Kinematics

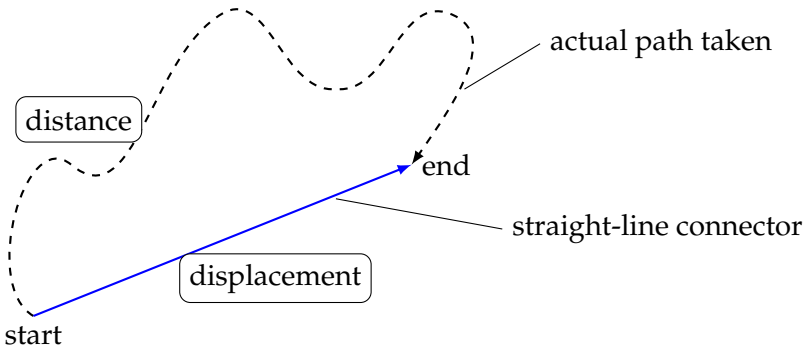
Name _____ Date _____

By the end of this topic I can…

- distinguish distance from displacement, and speed from velocity, and define acceleration
- read gradients and areas on displacement–time and velocity–time graphs
- use the equations of motion for constant acceleration, including free fall
- analyse projectile motion by treating the horizontal and vertical motion separately

Kinematic quantities

Distance and displacement



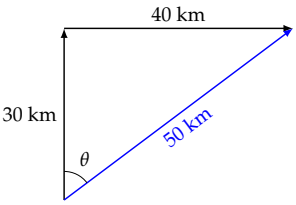
The path taken (dashed) and the displacement (straight arrow).

_____ is the _____: the straight-line distance from the start to the end.

• **distance** is a _____ (size only); **displacement** is a _____ (size and direction)

• both are measured in _____, but displacement can be zero for a journey with a long distance

Example 2.1 An athlete is running around a circular track of radius 60 m. When he completes one lap, what is the distance moved out? What about his displacement?



Example 2.2 A ship travels 30 km north, takes a right, and then travels 40 km east to reach its destination. Compare the distance and the displacement travelled.

Speed and velocity

_____ is the
_____:

$[v] =$ _____

- velocity is a _____; in one dimension we give direction with a _____: a negative velocity means motion in the opposite direction
- **average velocity**:
- **average speed**:
- **instantaneous velocity** is the average velocity over a very small Δt ($\Delta t \to 0$):

Example 2.3 A cyclist travels a distance of 3.0 km in 20 minutes. She rests for 15 minutes. She then covers a further distance of 5.1 km in a time of 40 minutes. Calculate the average speed of the cyclist in m s^{-1}: (a) during the first 20 minutes of the journey, (b) for the whole journey.

Example 2.4 A man walks along a straight road for a distance of 800 m in 5.0 minutes. He then turns around, and walks along the same road for a distance of 280 m in 3.0 minutes. What is the average speed and the average velocity of this man during the 8.0 minutes?

Acceleration

_____ is the
_____:

$[a] =$ _____

• acceleration is a _____. In linear motion take the direction of the initial velocity as _____

• $a > 0$: speeding up in the positive direction; $a < 0$: _____ (deceleration)

• velocity changes if the _____ changes or the _____ changes, so an object moving in a curve always has a non-zero acceleration

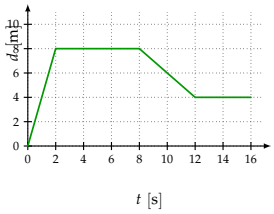
• **instantaneous acceleration**:

Example 2.5 A ball hits a barrier at right angles with a speed of 15 m s^{-1}. It makes contact with the barrier for 30 ms and then rebounds with a speed of 12 m s^{-1}. What is the average acceleration during the time of contact?

Question 2.1 Is it possible for an object moving at a steady speed to have acceleration?

Motion graphs

Displacement–time graphs

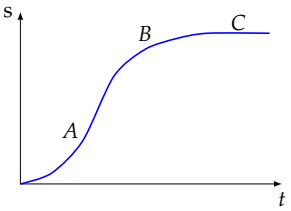


A generic displacement–time graph.

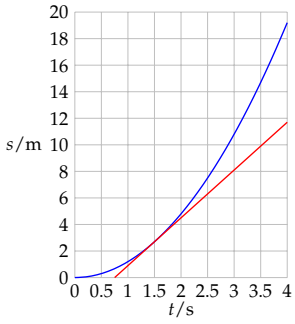
The gradient (tangent) of an *s*–*t* graph is the rate of change of displacement:

$$\text{velocity} = \square$$

- a straight line means _____; a horizontal line means the object is _____
- a curve means the velocity is _____: the steeper the line, the greater the speed
- a downward-sloping tangent means a _____ velocity



Example 2.6 Describing the motion from the displacement-time graph shown.



