d)$, then divide by 3.



Question 32.6 A 2×2 section is exposed from four directions (see figure). For any one direction, the detector readings total 19. The cumulative readings are shown. (a) Find the value of each pixel. (b) State the detector readings for each direction.



beam directions

34	28
40	31

cumulative reading

(a) Background $= 19$. Subtract it and divide by 3: top-left $(34-19)/3 = 5$, top-right $(28-19)/3 = 3$, bottom-left $(40-19)/3 = 7$, bottom-right $(31-19)/3 = 4$. Check: $5+3+7+4 = 19$.
(b) Pixels: $\begin{matrix} 5 & 3 \\ 7 & 4 \end{matrix}$.
from the left (horizontal beam): rows $5+3 = 8$ and $7+4 = 11$
from above (vertical beam): columns $5+7 = 12$ and $3+4 = 7$
from the upper left (diagonal): 3, $5+4 = 9$, 7
from the upper right (diagonal): 5, $3+7 = 10$, 4

Question 32.7 By reference to the principles of CT imaging, suggest why the dose from a CT scan is greater than from a simple X-ray image.

A CT scan takes many exposures from many directions for each of many slices, so the total radiation absorbed is much greater than for a single exposure.

Ultrasound scans

Nature of ultrasound

• sound is a longitudinal, mechanical wave: it needs a material medium. Ultrasound has $f >$ 20 000 Hz (human hearing is about 20 Hz to 20 kHz).

• for a given medium the speed v is constant, and $v = f\lambda$, so higher frequency means shorter wavelength and better resolution: fine detail can be seen

Question 32.8 Sound travels at 1600 m s^{-1} in soft tissue. Compare the wavelengths at (a) 500 Hz, (b) 10 MHz.

(a) $\lambda = \dfrac{1600}{500} = 3.2$ m (b) $\lambda = \dfrac{1600}{10 \times 10^6} = 1.6 \times 10^{-4}$ m $= 0.16$ mm
The 10 MHz wavelength is 20 000 times shorter, so it can resolve far smaller structures.

Production of ultrasound

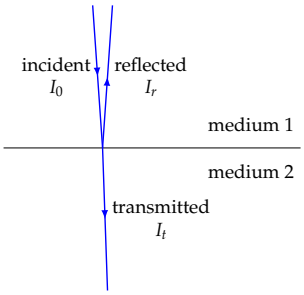
• a piezo-electric crystal vibrates when an a.c. voltage is applied, producing ultrasound with $f_{\text{sound}} = f_{\text{a.c.}}$

• the amplitude is largest (resonance) for crystal thickness $d =$ $\frac{1}{2}\lambda$

• the same transducer detects the echoes. After each pulse the vibration must stop quickly, so a damping block (epoxy resin) is used. The parts: piezo-electric crystal, acoustic window, damping block, electrodes.

Attenuation and reflection of ultrasound

• intensity falls exponentially: $I = I_0 e^{-\alpha x}$ (α is the absorption coefficient). Scans rely on reflected waves, so weak echoes from deep tissue are amplified.



• at a boundary $I_0 =$ $I_r + I_t$. For normal incidence:

$$\frac{I_r}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2 \qquad \frac{I_t}{I_0} = 1 - \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

• the acoustic impedance of a medium: $Z = \rho c$

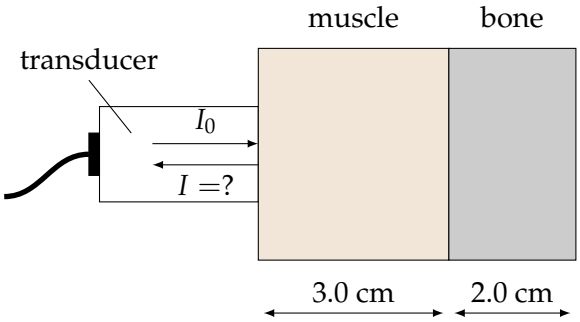
(density $\times$ speed of sound)

Example 32.3 Air: $\rho = 1.29$ kg m^{-3}, $c = 330$ m s^{-1}. Soft tissue: $\rho = 1060$ kg m^{-3}, $c = 1540$ m s^{-1}. What happens when ultrasound goes from air into the body at right angles?

$Z_{\text{air}} = 1.29 \times 330 = 4.26 \times 10^2$, $Z_{\text{tissue}} = 1060 \times 1540 = 1.63 \times 10^6$ kg m^{-2} s^{-1}

$$\frac{I_r}{I_0} = \left(\frac{4.26 \times 10^2 - 1.63 \times 10^6}{4.26 \times 10^2 + 1.63 \times 10^6}\right)^2 \approx 99.9\%$$

Almost all is reflected. A gel with impedance close to that of tissue is put between transducer and skin (impedance matching).



Example 32.4 An ultrasonic pulse of intensity I_0 enters a layer of muscle 3.0 cm thick, then reaches a muscle-bone boundary (bone 2.0 cm thick). Muscle: $c = 1600$ m s^{-1}, $Z = 1.7 \times 10^6$, $\mu = 23$ m^{-1}. Bone: $c = 4000$ m s^{-1}, $Z = 6.4 \times 10^6$, $\mu = 130$ m^{-1} (Z in kg m^{-2} s^{-1}). (a) Find the intensity of the echo back at the muscle surface. (b) Find its arrival time.

