Gravitational Fields

A Level Physics 9702

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By the end of this topic I can...

- use Newton's law of gravitation, and apply it to circular orbits and Kepler's third law
- explain apparent weight and the effect of the Earth's rotation
- define field strength and potential; use $g = GM/r^2$ and $\varphi = -GM/r$
- use $E_p = -GMm/r$ with energy conservation, including escape speed

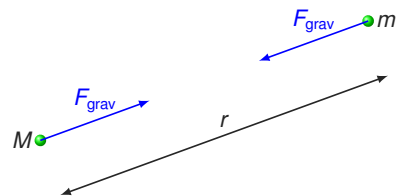
Newton's law of gravitation

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The gravitational force between two **point masses** is proportional to the **product of their masses** and inversely proportional to the **square of their distance**.

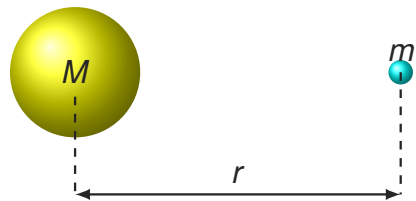
- $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
- always **attractive**; acts between **any** two masses

$$F_{\text{grav}} = \frac{GMm}{r^2}$$



Point masses and spheres

- Newton's law is for **point masses**: no size, so *r* is well defined
- a **uniform sphere** acts as a point mass at its centre
- *r* is the distance between **centres**
- shell theorem: no net force on anything inside a hollow shell



Example: gravity on a man

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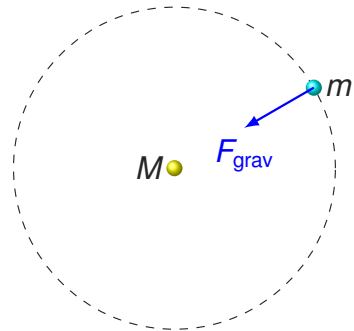
$$F = \frac{GMm}{R^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 60}{(6.4 \times 10^6)^2} \approx 586 \text{ N}$$

Orbits: gravity is the centripetal force

A satellite in circular orbit:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GMm}{r^2} = m\omega^2 r$$



Example: GPS satellites

Example: GPS satellites

(a)

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Rightarrow \quad v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^6 + 2.0 \times 10^7}} \approx 3.9 \times 10^3 \text{ m s}^{-1}$$

(b) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi \times (6.4 \times 10^6 + 2.0 \times 10^7)}{3.9 \times 10^3} \approx 4.3 \times 10^4$ s

Example: geostationary orbit

Example: geostationary orbit

$$\frac{GMm}{r^2} = m\omega^2 r = m\left(\frac{2\pi}{T}\right)^2 r \quad \Rightarrow \quad r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \left(\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (24 \times 3600)^2}{4\pi^2}\right)^{1/3} \approx 4.23 \times 10^7 \text{ m}$$

Example: Kepler's third law

Example: Kepler's third law

$$\frac{GMm}{r^2} = m\left(\frac{2\pi}{T}\right)^2 r \quad \Rightarrow \quad T^2 = \frac{4\pi^2}{GM} r^3$$

G and the Sun's mass M are constants, so $\frac{4\pi^2}{GM}$ is constant: $T^2 \propto r^3$.

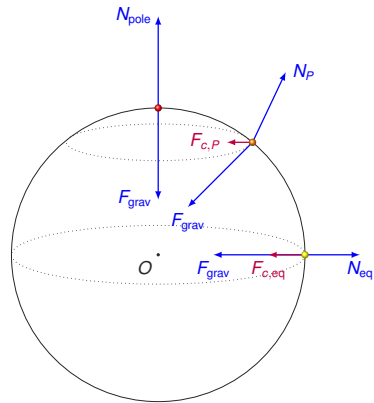
Kepler's third law

$$T^2 = \frac{4\pi^2}{GM} r^3$$

- $T^2 \propto r^3$ for all planets round the same star
- holds for elliptical orbits too

Apparent weight

- **actual weight**: the gravitational force on the object
- **apparent weight**: the upward contact force that stops it falling
- they differ with vertical acceleration, buoyancy, and the Earth's **rotation**
- at the equator: $N = \dfrac{GMm}{R^2} - m\omega^2 R$; at the poles: $N = \dfrac{GMm}{R^2}$



Example: a stone at the equator

Example: a stone at the equator

(a) $F_{\text{grav}} = \dfrac{GMm}{R^2} = \dfrac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 5.0}{(6.37 \times 10^6)^2} \approx 49.07$ N

(b) $F_c = m\omega^2 R = 5.0 \times \dfrac{4\pi^2}{(24 \times 3600)^2} \times 6.37 \times 10^6 \approx 0.17$ N

reading $= F_{\text{grav}} - F_c = 49.07 - 0.17 \approx 48.90$ N

Gravitational field strength

Gravitational field strength

The gravitational **force per unit mass**.

- unit: N kg^{-1} (= m s^{-2})
- a **vector**, pointing towards the source
- several sources: add as **vectors**

$$g = \frac{F_{\text{grav}}}{m}$$

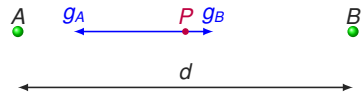
$$g = \frac{GM}{r^2}$$

Example: two stars

Star A of mass 6.0×10^{30} kg and star B of mass 1.5×10^{30} kg are 2.0×10^{12} m apart. (a) What is the field strength at the mid-point P? (b) What force acts on a 4.0×10^{6} kg comet at P?