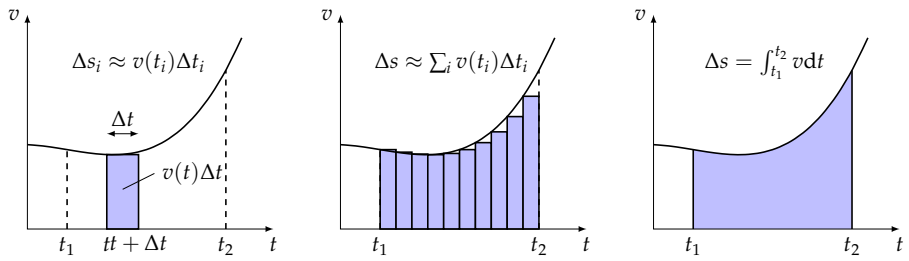
Example 2.7 The diagram shows the displacement-time graph for a vehicle travelling along a straight road. Use the graph to find, (a) the average velocity during the first 4.0 s of the motion, (b) the velocity of the vehicle *at* time $t = 1.5$ s.

Velocity–time graphs

The gradient of a v–t graph gives the acceleration, and the area under it gives the change in displacement:

acceleration =

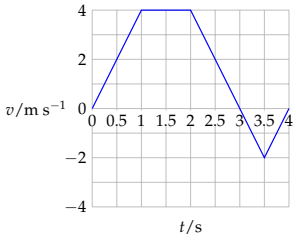
change in displacement =



(a) $\Delta s \approx v\,\Delta t$ (b) sum of many strips (c) area under graph

In a short interval v is almost constant, so $\Delta s \approx v\Delta t$ is the area of a thin strip. Add up all the strips and you get the area under the curve.

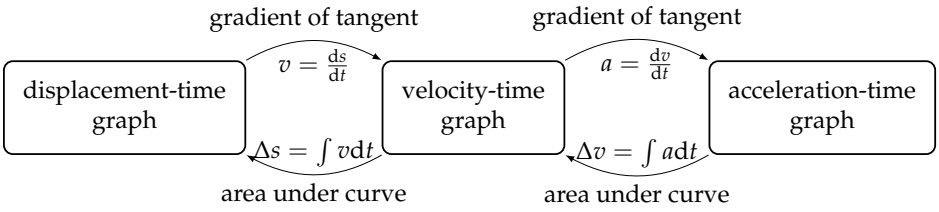
- a horizontal line: _____ (zero acceleration)
- a straight sloping line: _____
- area *below* the time axis counts as _____ displacement (motion in the reverse direction)



Example 2.8 The velocity of a toy car is shown. For the journey shown on the graph, use the graph to find (a) the total distance travelled, and (b) the total displacement travelled.

Acceleration–time graphs

The area under an a–t graph is the _____.



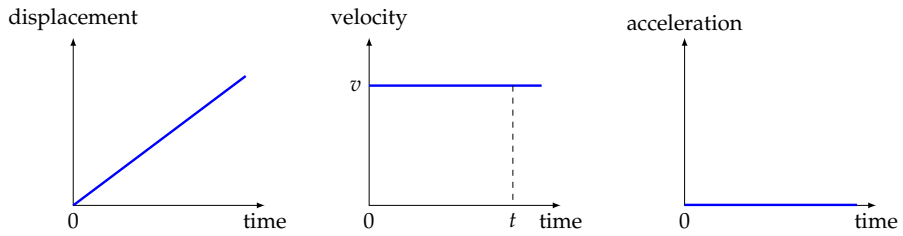
How the three graphs are linked: gradients go one way, areas the other.

Constant velocity and constant acceleration

Constant velocity

For $v =$ constant we have $a =$ ____ and

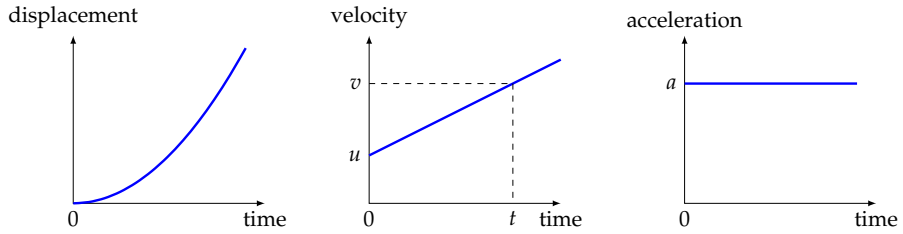
(from the origin; otherwise $s = s_0 + vt$). The s–t graph is a _____.



