Kinematics

AS Level Physics 9702

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By the end of this topic I can. . .

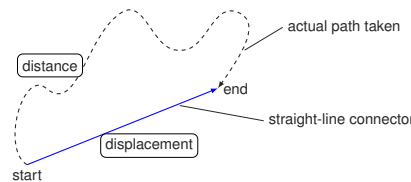
- distinguish distance from displacement, and speed from velocity, and define acceleration
- read gradients and areas on displacement–time and velocity–time graphs
- use the equations of motion for constant acceleration, including free fall
- analyse projectile motion using horizontal and vertical components

Distance and displacement

Displacement

The **distance moved in a specific direction**: the straight-line distance from start to end.

- distance is a **scalar**; displacement is a **vector**
- both are measured in metres
- a long journey can have zero displacement

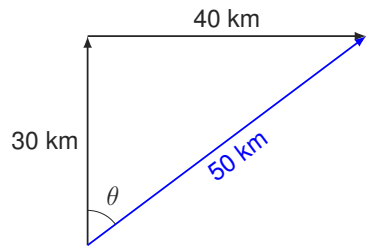


Example: distance and displacement

A ship travels 30 km north, takes a right, and then travels 40 km east to reach its destination. Compare the distance and the displacement travelled.

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Summing of all lengths gives distance:
$30 + 40 = 70$ km
The displacement vector is shown on the diagram.
The magnitude of displacement $= \sqrt{30^2 + 40^2} =$ 50 km
at an angle of $\theta = \tan^{-1} \frac{40}{30} \approx 53^\circ$ east of north

Velocity

Velocity

The **change in displacement per unit time**.

- a **vector**; unit m s^{-1}
- in one dimension the sign gives the direction

$$v = \frac{\Delta s}{\Delta t}$$

Average and instantaneous velocity

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t}$$

Example: average speed and average velocity

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Total distance travelled $= 800 + 280 = 1080$m, so average speed:
$v = \frac{1080}{8.0 \times 60} = 2.25$ m s^{-1}
change of displacement $= 800 + (-280) = 520$m, so average velocity:
$v = \frac{520}{8.0 \times 60} \approx 1.08$ m s^{-1}

Acceleration

Acceleration

The **change in velocity per unit time**.

- a **vector**; unit m s^{-2}
- $a < 0$: slowing down (deceleration)
- velocity changes if the speed **or** the direction changes

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

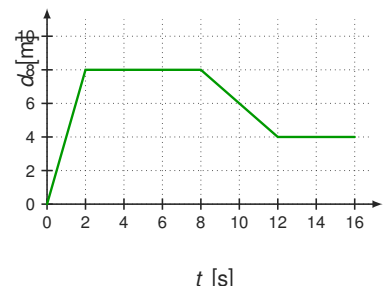
Example: rebound from a barrier

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note that direction of velocity changed during rebound, so $\Delta v = 15 - (-12) = 27$ m s^{-1}
average acceleration: $a = \frac{\Delta v}{\Delta t} = \frac{27}{30 \times 10^{-3}} = 900$ m s^{-2}

Displacement–time graphs

- **gradient = velocity**
- straight line: constant velocity; horizontal: at rest
- curve: the velocity is changing
- downward-sloping tangent: negative velocity



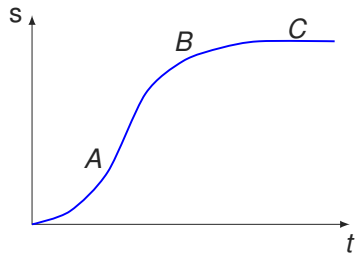
Example: describing an s–t graph

Describing the motion from the displacement-time graph shown.

Example: describing an s–t graph

Describing the motion from the displacement-time graph shown.

Stage *A*: the gradient is increasing, so the object is speeding up.
Stage *B*: the gradient decreases, so it gradually slows down.
Stage *C*: the curve is horizontal, the gradient is zero, so the object has come to a stop.

