Ideal Gases

A Level Physics 9702

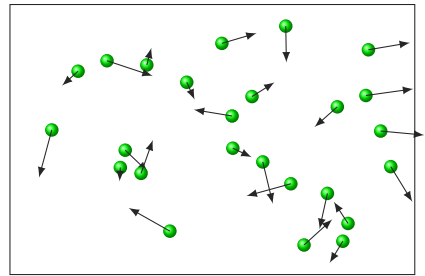
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By the end of this topic I can...

- use the mole, the Avogadro constant and molar mass to count molecules
- explain gas pressure in terms of collisions of molecules with the container
- use Boyle's, Charles's, Gay-Lussac's and Avogadro's laws
- use $pV = nRT$ and $pV = NkT$, and state the assumptions of an ideal gas
- derive and use $p = \frac{Nm\langle v^2 \rangle}{3V}$ and $\langle E_k \rangle = \frac{3}{2}kT$

Motion of gas particles

- a very large number of molecules moving **randomly** at high speed
- randomness comes from **collisions**
- at any instant: a **range** of speeds
- **Brownian motion**: smoke particles jerk about because of unseen molecular collisions
- faster at higher temperature



The mole and the Avogadro constant

Mole

The amount of substance in **12 g of carbon-12**. One mole of anything contains 6.02×10^{23} particles.

- unit: mol $\quad N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$

$$N = nN_A$$

Molar mass

Molar mass M

The **mass of a sample divided by the amount of substance**: $M = \frac{m}{n}$

amount of substance

$$n = \frac{m}{M}$$

mass of one molecule

$$m_0 = \frac{M}{N_A}$$

Example 21.1

Find the number of molecules in 160 g of argon-40 gas.

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$$n = \frac{m}{M} = \frac{160}{40} = 4.0 \text{ mol}$$
$$N = nN_A = 4.0 \times 6.02 \times 10^{23} \approx 2.41 \times 10^{24}$$

Pressure: the qualitative picture

- molecules rebound from the wall: their **momentum** changes
- Newton 2: the wall exerts a **force** on them
- Newton 3: they exert an equal and opposite force on the wall
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Heat a gas at fixed volume: faster molecules hit harder *and* more often, so p increases.

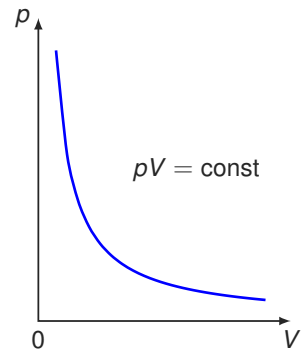
Boyle's law

Constant temperature (isothermal):

$$pV = \text{const}$$

$$p \propto \frac{1}{V}$$

$p_1 V_1 = p_2 V_2$



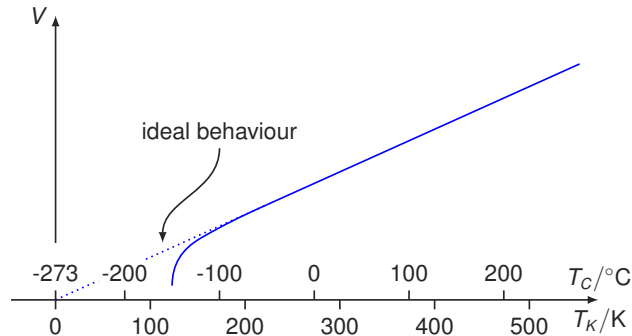
Charles's law

Constant pressure (isobaric):

$$\frac{V}{T} = \text{const}$$

$$V \propto T$$

Kelvin only!



