Magnetic Fields

Name _____ Date _____

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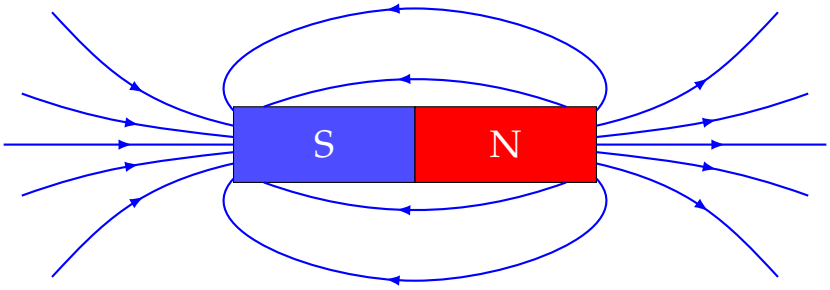
- sketch magnetic field patterns and use the right-hand grip rule and Fleming's left-hand rule
- use $F = BIL\sin\theta$ and $F = Bqv\sin\theta$, and define magnetic flux density B
- analyse circular motion of charged particles: mass spectrometer, cyclotron, velocity selector
- explain the Hall effect and use $V_H = \frac{BI}{ntq}$

Magnets and magnetic fields

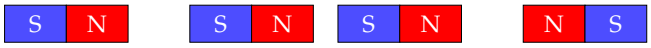
Magnets

A magnet creates a magnetic field. All magnets have a north pole and a south pole (no monopoles at A level). A freely suspended magnet's north pole points to the Earth's geographic north: a north-seeking pole.

- like poles repel; opposite poles attract
- magnetic field lines: emerge from the north pole and enter the south pole. Closely spaced lines mean a strong field. A small compass lines up along the line, its north pole pointing in the direction of the field.
- the strength of the field is the magnetic flux density, B (defined in §)



The field of a bar magnet.

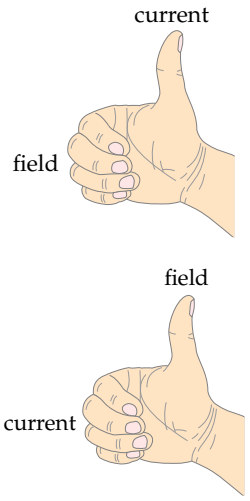


(a) attract (b) repel

Question 25.1 Two identical bar magnets are placed side by side as shown. Sketch the magnetic field lines for each case.

Lines run from N to S and never cross.
(a) Facing opposite poles: the lines go **straight across the gap**, closely spaced (strong field).
(b) Facing like poles: the lines are pushed apart and bend away; there is a point of zero field midway between.

Magnetic field due to currents



Hans Christian Orsted discovered in 1820 that a current deflects a nearby compass: a ___current___ produces a magnetic field. The direction is given by the **right-hand grip rule**.

- **straight wire**: thumb along the ___current___, fingers curl in the direction of the ___field___. Field lines are ___concentric circles___ around the wire.
- **loop or coil**: fingers curl along the ___current___, thumb points along the ___field___ inside the loop
- $B \propto I$: a larger current gives a ___stronger___ field

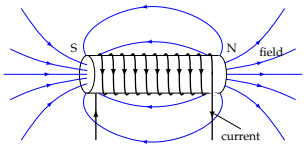
Long straight wire: $B = \dfrac{\mu_0 I}{2\pi r}$ where r is the ___perpendicular distance___ from the wire and $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1} is the ___permeability of free space___.

- at the centre of a long coil of N turns, length L:

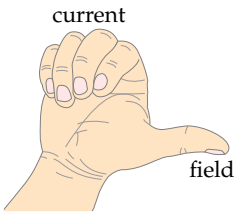
$$B = \frac{\mu_0 N I}{L}$$

- a ___soft iron___ core strengthens the field: ferromagnetic materials (iron, cobalt, nickel) concentrate the field lines. A material of relative permeability μ_r multiplies B by μ_r.

Solenoids and electromagnets



A current-carrying coil is a ___solenoid___. Its field is like that of a ___bar magnet___: use the grip rule to find which end is N and which is S. Inside, the field is ___uniform___.