(a) The pulse crosses the muscle twice (2×0.030 m), and is reflected once:

$$I = I_0 \times e^{-23 \times 2 \times 0.030} \times \left(\frac{6.4 - 1.7}{6.4 + 1.7}\right)^2 \approx I_0 \times 0.252 \times 0.337 \approx 0.085 I_0$$

(b) $\Delta t = \dfrac{2x_{\text{m}}}{c_{\text{m}}} = \dfrac{2 \times 0.030}{1600} \approx 3.75 \times 10^{-5}$ s

Question 32.9 A gel has $Z = 1.65 \times 10^6$ kg m^{-2} s^{-1}. Using the soft-tissue value above, find the reflection coefficient from gel into the body, and hence explain why gel is used.

$\frac{I_r}{I_0} = \left(\frac{1.65 \times 10^6 - 1.63 \times 10^6}{1.65 \times 10^6 + 1.63 \times 10^6}\right)^2 \approx 3 \times 10^{-5}$ (0.003%)
Almost all is transmitted into the body, compared with 99.9% reflected from air.

Question 32.10 Is it a good idea to use ultrasound to image the lungs? Give your reasons.

No: the lungs contain air, whose impedance is very different from tissue, so almost all the ultrasound is reflected at the boundary and very little reaches (or returns from) deeper structures.

Question 32.11 In the muscle-bone example, a detector is coupled to the far side of the bone. What intensity would be measured?

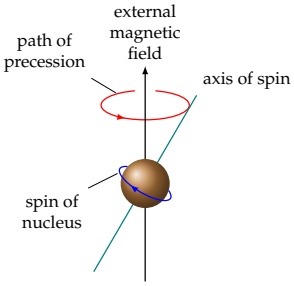
Transmitted fraction at the boundary: $1 - 0.337 = 0.663$.
$I = I_0\,\mathrm{e}^{-23\times0.030} \times 0.663 \times \mathrm{e}^{-130\times0.020} = I_0 \times 0.502 \times 0.663 \times 0.0743 \approx 0.025 I_0$

Ultrasound imaging

- pulses from a piezo-electric transducer enter the body and are reflected at boundaries. The same transducer detects the echoes, and a computer builds the image.
- arrival time gives the depth/thickness of structures; echo intensity depends on Z and attenuation, so gives the nature of structures
- **advantages**: safe (no ionising radiation, can image a fetus); real-time images; low cost; portable
- **disadvantages**: structures behind bone or air are hard to image; needs a skilled operator

MRI

Nuclear magnetic resonance



- nuclei of some atoms have a property called spin : they behave like tiny magnets
- **step 1: energy levels split.** With no field the orientations are random and have equal energy. In a strong field B_0 most nuclei line up parallel (low energy) and a few anti-parallel (high energy).
- **step 2: precession.** The spin axis is slightly tilted and precesses about the field direction at the Larmor frequency : $\omega_0 = \gamma B_0$

(γ is the gyromagnetic ratio , which depends on the nucleus)

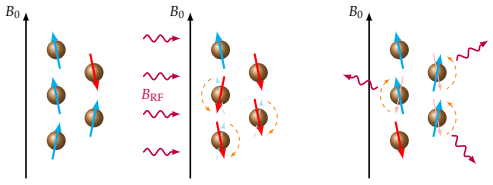
- **step 3: resonance.** Radio-frequency pulses at the Larmor frequency are applied. Low-energy nuclei absorb photons and flip to the higher state: nuclear magnetic resonance .

Example 32.5 For protons $\gamma = 2.68 \times 10^8$ rad s^{-1}T^{-1}. What frequency of pulse causes resonance of hydrogen nuclei in a field of 1.8 T?

$\omega_0 = \gamma B_0 = 2.68 \times 10^8 \times 1.8 = 4.82 \times 10^8$ rad s^{-1}

$f = \dfrac{\omega_0}{2\pi} = \dfrac{4.82 \times 10^8}{2\pi} \approx 7.7 \times 10^7$ Hz: a radio frequency

- **step 4: relaxation.** When the pulse stops the nuclei flip back and release RF photons . The relaxation time depends on the surroundings, so it identifies the type of tissue . Two processes: spin-lattice (T_1): energy passes to the surrounding lattice ; spin-spin (T_2): energy passes to other spinning nuclei.



1. aligned in B_0 2. resonance with B_{RF} 3. relaxation: RF photons emitted

Magnetic resonance imaging

• a large superconducting electromagnet gives a strong uniform field B_0; the nuclei align and precess at the Larmor frequency

• gradient coils add a non-uniform field, so nuclei at different positions have different precession frequencies: this gives the location

• RF coils transmit pulses at the Larmor frequency; other RF coils detect the signals emitted on relaxation. A computer builds the image. The relaxation time gives the nature of the tissue.

• **advantages**: radio waves, so no known biological hazard; good contrast between soft tissues; 3D images, any slice

• **disadvantages**: very expensive (superconducting coils at very low temperature); unsuitable with metal implants or pacemakers; patient must lie still; bones show up poorly

Question 32.12 State the main components of an MRI scanner and describe their functions.

Superconducting magnet: strong uniform field B_0 to align the nuclei. Gradient coils: non-uniform field so resonant frequency depends on position. RF transmitter coils: send pulses at the Larmor frequency. RF receiver coils: detect the signal emitted on relaxation. Computer: processes signals and builds the image.