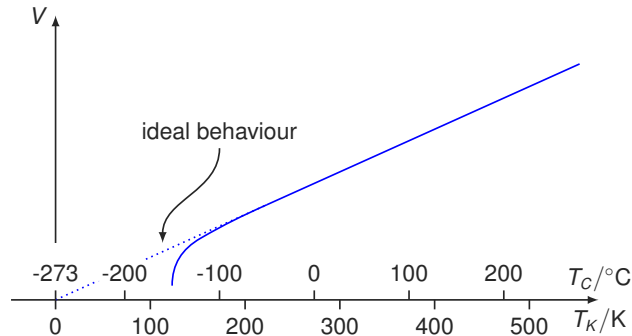
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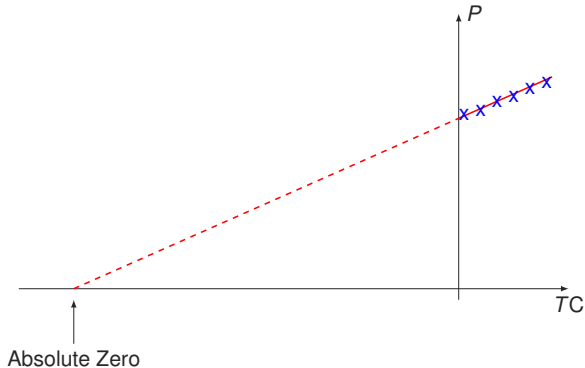
Volume $\rightarrow$ 0 at a certain temperature: the idea of **absolute zero**

Gay-Lussac's law

Constant volume (isochoric):

$$\frac{p}{T} = \text{const}$$

$$p \propto T$$



Avogadro's law

Avogadro's law

Equal volumes of **all gases**, at the same temperature and pressure, contain the **same number of molecules**.

$$\frac{V}{n} = \text{const}$$

(at constant T and p)

The ideal gas

- numerous, identical, **point-like** molecules, far apart: intermolecular forces **negligible**
- random motion, obeying **Newton's laws**
- **elastic** collisions with the walls
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Real gases behave ideally at **high temperature** and **low pressure**.

The ideal gas equation

$$pV = nRT$$

$$pV = NkT$$

$$R = kN_A$$

$R = 8.31$ J mol^{-1} K^{-1}
$k = 1.38 \times 10^{-23}$ J K^{-1}

Use **kelvin**: $T_K = T_C + 273$

Example 21.3: sealed cylinder

0.050 m^3 holds 75 g of air ($M = 29$ g mol^{-1}). (a) Pressure at 30°C? (b) Temperature when the volume doubles at constant pressure?

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$$n = \frac{75}{29} \approx 2.59 \text{ mol}$$

$$\text{(a)}\quad p = \frac{nRT}{V} = \frac{2.59 \times 8.31 \times 303}{0.050} \approx 1.30 \times 10^5 \text{ Pa}$$

$$\text{(b) } V \propto T\text{:}\quad T_2 = 2 \times 303 = 606 \text{ K} = 333^\circ\text{C}$$

Example 21.4: filling balloons

5000 cm^3 of air at 27°C and 6.0×10^5 Pa fills balloons of 1000 cm^3 at 1.0×10^5 Pa. How many balloons?

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Initial: $n_0 = \dfrac{6.0 \times 10^5 \times 5000 \times 10^{-6}}{8.31 \times 300} \approx 1.203$ mol

Left in cylinder (pressure has dropped to 1.0×10^5 Pa): $n_{\text{remain}} \approx 0.201$ mol

Each balloon: $n_b = \dfrac{1.0 \times 10^5 \times 1000 \times 10^{-6}}{8.31 \times 300} \approx 0.040$ mol

$$N = \frac{1.203 - 0.201}{0.040} \approx 25 \text{ balloons}$$

Example 21.5: a leaking cylinder

5.0×10^{-4} m^3 at 300 K and 4.0×10^6 Pa; molecules leak at 1.6×10^{16} s^{-1}. How long until the pressure falls 5.0%?

