Magnetic Fields

A Level Physics 9702

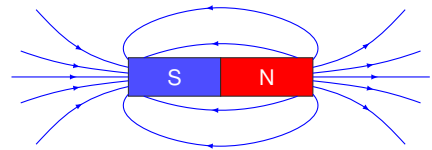
J. Rowing • Vijay International School Praslin

By the end of this topic I can...

- sketch field patterns; use the right-hand grip rule and Fleming's left-hand rule
- use $F = BIL\sin\theta$ and $F = Bqv\sin\theta$; define flux density B
- circular motion of charged particles: mass spectrometer, cyclotron, velocity selector
- the Hall effect: $V_H = BI/ntq$

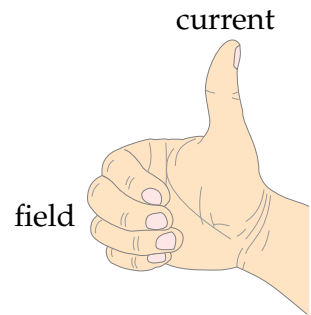
Magnets and field lines

- like poles **repel**, opposite poles **attract**
- field lines leave **N**, enter **S**
- close lines: **strong** field
- strength is the **magnetic flux density** *B*



Field due to a current

- a current produces a magnetic field (Orsted, 1820)
- **right-hand grip rule**: thumb along current, fingers curl along field
- straight wire: **concentric circles**
- $B \propto I$



Long straight wire and coil

Field of a current

A straight wire gives **circular** field lines; a long coil gives a **uniform** field inside.

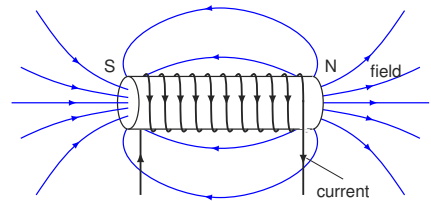
- $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}
- a **soft iron** core strengthens the field

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \frac{\mu_0 N I}{L}$$

Solenoids and electromagnets

- a solenoid's field is like a **bar magnet**
- use the grip rule to find the **N** and **S** ends
- larger *I* or more turns: **stronger** field
- iron core: **electromagnet**



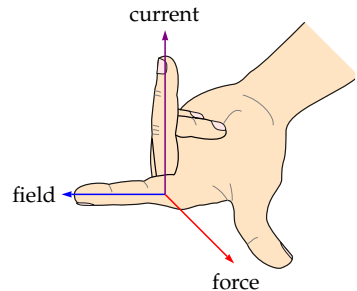
Force on a current

Force on a conductor

Fleming's **left-hand rule**: First finger **F**ield, seCond finger **C**urrent, thuMb **M**otion.

- $B \perp I$: $F = BIL$ (maximum)
- $B \parallel I$: $F = 0$
- force is perpendicular to both B and I

$$F = BIL \sin\theta$$

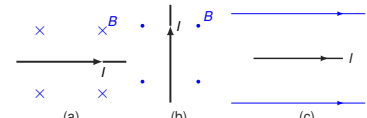


Example: direction of the force

Use the left-hand rule to find the direction of the magnetic force on each wire. In (a) B is into the page, in (b) B is out of the page, and in (c) B is to the right.

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(a) B into the page, I to the right: force is **up the page**.
(b) B out of the page, I up the page: force is **to the right**.
(c) I is parallel to B: **no force**.

Magnetic flux density

Magnetic flux density

The **force per unit length** on a conductor carrying **unit current** at **right angles** to the field.

- unit: **tesla**, 1 T = 1 N A^{-1} m^{-1}
- B is a **vector**

$$B = \frac{F}{IL \sin \theta}$$

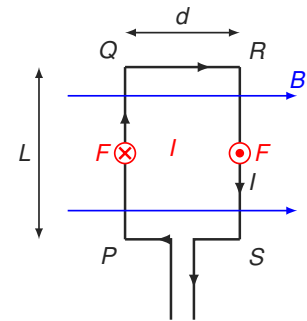
Example: the force on a frame

A rectangular frame *PQRS* carries current *I* in a uniform field *B*, with the plane of the frame *parallel* to the field. Explain why it turns.

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QR and *PS* are parallel to *B*: no force.
PQ and *RS* are perpendicular to *B*: force $F = BIL$ each. By the left-hand rule they act in **opposite directions** (into and out of the page).
An equal and opposite pair of forces is a **couple**, with torque $\tau = Fd = BILd$ about the central axis, so the frame rotates. This is the principle of the electric motor.