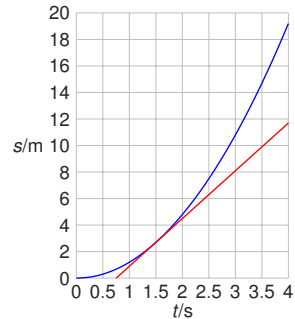


Example: reading an s–t graph

The diagram shows the displacement-time graph for a vehicle travelling along a straight road. Use the graph to find, (a) the average velocity during the first 4.0 s of the motion, (b) the velocity of the vehicle *at* time $t = 1.5$ s.

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during first 4.0 s, average velocity is

$$v = \frac{\Delta s}{\Delta t} = \frac{19.2}{4.0} \approx 4.8 \text{ m s}^{-1}$$

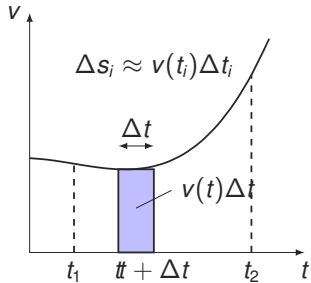
to find velocity at $t = 1.5$ s, a tangent is drawn

gradient of tangent gives instantaneous velocity:

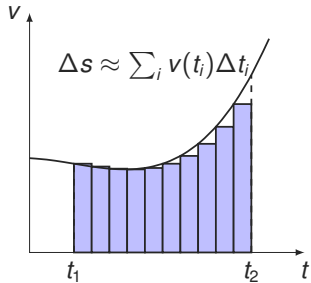
$$v = \frac{11.6-0}{4.0-0.75} \approx 3.6 \text{ m s}^{-1}$$

Velocity–time graphs

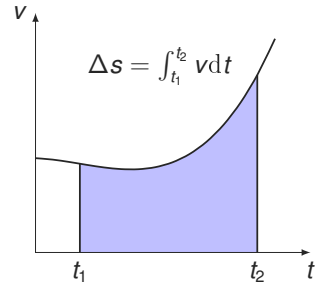
- **gradient = acceleration**
- **area under the graph = change in displacement**
- area below the time axis counts as negative
- area under an *a*–*t* graph = change in velocity



(a) $\Delta s \approx v\,\Delta t$



(b) sum of many strips



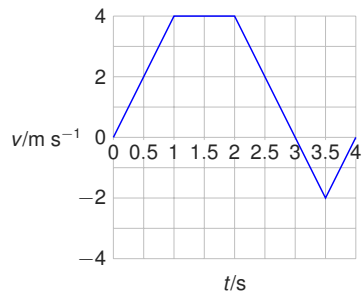
(c) area under graph

Example: distance and displacement from a v–t graph

The velocity of a toy car is shown. For the journey shown on the graph, use the graph to find (a) the total distance travelled, and (b) the total displacement travelled.

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Distance is estimated using area under v-t graph

0∼3.0 s: $s_1 = \frac{1}{2} \times 1.0 + 3.0 \times 4.0 = 8.0$ m

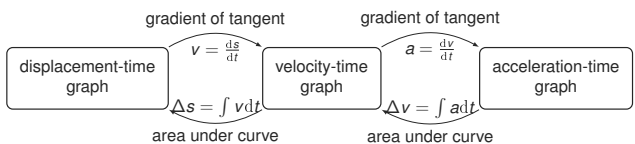
3.0∼4.0 s: $s_2 = \frac{1}{2} \times 1.0 \times 2.0 = 1.0$ m

total distance $= 8.0 + 1.0 = 9.0$ m

total displacement $= (+8.0) + (-1.0) = 7.0$ m

Linking the three graphs

- gradient of s–t gives v; gradient of v–t gives a
- area under a–t gives Δv; area under v–t gives Δs



Equations of motion (SUVAT)

Constant acceleration only

s displacement, u initial velocity, v final velocity, a acceleration, t time.

$$v = u + at$$

$$s = \tfrac{1}{2}(u + v)t$$

$$s = ut + \tfrac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Example: an accelerating car

Example: an accelerating car

$$v = u + at \quad \Rightarrow \quad v = 0 + 5.0 \times 6.0 = 30 \text{ m s}^{-1}$$

$$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad s = 0 + \frac{1}{2} \times 5.0 \times 6.0^2 = 90 \text{ m}$$

Example: braking distance

Example: braking distance

$$2as = v^2 - u^2 \quad \Rightarrow \quad s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 30^2}{2 \times (-5.0)} = 90 \text{ m}$$