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(a) g_A points towards A and g_B towards B: opposite directions.

$$g_P = g_A - g_B = \frac{GM_A}{r^2} - \frac{GM_B}{r^2}$$

$$= 6.67\times10^{-11}\,\frac{6.0 \times 10^{30} - 1.5 \times 10^{30}}{(1.0 \times 10^{12})^2} \approx 3.0\times10^{-4}\ \text{N kg}^{-1}$$

(b) $F = mg = 4.0 \times 10^{6} \times 3.0 \times 10^{-4} \approx 1.2 \times 10^{3}$ N

Acceleration of free fall

If gravity is the only force, $ma = mg$:

$$a = g$$

- field strength **is** the acceleration of free fall

Example: Mount Everest

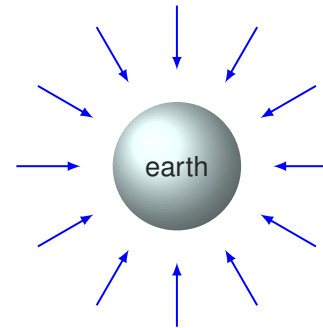
Example: Mount Everest

(a) at the surface $g_s = \dfrac{GM}{R^2} \quad \Rightarrow \quad 9.81 = \dfrac{6.67 \times 10^{-11} \times M}{(6.37 \times 10^6)^2} \quad \Rightarrow \quad M \approx 5.97 \times 10^{24}$ kg

(b) $g = \dfrac{GM}{(R+H)^2} = \dfrac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6 + 8.8 \times 10^3)^2} \approx 9.79 \text{ N kg}^{-1}$

Gravitational field lines

- **direction** of lines = direction of the field
- **spacing** shows strength: closer = stronger
- always **end on a mass**
- around a planet: **radial**; near the surface: **uniform**



Potential energy

Potential energy

Energy due to **position in a force field**. Work done **by** the field decreases P.E.; work done **against** it increases P.E.

$$W = -\Delta E_p$$

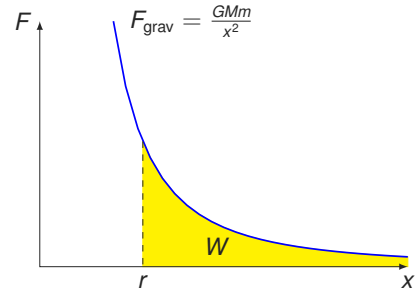
$$\Delta E_p = mg\,\Delta h$$

Gravitational potential energy

Work done by gravity bringing m in from infinity (area under the F–x graph):

$$E_p = -\frac{GMm}{r}$$

- always **negative**, a **scalar**; zero at infinity
- $E_p = mgh$ only near the surface (uniform field)



Example: escape velocity

Example: escape velocity

Energy at the surface = energy at infinity (the minimum case has zero K.E. there):

$$\tfrac{1}{2}mu^2 - \frac{GMm}{R} = \tfrac{1}{2}mv^2 + 0 \quad \text{with } v \geq 0 \;\Rightarrow\; u^2 \geq \frac{2GM}{R}$$

$$u_{\text{min}} = \sqrt{\frac{2GM}{R}}$$

$$u_{\text{min}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^{6}}} \approx 1.12 \times 10^{4} \text{ m s}^{-1}$$

Gravitational potential

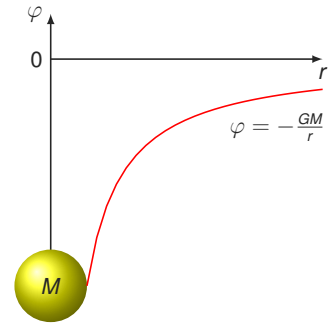
Gravitational potential

The **work done by gravity** bringing a **unit mass** from **infinity** to the point.

- unit: J kg^{-1}; always **negative**; a **scalar**
- gain in K.E. = loss in G.P.E.: $\frac{1}{2}v^2 = \Delta\varphi$

$$\varphi = \frac{E_p}{m}$$

$$\varphi = -\frac{GM}{r}$$



Example: potential at a point

Example: potential at a point

(a) $\varphi_P = \varphi_A + \varphi_B = -6.67 \times 10^{-11} \left(\dfrac{1.5 \times 10^{30}}{2.0 \times 10^{12}} + \dfrac{6.0 \times 10^{26}}{8.0 \times 10^{10}} \right) \approx -5.05 \times 10^{7}\ \text{J kg}^{-1}$

(b) gain in K.E. = loss in G.P.E.: $\frac{1}{2}v^2 = \varphi_\infty - \varphi_P = -\varphi_P$

$$v = \sqrt{2 \times 5.05 \times 10^{7}} \approx 1.01 \times 10^{4}\ \text{m s}^{-1}$$

Example: the Moon

Example: the Moon

(a) $\varphi(R) = -\dfrac{GM}{R} \quad \Rightarrow \quad M = \dfrac{2.82 \times 10^6 \times 1.74 \times 10^6}{6.67 \times 10^{-11}} \approx 7.36 \times 10^{22}$ kg

(b) $\Delta\varphi = -\dfrac{GM}{3R} + \dfrac{GM}{2R} = \dfrac{GM}{6R} = \dfrac{2.82 \times 10^6}{6} \approx 4.70 \times 10^5$ J kg^{-1} (a *decrease*)

(c) $\frac{1}{2}v^2 = \Delta\varphi \quad \Rightarrow \quad v = \sqrt{2 \times 4.70 \times 10^5} \approx 970$ m s^{-1}

Key equations

force	$F_{\text{grav}} = \dfrac{GMm}{r^2}$	energy	$E_p = -\dfrac{GMm}{r}$
orbit	$\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$	potential	$\varphi = -\dfrac{GM}{r}$
Kepler	$T^2 = \dfrac{4\pi^2}{GM} r^3$	escape	$u_{\text{min}} = \sqrt{\dfrac{2GM}{R}}$
field strength	$g = \dfrac{GM}{r^2}$		