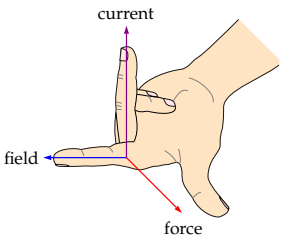
- stronger field with larger ___current___ I or more ___turns___ N per unit length
- an iron core makes an ___electromagnet___

Question 25.2 Describe the magnetic field due to an alternating current through a solenoid.

The field has the same **frequency** as the current: its strength rises and falls, and its direction (N and S poles) **reverses** every half cycle.

Magnetic force

Force on a current-carrying conductor



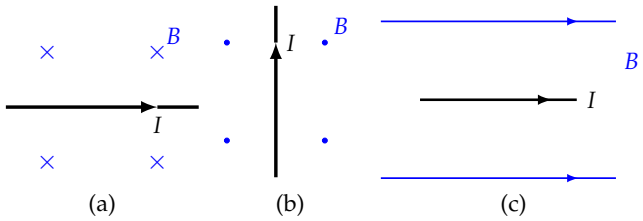
Fleming's left-hand rule.

A wire carrying current in an external field feels a ___magnetic force___, because the wire's field interacts with the external field.

- **Fleming's left-hand rule**: First finger = ___Field___, seCond finger = ___Current___, thuMb = ___Motion___ (force). The three are mutually ___perpendicular___.

Force on a conductor of length L: $F = BIL\sin\theta$ θ is the angle between ___B and the current___.

- $B \perp I$: $F =$ ___BIL___ (maximum)
- $B \parallel I$: $F =$ ___0___
- in general only the ___component of B perpendicular to the wire___ contributes, giving the $\sin\theta$. The force is perpendicular to both B and I.



Example 25.1 Use the left-hand rule to find the direction of the magnetic force on each wire. In (a) B is into the page, in (b) B is out of the page, and in (c) B is to the right.

(a) B into the page, I to the right: force is **up the page**.
(b) B out of the page, I up the page: force is **to the right**.
(c) I is parallel to B: **no force**.

Magnetic flux density

Rearranging $F = BIL\sin\theta$ gives the definition of B:

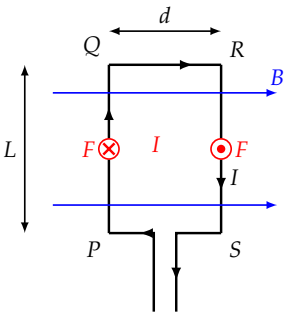
Magnetic flux density B is the force per unit length on a conductor carrying unit current at right angles to the field:

$$B = \frac{F}{IL\sin\theta}$$

- unit: the tesla (T), $1\text{ T} = 1\text{ N A}^{-1}\text{ m}^{-1}$. A 1 m wire at right angles to the field carrying 1 A feels 1 N when $B = 1$ T.
- B is a vector. (A fridge magnet is about 10^{-3} T; an MRI scanner 1–3 T.)

Example 25.2 A wire of length 0.80 m carrying 5.0 A lies at right angles to a magnetic field and experiences a force of 0.60 N. What is the flux density?

$$B = \frac{F}{IL} = \frac{0.60}{5.0 \times 0.80} = 0.15\text{ T}$$



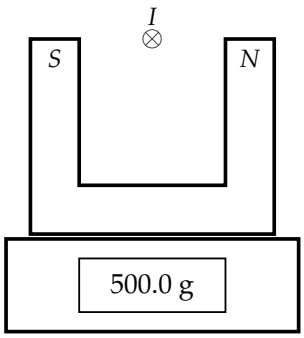
Example 25.3 A rectangular frame $PQRS$ carries current I in a uniform field B, with the plane of the frame *parallel* to the field. Explain why it turns.

QR and PS are parallel to B: no force.
PQ and RS are perpendicular to B: force $F = BIL$ each. By the left-hand rule they act in **opposite directions** (into and out of the page).
An equal and opposite pair of forces is a **couple**, with torque $\tau = Fd = BILd$ about the central axis, so the frame rotates. This is the principle of the electric motor.

Question 25.3 If the plane of the frame is at right angles to the field, describe the forces on each side and what happens to the frame.

Every side is perpendicular to B and feels BIL, but the forces lie in the plane of the frame and point radially (all outwards or all inwards).
They cancel in pairs: **no net force and no torque**. The frame does not rotate (it is only stretched or squeezed).