Example: overtaking

Example: overtaking

Lets suppose overtake occurs at time t after motorcycle starts to accelerate, so the distance travelled by motorcycle:

$$s_m = u_m t + \frac{1}{2} a t^2 \quad \Rightarrow \quad s_m = 0 + \frac{1}{2} \times 2.0 \times t^2$$

In the same time the distance travelled by truck:

$$s_t = v_t t \quad \Rightarrow \quad s_t = 16t$$

The overtake when:

$$s_m = s_t \quad \Rightarrow \quad \frac{1}{2} \times 2.0 \times t^2 = 16t \quad \Rightarrow \quad t = 16 \text{ s}$$

Substitute t into either s_m or s_t, one finds distance travelled: $s = 256$ m

Free fall

No air resistance: every object falls with $g \approx 9.81$ m s^{-2}.

$$v = gt$$

$$s = \tfrac{1}{2}gt^2$$

- released from rest: $u = 0$, $a = g$
- independent of mass

Example: a falling object

Example: a falling object

$$h = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 24}{9.81}} \approx 2.21 \text{ s}$$

$$v = gt = 9.81 \times 2.21 \quad \Rightarrow \quad v \approx 21.7 \text{ m s}^{-1}$$

to find final velocity, one can also use $v^2 = u^2 + 2as$:

$$v^2 = \cancelto{0}{u^2} + 2gh = 2 \times 9.81 \times 24 \quad \Rightarrow \quad v \approx 21.7 \text{ m s}^{-1}$$

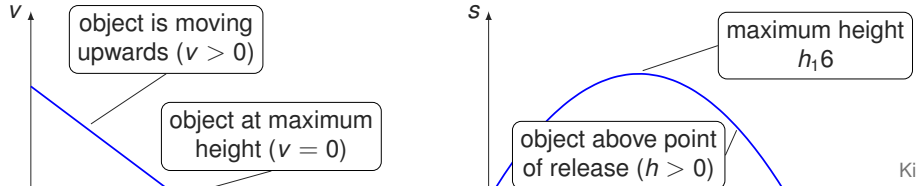
Upward projection

Take upwards as positive, so $a = -g$:

$$v = u - gt$$

$$s = ut - \tfrac{1}{2}gt^2$$

- $v = 0$ at the highest point
- time up = time down; return speed = launch speed



Example: ball thrown upwards

Example: ball thrown upwards

maximum height is reached when $v = 0$, so

$$v = u - gt = 0 \quad \Rightarrow \quad t = \frac{u}{g} = \frac{12}{9.81} \approx 1.22 \text{ s}$$

$$H_{\max} = ut - \tfrac{1}{2}gt^2 = 12 \times 1.22 - \tfrac{1}{2} \times 9.81 \times 1.22^2 \approx 7.34 \text{ m}$$

It is also possible to use $v^2 - u^2 = 2as$ to find $H_{\max}$:

$$0^2 - u^2 = 2(-g)H_{\max} \quad \Rightarrow \quad H_{\max} = \frac{u^2}{2g} = \frac{12^2}{2 \times 9.81} \approx 7.34 \text{ m}$$

Example: stone thrown from a cliff

Example: stone thrown from a cliff

Take positive direction to point upwards, we use $v^2 - u^2 = 2as$ to find $v^2 = 14.0^2 + 2 \times (-9.81) \times (-35) \approx 883 \text{ m}^2 \text{ s}^{-2} \quad \Rightarrow \quad v \approx -29.7 \text{ m s}^{-1}$. To find time, we can use $v = u - gt$, hence: $t = \frac{v-u}{-g} = \frac{-29.7-14.0}{-9.81} \approx 4.46$ s.

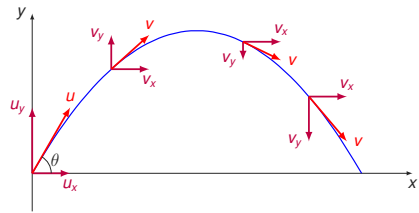
One can also attempt $s = ut - \frac{1}{2}gt^2$, this leads to the equation: $-35 = 14.0t - \frac{1}{2} \times 9.81t^2$.

This quadratic equation in t gives two roots: $t_1 \approx 4.46$ s, and $t_2 \approx -1.60$ s.