Constant velocity: s–t, v–t and a–t.

Constant acceleration

Here a is constant, the v–t graph is a _____ and the s–t graph is a _____.



Constant acceleration: s–t, v–t and a–t.

Equations of motion (SUVAT)

s = displacement, u = _____, v = _____, a = acceleration, t = time.

- from $a = \dfrac{v-u}{t}$:

- the area under the v–t graph (average velocity $\times$ time):

- put the first into the second to remove v:

- eliminate t:

Using the equations: write down the five quantities, mark the three you know and the one you want, then pick the equation that has _____. They only apply when the acceleration is _____.

Example 2.9 A car starts from rest and accelerates uniformly at 5.0 m s^{-2} for 6.0 s.
(a) How fast is the car travelling at $t = 8.0$ s?
(b) What is the distance travelled by the car in this time?

Example 2.10 A car is travelling at 30 m s^{-1}. A hazard appears in front of the car, and the driver takes immediate action to stop the car. When brakes are applied, deceleration of the car is 5.0 m s^{-2}. What is the braking distance?

Example 2.11 At the instant the traffic light turns green, a motorcycle waiting at the stop line starts with a constant acceleration of 2.0 m s^{-2}. At the same instant, a truck at a constant speed of 16 m s^{-1} overtakes and passes the motorcycle. How far beyond the stop line will the motorcycle overtake the truck?

Free fall and projectiles

Free fall

Ignoring air resistance, every object near the Earth's surface falls with the same constant acceleration $g \approx$ _____ downwards, independent of its mass.

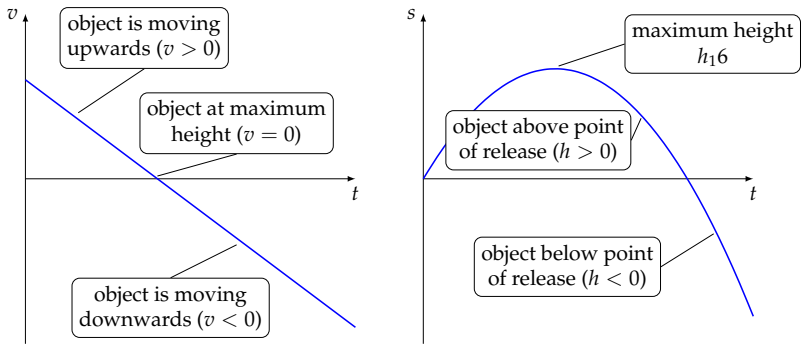
Released from rest ($u = 0$, $a = g$):

Example 2.12 An object is released from rest from a height of $h = 24$ m and falls freely under gravity. Air resistance is negligible.
(a) How long does it take to hit the ground?
(b) What is its speed when hitting the ground?

Upward projection

Take _____ as positive, so $a =$ _____:

- $v > 0$: moving up; $v < 0$: moving _____; at the highest point $v =$ ____
- $s > 0$: above the point of release; $s < 0$: _____ it
- the time up equals the time _____ back to the same height, and the speed on return equals the initial speed



v–t and s–t graphs for an object thrown upwards.

Example 2.13 A ball is projected vertically upwards at 12 m s^{-1}. Air resistance is negligible. (a) Find the time taken for the ball to reach the highest position. (b) Find the greatest height.

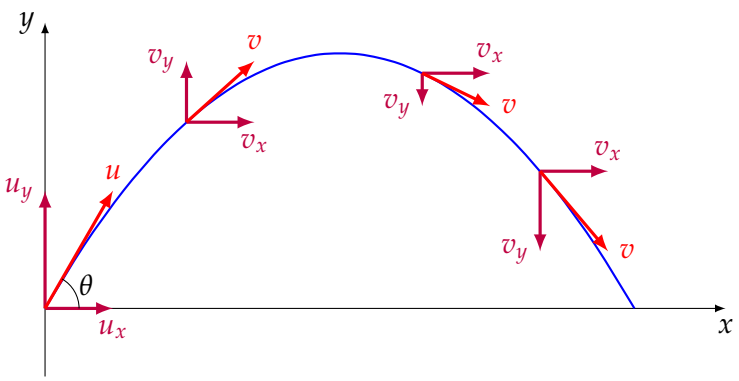
Example 2.14 A stone is thrown vertically upwards with an initial velocity of 14.0 m s^{-1} from the edge of a cliff that is 35 m from the sea below.
(a) Find the speed at which it hits the sea.
(b) Find the time taken for the stone to hit the sea.

Projectile motion

A projectile moves under gravity alone. Its horizontal and vertical motions are _____:

- horizontally: $a_x =$ ____, so the horizontal velocity is _____
- vertically: $a_y =$ _____ (upwards positive), constant acceleration

The path is a _____.