Example 25.4 A U-shaped magnet sits on a balance reading 500.0 g, with a wire above it carrying 10 A into the page. The flux density between the poles is 0.50 T and 8.0 cm of wire is in the field. What does the balance now read?



Left-hand rule: the force on the wire is **upwards**. By Newton's third law the force on the magnet is **downwards**, so the reading increases.

$$F = BIL = 0.50 \times 10 \times 8.0 \times 10^{-2} = 0.40 \text{ N}$$

$$\Delta m = \frac{F}{g} = \frac{0.40}{9.81} \approx 0.041 \text{ kg} = 40.8 \text{ g}$$

New reading $= 500.0 + 40.8 = 540.8$ g

Question 25.4 The current in the example above is replaced by 6.0 A flowing out of the page. What does the balance read?

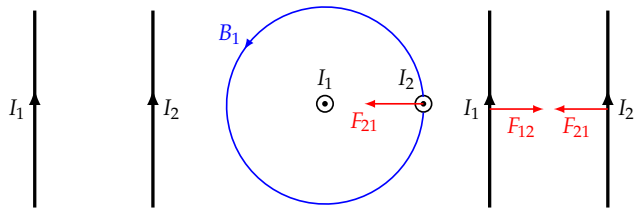
The force on the wire is now downwards, so the force on the magnet is **upwards**.
$F = 0.50 \times 6.0 \times 8.0 \times 10^{-2} = 0.24$ N, equivalent to $0.24/9.81 = 24.5$ g
New reading $= 500.0 - 24.5 = 475.5$ g

Force between parallel current-carrying wires

Wire 1 produces a field B_1 at wire 2 (use the grip rule). The left-hand rule then gives the force F_{21} on wire 2 (pointing towards wire 1 for currents in the ___same___ direction). By Newton's third law F_{12} is equal and ___opposite___.

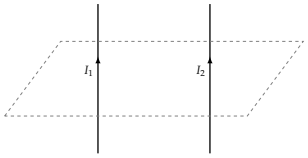
Parallel currents ___attract___; anti-parallel currents ___repel___.

• the ampere is defined using this: two long parallel wires 1 m apart each carrying 1 A experience a force of 2×10^{-7} N per metre



Question 25.5 A light coil of several loops is hung from a fixed point. A current is switched on. State and explain the change in the separation of the loops.

Neighbouring turns carry current in the **same direction**, so they **attract**: the coil contracts and the loops move closer together.



Question 25.6 Two long parallel wires carry $I_1 = 3.0$ A and $I_2 = 5.0$ A in the same direction, 4.0 cm apart. (a) Draw at least three field lines due to I_1. (b) State the direction of the force on I_2. (c) Find the flux density due to I_1 at I_2. (d) Find the force per unit length on I_2. (e) Find the force per unit length on I_1.

(a) Concentric circles around I_1 in a plane perpendicular to the wire, with direction from the grip rule, spaced further apart as r increases.
(b) I_2 is attracted **towards** I_1 (parallel currents attract).
(c) $B = \dfrac{\mu_0 I_1}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 3.0}{2\pi \times 0.040} = 1.5 \times 10^{-5}$ T
(d) $\dfrac{F}{L} = BI_2 = 1.5 \times 10^{-5} \times 5.0 = 7.5 \times 10^{-5}$ N m^{-1}
(e) The same magnitude, 7.5×10^{-5} N m^{-1}, directed **towards** I_2 (Newton's third law).

Question 25.7 If one of the wires carries an alternating current, the wires should vibrate but are not seen to move. Make reasonable estimates and explain why.

The force per metre is of order 10^{-4} N m^{-1} (for a few amperes about a centimetre or so apart), so for a 1 m wire the force is $\sim 10^{-4}$ N, tiny compared with the wire's weight and tension.
The force also alternates at 50–100 Hz, so the wire has no time to move far: the amplitude is far too small to see.