Note that a negative root should be discarded since it means stones hits the sea before it is thrown so time taken for stone to hit the sea is $t \approx 4.46$ s

Projectile motion

- horizontal: $a_x = 0$, constant velocity
- vertical: $a_y = -g$
- path is a **parabola**
- at the top $v_y = 0$



Projectile equations

$u_x = u\cos\theta$, $u_y = u\sin\theta$

$$v_x = u\cos\theta$$

$$x = u\cos\theta\ t$$

$$v_y = u\sin\theta - gt$$

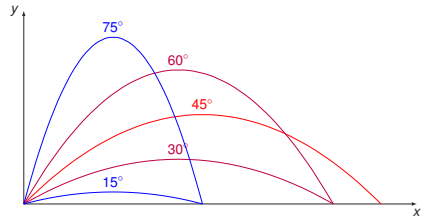
$$y = u\sin\theta\ t - \tfrac{1}{2}gt^2$$

Height, time of flight and range

$$H_{\max} = \frac{u^2 \sin^2 \theta}{2g}$$

$$T = \frac{2u \sin \theta}{g}$$

$$X = \frac{u^2 \sin 2\theta}{g}$$



- greatest range at 45°
- complementary angles give the same

Example: a thrown ball

Example: a thrown ball

At the highest point: $v_y = u_y - gt = 0 \quad \Rightarrow \quad t = \frac{u \sin\theta}{g} = \frac{15 \times \sin 40^\circ}{9.81} \approx 0.983$ s

Vertical displacement:

$y = u_y t - \frac{1}{2} g t^2 = 15 \sin 40^\circ \times 0.983 - \frac{1}{2} \times 9.81 \times 0.983^2 \approx 4.74$ m

Horizontal displacement: $x = u_x t = 15 \cos 40^\circ \times 0.983 \approx 11.3$ m

Resultant displacement: $|OP| = \sqrt{x^2 + y^2} = \sqrt{11.3^2 + 4.74^2} \approx 12.2$ m

Example: horizontal projection

Example: horizontal projection

Vertically, take downward as positive: $h = \cancelto{0}{u_y t} + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 1.2}{9.81}} \approx 0.495$ s

Horizontal range: $x = u_x t = 7.20 \times 0.495 \approx 3.56$ m

Final vertical velocity: $v_y = \cancelto{0}{u_y} + gt = 9.81 \times 0.495 \approx 4.85$ m s^{-1}

Magnitude of resultant velocity: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{7.20^2 + 4.85^2} \approx 8.68$ m s^{-1}

Angle to which resultant velocity makes with horizontal: $\phi = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{4.85}{7.20} \approx 34^\circ$

Question: ball kicked at a barrier

Question: ball kicked at a barrier

(a) Horizontal: $u_x = \dfrac{18}{1.5} = 12.0 \text{ m s}^{-1}$. Vertical:

$2.5 = u_y(1.5) - \frac{1}{2}(9.81)(1.5)^2 \quad \Rightarrow \quad u_y = 9.02 \text{ m s}^{-1}$.

$u = \sqrt{12.0^2 + 9.02^2} \approx 15 \text{ m s}^{-1}$ at $\tan^{-1} \dfrac{9.02}{12.0} \approx 37^\circ$ above the horizontal.

(b) $v_x = 12.0 \text{ m s}^{-1}$, $v_y = 9.02 - 9.81 \times 1.5 = -5.69 \text{ m s}^{-1}$ (moving downwards).

$v = \sqrt{12.0^2 + 5.69^2} \approx 13 \text{ m s}^{-1}$ at $\tan^{-1} \dfrac{5.69}{12.0} \approx 25^\circ$ below the horizontal.

Key equations

velocity	$v = \dfrac{\Delta s}{\Delta t}$	SUVAT 3	$s = ut + \frac{1}{2}at^2$
acceleration	$a = \dfrac{\Delta v}{\Delta t}$	SUVAT 4	$v^2 = u^2 + 2as$
SUVAT 1	$v = u + at$	max height	$H_{\text{max}} = \dfrac{u^2 \sin^2 \theta}{2g}$
SUVAT 2	$s = \frac{1}{2}(u + v)t$	range	$X = \dfrac{u^2 \sin 2\theta}{g}$