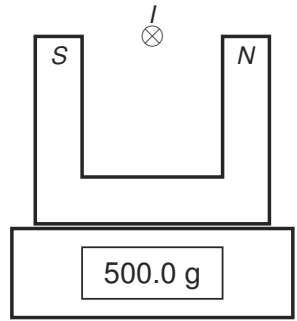


Example: a balance

A U-shaped magnet sits on a balance reading 500.0 g, with a wire above it carrying 10 A into the page. The flux density between the poles is 0.50 T and 8.0 cm of wire is in the field. What does the balance now read?

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Left-hand rule: the force on the wire is **upwards**. By Newton's third law the force on the magnet is **downwards**, so the reading increases.

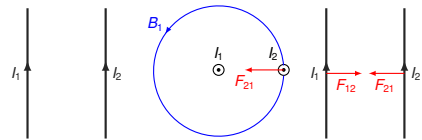
$F = BIL = 0.50 \times 10 \times 8.0 \times 10^{-2} = 0.40$ N

$\Delta m = \dfrac{F}{g} = \dfrac{0.40}{9.81} \approx 0.041$ kg $= 40.8$ g

New reading $= 500.0 + 40.8 = 540.8$ g

Parallel current-carrying wires

- wire 1's field acts on wire 2 (and vice versa)
- forces are **equal and opposite** (Newton 3)
- **parallel** currents **attract**
- **anti-parallel** currents **repel**
- this defines the **ampere**

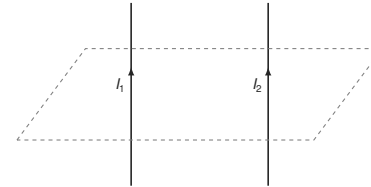


Question: two parallel wires

Two long parallel wires carry $I_1 = 3.0$ A and $I_2 = 5.0$ A in the same direction, 4.0 cm apart. (a) Draw at least three field lines due to I_1. (b) State the direction of the force on I_2. (c) Find the flux density due to I_1 at I_2. (d) Find the force per unit length on I_2. (e) Find the force per unit length on I_1.

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(a) Concentric circles around I_1 in a plane perpendicular to the wire, with direction from the grip rule, spaced further apart as r increases.

(b) I_2 is attracted **towards** I_1 (parallel currents attract).

(c) $B = \dfrac{\mu_0 I_1}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 3.0}{2\pi \times 0.040} = 1.5 \times 10^{-5}$ T

(d) $\dfrac{F}{L} = BI_2 = 1.5 \times 10^{-5} \times 5.0 = 7.5 \times 10^{-5}$ N m^{-1}

(e) The same magnitude, 7.5×10^{-5} N m^{-1}, directed

Force on a moving charge

Force on a charge

$F = Bqv \sin\theta$: perpendicular to **both** v and B.

- for $-q$ the equivalent current is **opposite** to v
- $v \parallel B$: no force
- does **no work**: changes direction, not speed

$$F = Bqv \sin\theta$$

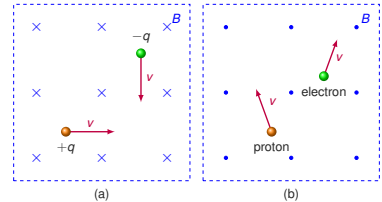
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Find the direction of the magnetic force on each moving charge. In (a) B is into the page and in (b) B is out of the page.

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(a) $+q$ moving right: force is **up the page**. $-q$ moving down: force is **to the left**.
(b) The proton moving up and slightly left (at 110°) feels a force to the right, perpendicular to v, at about 20° above the horizontal.
The electron moving up and slightly right (at 70°) is negative, so the force is reversed: to the left, at about 160°.



Electric and magnetic fields

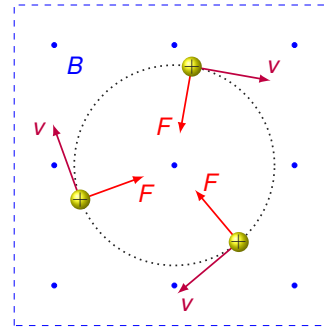
- electric: $F = Eq$, acts on **all** charges, along E
- magnetic: $F = Bqv \sin\theta$, **moving** charges only, perpendicular to B and v
- electric force **can** do work; magnetic force **cannot**