Force on a moving charged particle

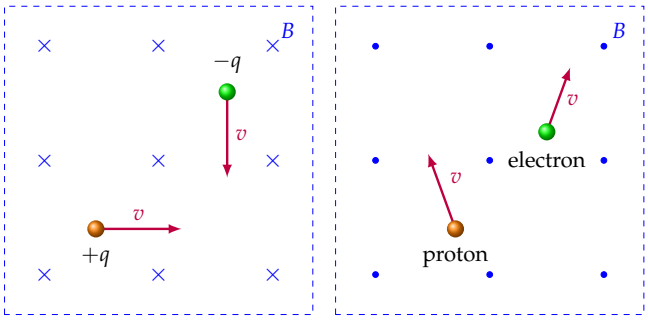
A current is moving charges. Substituting $I = nAqv$ into $F = BIL\sin\theta$, and noting that nAL is the number of charge carriers in the wire, the force on __one__ charge is

$$F = Bqv\sin\theta$$

where θ is the angle between $\vec{v}$ and $\vec{B}$.

- direction: left-hand rule. For $+q$ the current is along v; for $-q$ the current is __opposite__ to v
- $v \perp B$: $F =$ __Bqv__ ; $v \parallel B$: $F =$ __0__

- F is always perpendicular to both v and B



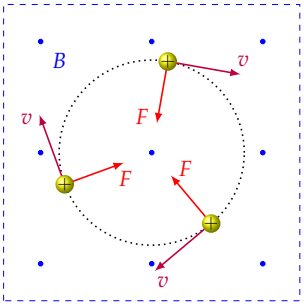
Example 25.5 Find the direction of the magnetic force on each moving charge. In (a) B is into the page and in (b) B is out of the page.

(a) $+q$ moving right: force is **up the page**. $-q$ moving down: force is **to the left**.
(b) The proton moving up and slightly left (at 110°) feels a force to the right, perpendicular to v, at about 20° above the horizontal.
The electron moving up and slightly right (at 70°) is negative, so the force is reversed: to the left, at about 160°.

Electric and magnetic fields compared

	electric field	magnetic field
field strength	E	magnetic flux density B
force	$F = Eq$	$F =$ $Bqv \sin\theta$
depends on velocity?	no: acts on stationary and moving charges	yes : only moving charges, perpendicular component
direction	along E (same for $+q$, opposite for $-q$)	perpendicular to B and v
effect on motion	changes speed and direction	changes direction only , not speed
work done	yes	none

Charged particles in magnetic fields



Circular motion in a uniform field

A particle moving at right angles to a uniform field feels a force that is always perpendicular to its velocity and constant in size: it moves in a circle. The magnetic force provides the centripetal force:

$$Bqv = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{Bq}$$

- larger v: larger radius; larger B: smaller radius; larger m: larger radius
- the specific charge $q/m = \dfrac{v}{Br}$ can be found from measurements
- period: $T = \dfrac{2\pi r}{v}$, so $T = \dfrac{2\pi m}{Bq}$, independent of v and r

Example 25.6 An α-particle travelling at 2.5×10^4 m s^{-1} enters a uniform field of flux density 5.4 mT, normal to its velocity. What is the radius of its path?

$$r = \frac{mv}{Bq} = \frac{4 \times 1.66 \times 10^{-27} \times 2.5 \times 10^4}{5.4 \times 10^{-3} \times 2 \times 1.60 \times 10^{-19}} \approx 0.096 \text{ m} = 9.6 \text{ cm}$$

Question 25.8 An α-particle and a β^- particle enter the same uniform field at the same speed. Compare the radii of their paths.

$r \propto \dfrac{m}{q}$. For the α: $\dfrac{6.64 \times 10^{-27}}{3.2 \times 10^{-19}} = 2.1 \times 10^{-8}$; for the β: $\dfrac{9.11 \times 10^{-31}}{1.60 \times 10^{-19}} = 5.7 \times 10^{-12}$.
$\dfrac{r_\alpha}{r_\beta} \approx 3.6 \times 10^3$: the α path is about 3600 times bigger, and they curve in **opposite** directions.

Question 25.9 Show that the angular velocity of a charged particle in a uniform field is independent of the radius of its path.

$Bqv = m\omega^2 r$ with $v = \omega r$ gives $Bq\omega r = m\omega^2 r$, so $\omega = \dfrac{Bq}{m}$, which does not contain r.

Question 25.10 A charged particle enters a uniform field at an angle $\theta \neq 90°$. State and explain its path.