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$$N_0 = \frac{pV}{kT} = \frac{4.0 \times 10^6 \times 5.0 \times 10^{-4}}{1.38 \times 10^{-23} \times 300} \approx 4.83 \times 10^{23}$$

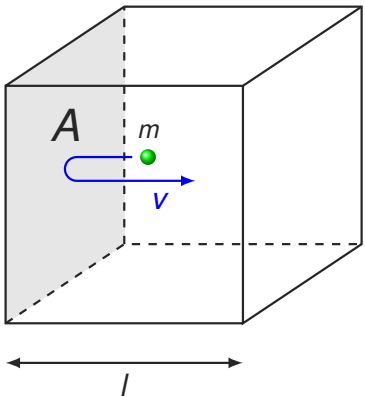
$N \propto p$: $\quad \Delta N = 0.05 \times 4.83 \times 10^{23} \approx 2.42 \times 10^{22}$

$$t = \frac{2.42 \times 10^{22}}{1.6 \times 10^{16}} \approx 1.51 \times 10^6 \text{ s} \approx 17.5 \text{ days}$$

The kinetic model

- constant **random** motion
- separation $\gg$ molecular size: molecular volume negligible
- intermolecular **forces** negligible
- **elastic** collisions: no kinetic energy lost
- **straight lines** between collisions

Pressure from one molecule



$$\Delta P_x = mv_x - (-mv_x) = 2mv_x$$

$$\Delta t = \frac{2l}{v_x}$$

$$F_x = \frac{\Delta P_x}{\Delta t} = \frac{mv_x^2}{l}$$

$$p_x = \frac{F_x}{A} = \frac{mv_x^2}{V}$$

From one molecule to N molecules in 3-D

- N molecules: multiply by N, and average over the distribution of speeds
- $v^2 = v_x^2 + v_y^2 + v_z^2$ and no preferred direction: $\langle v_x^2 \rangle = \dfrac{\langle v^2 \rangle}{3}$

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$$p = \frac{Nm \langle v^2 \rangle}{3V}$$

with $\dfrac{Nm}{V} = \rho$:

$$p = \frac{1}{3} \rho \langle v^2 \rangle$$

Kinetic energy of a gas

Compare $pV = NkT$ with $pV = \frac{1}{3}Nm\langle v^2\rangle$: $\quad m\langle v^2\rangle = 3kT$

$$\langle E_k\rangle = \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT$$

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$$v_{\text{rms}}^2 \propto T$$

- translational K.E. only; depends on T alone
- in a mixture, K.E. is shared equally: heavier molecules move **more slowly**

Example 21.7: oxygen and nitrogen at 300 K

Mean translational K.E.? Typical speed of O_2 (32 g mol^{-1}) and N_2 (28 g mol^{-1})?

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$$\langle E_k \rangle = \tfrac{3}{2} kT = \tfrac{3}{2} \times 1.38 \times 10^{-23} \times 300 \approx 6.21 \times 10^{-21} \text{ J}$$

$$\langle v^2 \rangle = \frac{3RT}{M} \quad \Rightarrow \quad v_{O_2} \approx 483 \text{ m s}^{-1}, \quad v_{N_2} \approx 517 \text{ m s}^{-1}$$

Example 21.8: helium-4 heated

Helium-4 at 54°C. Mean square speed? R.m.s. speed when heated to 540°C?

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$$\left\langle v^2 \right\rangle = \frac{3kT}{m} = \frac{3 \times 1.38 \times 10^{-23} \times 327}{6.64 \times 10^{-27}} \approx 2.04 \times 10^6 \text{ m}^2 \text{ s}^{-2}$$

$$v'_{\text{rms}} = \sqrt{\frac{T'}{T}}\, v_{\text{rms}} = \sqrt{\frac{813}{327}} \times \sqrt{2.04 \times 10^6} \approx 2.25 \times 10^3 \text{ m s}^{-1}$$

Key equations

amount	$N = nN_A$	ideal gas	$pV = nRT$
molar mass	$n = \frac{m}{M}$		$pV = NkT$
Boyle	$pV = \text{const}$	gas constants	$R = kN_A$
Charles	$\frac{V}{T} = \text{const}$	pressure	$p = \frac{Nm\langle v^2 \rangle}{3V}$
Gay-Lussac	$\frac{p}{T} = \text{const}$	mean K.E.	$\langle E_k \rangle = \frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}kT$