Circular motion in a field

Magnetic force provides the **centripetal force**:

$$Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq}$$



- larger v or m: larger radius
- larger B: smaller radius

Example: alpha particle

Example: alpha particle

$$r = \frac{mv}{Bq} = \frac{4 \times 1.66 \times 10^{-27} \times 2.5 \times 10^{4}}{5.4 \times 10^{-3} \times 2 \times 1.60 \times 10^{-19}} \approx 0.096 \text{ m} = 9.6 \text{ cm}$$

The period

Time for one circle:

$$T = \frac{2\pi m}{Bq}$$

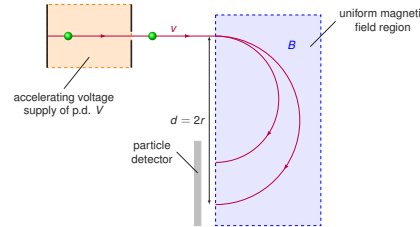
- **independent** of speed and radius
- used in the cyclotron

The mass spectrometer

Accelerate through V, then bend in a field:

$$\frac{q}{m} = \frac{2V}{B^2 r^2}$$

- different q/m gives different radius
- identifies unknown particles



Question: unknown particle

Question: unknown particle

(a) $r = 0.33$ m. $m = \dfrac{Bqr}{v} = \dfrac{8.8 \times 10^{-3} \times 1.60 \times 10^{-19} \times 0.33}{1.4 \times 10^{5}} = 3.3 \times 10^{-27}$ kg

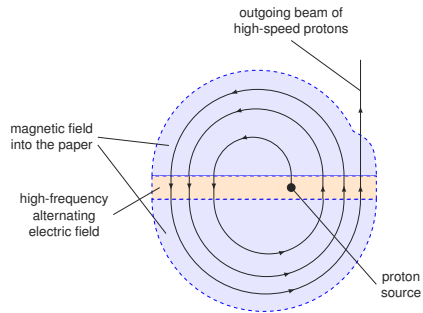
(b) $\dfrac{3.3 \times 10^{-27}}{1.66 \times 10^{-27}} \approx 2$ u with charge $+e$: a **deuteron** (nucleus of hydrogen-2: one proton and one neutron).

The cyclotron

Alternating p.d. at the cyclotron frequency:

$$f = \frac{Bq}{2\pi m}$$

- each gap crossing gives energy qV
- particles spiral outwards
- period does not depend on radius



Question: protons in a cyclotron

Question: protons in a cyclotron

(a) $f = \dfrac{Bq}{2\pi m} = \dfrac{1.5 \times 1.60 \times 10^{-19}}{2\pi \times 1.67 \times 10^{-27}} = 2.3 \times 10^{7}$ Hz $= 23$ MHz

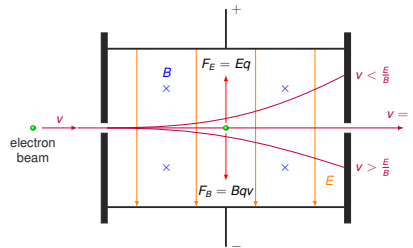
(b) Two gap crossings per circle, each adding $qV = 20$ keV: 40 keV per circle. Circles $= \dfrac{10 \times 10^{3} \text{ keV}}{40 \text{ keV}} = 250.$

The velocity selector

Undeflected when the electric and magnetic forces balance:

$$v = \frac{E}{B}$$

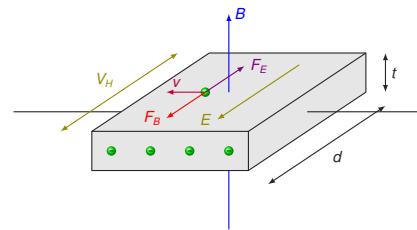
- independent of q and m
- faster or slower particles are deflected



The Hall effect

Hall voltage across a conductor in a field:

$$V_H = \frac{BI}{ntq}$$



- small n (semiconductor) and thin slice: larger V_H
- sign depends on electrons or holes
- $V_H \propto B$: the Hall probe

Key equations

wire	$B = \dfrac{\mu_0 I}{2\pi r}$	radius	$r = \dfrac{mv}{Bq}$
coil	$B = \dfrac{\mu_0 N I}{L}$	selector	$v = \dfrac{E}{B}$
force	$F = BIL\sin\theta$	Hall	$V_H = \dfrac{BI}{ntq}$
charge	$F = Bqv\sin\theta$	period	$T = \dfrac{2\pi m}{Bq}$