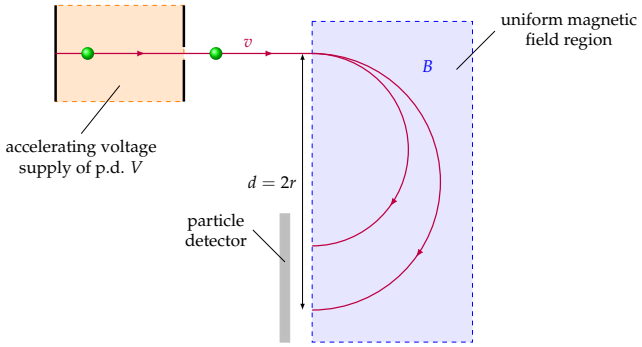
Split v into $v\sin\theta$ perpendicular to B and $v\cos\theta$ parallel to B.
The perpendicular component gives circular motion; the parallel component feels no force and stays constant. The path is a **helix** (spiral) about the field direction.

The mass spectrometer

A mass spectrometer measures the specific charge of ions. Particles are accelerated through a p.d. V: $\frac{1}{2}mv^2 = qV$, then move in a semicircle of radius r in the field: $v = \dfrac{Bqr}{m}$.

Eliminating v: $\dfrac{q}{m} = \dfrac{2V}{B^2r^2}$

- different particles have different q/m, so different radii, which identifies them



Question 25.11 Two paths of deflected particles are shown in the figure. Give reasons why their radii can differ.

$r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$, so r differs if the particles have different **masses** (for example isotopes) or different **charges**.

Question 25.12 The particles entering are positively charged and move to the right (as in the figure), curving downwards. In which direction should the field be applied?

Force is down for $+q$ moving right: left-hand rule gives B **out of the page**.

Question 25.13 A particle with charge $+e$ enters a uniform field of 8.8 mT at right angles with speed 1.4×10^5 m s^{-1}. It describes a semicircle of diameter 66 cm. (a) Find its mass. (b) Suggest what it could be.

(a) $r = 0.33$ m. $m = \frac{Bqr}{v} = \frac{8.8 \times 10^{-3} \times 1.60 \times 10^{-19} \times 0.33}{1.4 \times 10^5} = 3.3 \times 10^{-27}$ kg

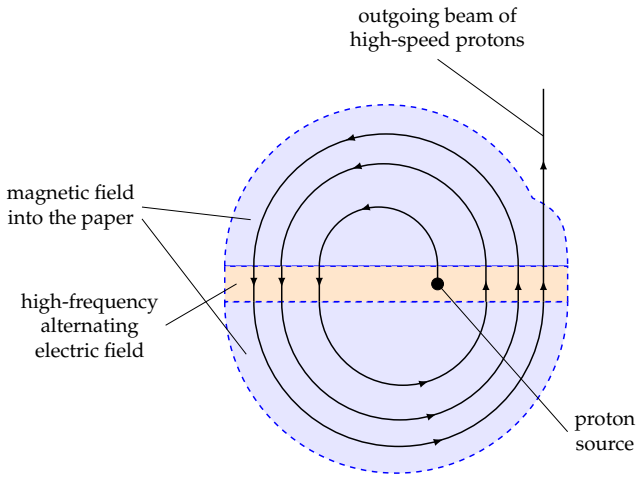
(b) $\frac{3.3 \times 10^{-27}}{1.66 \times 10^{-27}} \approx 2$ u with charge $+e$: a **deuteron** (nucleus of hydrogen-2: one proton and one neutron).

The cyclotron

A cyclotron is a ___particle accelerator___. Particles spiral outwards: in each ___semicircle___ in the magnetic field the direction is turned round; each time they cross the ___gap___ the alternating electric field accelerates them, adding energy ___qV___.

• period $T = \dfrac{2\pi m}{Bq}$ does not depend on ___radius___, so a fixed-frequency supply stays in step:

$$f = \frac{Bq}{2\pi m}$$ (the cyclotron frequency)



Question 25.14 Protons are accelerated in a cyclotron with a uniform field of 1.5 T and a gap voltage of 20 kV. (a) Find the frequency of the voltage supply needed. (b) How many circles must the protons make to gain 10 MeV?