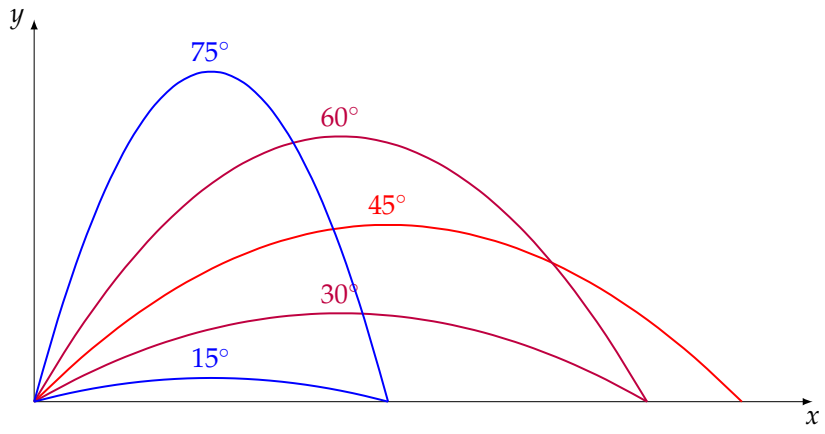
Velocity components at different points of a projectile's path.

Launched at speed u at angle θ above the horizontal, $u_x = u\cos\theta$ and $u_y = u\sin\theta$:

The resultant speed is $v = \sqrt{v_x^2 + v_y^2}$, at angle $\tan^{-1}(v_y/v_x)$ to the horizontal.

- at the highest point $v_y =$ ____, so the time to the top is $t = \dfrac{u\sin\theta}{g}$ and

- back at launch height $y = 0$, so the total time of flight and the range are



Trajectories for the same launch speed at different angles.

The range is greatest at an angle of _____ (for the same launch and landing height); the time of flight is greatest for $\theta = 90°$. Angles that add to $90°$ give the same range.

Example 2.15 A ball is thrown from a point O at 15 m s^{-1} at an angle of 40° to the horizontal. The ball reaches its highest position at point P. Ignore the effects of air resistance. (a) How long does it take to reach P? (b) What is the magnitude of the displacement OP?

Example 2.16 A small object is horizontally projected at 7.20 ms^{-1} from a surface at a height of $h = 1.2$ m above the ground. Assume there is no air resistance. (a) What is the time taken for the object to hit the ground? (b) What is the horizontal range? (c) Find the velocity at which the object hits the ground.

Practice questions

Question 2.2 What is the distance covered for a car that travels half a lap along a circular path of radius of 200 m. What about the displacement?

Question 2.3 A car travels 2400 m east in 3.0 minutes, then takes a left turn, and then travels 700 m north in 1.5 minutes. What is the average speed and the average velocity for this journey?

Question 2.4 A vehicle initially travels at a steady speed of 15 m s^{-1}. It accelerates uniformly for 10 s to reach a higher speed of 20 m s^{-1}. It maintains at this speed for 20 s, and then decelerates uniformly to a stop in the last 10 s. (a) Sketch the velocity-time graph for this motion. (b) Sketch the acceleration-time graph. (c) Find the distance travelled during the 40 s.

Question 2.5 A vehicle moving at 14 m s^{-1} accelerate uniformly to 26 m s^{-1} in 6.0 s. (a) What is the average velocity during this time? (b) What is the acceleration during this time (c) What is the distance travelled by the vehicle? (d) The vehicle then braked with constant deceleration to stop in another 8.0 s. What is the distance travelled during the time when brakes are applied?

Question 2.6 Two balls are dropped from rest from the same height. The second ball is released 0.80 s after the first one. What is their separation 1.5 s after the second ball is dropped?

Question 2.7 The acceleration of free fall on Pluto is about one-fifteenth of that on Earth. If it takes a time of T for a rock to fall from rest a distance of S, what is the time taken, in terms of T, for a rock to fall from rest through the same distance S on Pluto?

Question 2.8 Someone wants to toss a ball onto a platform that is at a height of 20 m above him. What is the minimum initial velocity needed to launch the ball?

Question 2.9 A ball rolls off a table and lands at a position of a horizontal distance of 1.2 m from the table. The table is 0.95 m high. Find the speed at which the ball leaves the table.

Question 2.10 A ball is kicked from the ground towards a vertical barrier. The barrier is at a horizontal distance of 18 m from the initial position of the ball. The ball strikes the barrier after 1.5 s at a height of 2.5 m above the ground. (a) Find the magnitude and the direction of the initial velocity. (b) Find the magnitude and the direction of the velocity at which the ball hits the barrier.