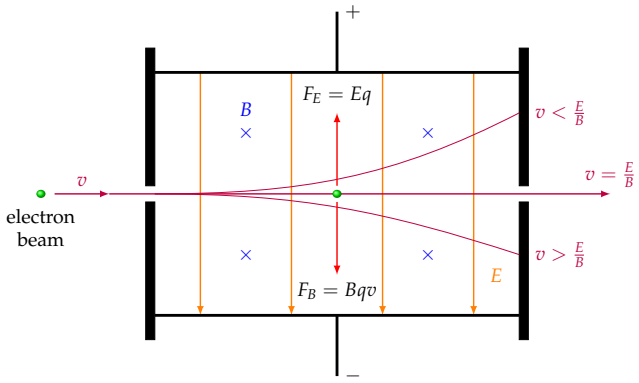
(a) $f = \dfrac{Bq}{2\pi m} = \dfrac{1.5 \times 1.60 \times 10^{-19}}{2\pi \times 1.67 \times 10^{-27}} = 2.3 \times 10^{7}$ Hz $= 23$ MHz
(b) Two gap crossings per circle, each adding $qV = 20$ keV: 40 keV per circle.
Circles $= \dfrac{10 \times 10^{3}\ \text{keV}}{40\ \text{keV}} = 250.$

The velocity selector

Beam of charged particles passes through crossed electric and magnetic fields. Particles of the right speed are ___undeflected___: the electric and magnetic forces balance:

$$Eq = Bqv \Rightarrow \boxed{v = \frac{E}{B}}$$

• faster particles: $F_B > F_E$ and they deflect one way; slower particles: $F_B < F_E$ and they deflect the ___other___ way



Question 25.15 Electrons at speed v pass undeflected through a velocity selector. What happens to α-particles of the same speed?

$v = E/B$ does not contain q or m, so the α-particles are also **undeflected**.

Question 25.16 A uniform field of 6.0×10^{-2} T is directed out of the page. A beam of protons moves from left to right across the page at 3.5×10^4 m s^{-1}. (a) In which direction do they deflect? (b) What electric field makes them undeviated?

(a) Left-hand rule: $+q$ moving right, B out of the page: force is **down the page**.
(b) The electric force must be **up the page**, so E points up the page: $E = vB = 3.5 \times 10^4 \times 6.0 \times 10^{-2} = 2.1 \times 10^3$ V m^{-1}

The Hall effect

A current in a conductor in a perpendicular field produces a transverse p.d., the Hall voltage V_H (Edwin Hall, 1879).

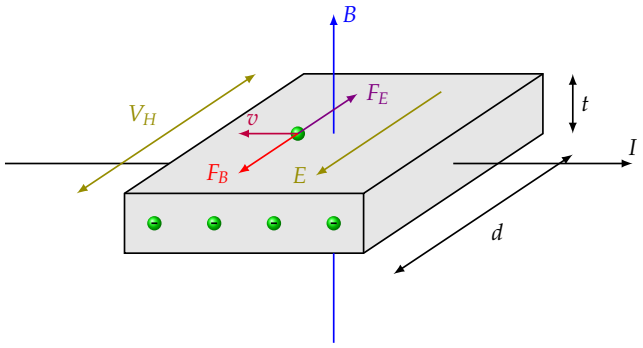
The magnetic force pushes the charge carriers to one side of the conductor. Charge builds up, creating an internal electric field E and p.d. V_H.

- in the steady state $F_E = F_B$: $Eq = Bqv$, with $E = \frac{V_H}{d}$ and $I = nAqv$ where $A = td$
- so $\frac{V_H}{d} = B\frac{I}{nAq}$, giving

$$V_H = \frac{BI}{ntq}$$

where n is the number density of carriers and t the thickness

- a large V_H needs small n (so use a semiconductor) and a thin slice
- the sign of V_H depends on whether the carriers are electrons or holes
- at fixed current, $V_H \propto$ B, so a Hall probe measures flux density



Question 25.17 Why is it difficult to detect the Hall voltage in a thin slice of copper?

Copper has a very large n (about 10^{29} m^{-3}), so $V_H = \frac{BI}{ntq}$ is tiny.

Question 25.18 A Hall probe is near one end of a strong magnet and is rotated through one full revolution. State and explain how the Hall voltage varies.

Only the component of B perpendicular to the slice matters, so $V_H \propto \cos\phi$ (ϕ the angle between B and the normal to the slice).
V_H varies sinusoidally: maximum when B is perpendicular to the slice, zero twice (when B is parallel to the slice), and reverses sign when the field passes through the slice the other way.