Gravitational Fields

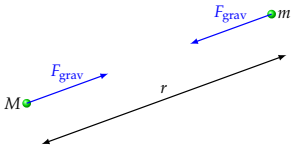
Name _____ Date _____

By the end of this topic I can…

- use Newton's law of gravitation, and apply it to circular orbits and Kepler's third law
- explain apparent weight and the effect of the Earth's rotation
- define field strength and potential, and use $g = GM/r^2$ and $\varphi = -GM/r$
- use $E_p = -GMm/r$ with energy conservation, including escape speed

Gravitational forces

Newton's law of gravitation



Gravitational attraction between M and m.

Any object with mass attracts any other massive object through the _____.

_____: the gravitational force between two _____ is proportional to the _____ and inversely proportional to the _____.

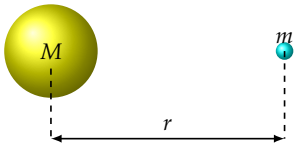
This law appeared in Newton's *Principia* (1687). In symbols:

where $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} is the _____.

• gravitational force is always _____, and acts between _____ two masses (it is universal)

• the law refers to _____: particles with no size, so the distance r is well defined

A sphere with uniform mass distribution (a star or planet) can be treated as a _____, so r is the distance between _____.[1]



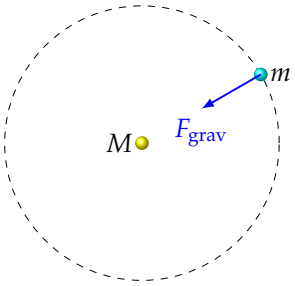
A uniform sphere acts as a point mass at its centre.

Example 19.1 The Earth is a uniform sphere of radius $R = 6.4 \times 10^6$ m and mass $M = 6.0 \times 10^{24}$ kg. Estimate the gravitational force on a 60 kg man at sea level.

[1] This is the *shell theorem*: a spherically symmetric shell attracts outside objects as if all its mass were at its centre, and exerts no net force on anything inside it.

Question 19.1 Estimate the gravitational force between you and the person nearest to you as you read this.

Planetary motion



A satellite in circular orbit.

A planet or satellite moves in a circular orbit when gravity provides the _____:

$$F_{grav} = F_c \quad \Rightarrow$$

or, using $v = \omega r$:

Example 19.2 GPS satellites move in circular orbits about 20 000 km above the Earth's surface. ($R = 6.4 \times 10^6$ m, $M = 6.0 \times 10^{24}$ kg.) (a) Find the speed of a GPS satellite. (b) Find its orbital period.

Example 19.3 A **geostationary satellite** stays above the same point on the equator, so its period is 24 hours. Find the radius of its orbit.

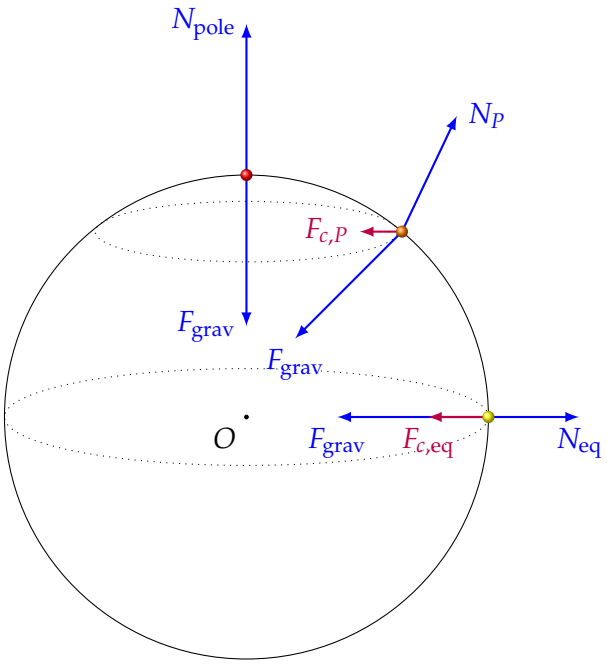
Example 19.4 Assuming circular orbits, show that for the planets the square of the orbital period is proportional to the cube of the orbital radius.[2]

[2] Kepler's third law (early 1600s). Newton showed that all three of Kepler's laws follow from his law of gravitation, and $T^2 \propto r^3$ holds for elliptical orbits too.

Question 19.2 Light takes about 8.0 minutes to travel from the Sun to the Earth. (a) What is the mass of the Sun? (b) At what speed does the Earth move round the Sun?

Apparent weight

The _____ of an object is the gravitational force on it. Its _____ is the upward _____ (from the ground or a spring balance) that opposes gravity and stops it falling. The two differ when there is vertical acceleration, or buoyancy, or because of the Earth's _____.



Apparent weight at the equator, at latitude P and at the pole (not to scale).

An object on the ground rotates with the Earth, so the resultant of gravity and the

contact force provides the _____. At the equator:

$$F_{\text{grav}} - N_{\text{eq}} = m\omega^2 R \quad \Rightarrow \quad N_{\text{eq}} = \frac{GMm}{R^2} - m\omega^2 R$$

At the poles there is no circular motion, so $N_{\text{pole}} =$ _____.

Nearer the equator the circle is larger, so more centripetal force is needed and the apparent weight is _____.

Example 19.5 A 5.0 kg stone hangs from a newton-meter at the equator. ($R = 6370$ km, $M = 5.97 \times 10^{24}$ kg.) (a) What is the gravitational force on the stone? (b) What does the meter read?

Question 19.3 Why are astronauts in a space station said to be *weightless*?

Question 19.4 How do you find the apparent weight of an object at an arbitrary latitude P? Does it act vertically downwards? Give your reasons.

Gravitational fields

To explain how masses attract at a distance we use the idea of a _____.

A _____ is a region of space where a _____.

Any mass M produces a field around it; a small _____ m placed in the field feels a force. We describe the field with _____ (to find forces) and _____ (to find energies).

Gravitational field strength

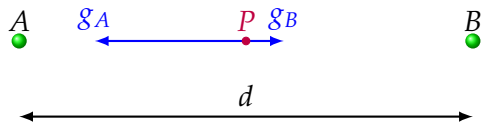
_____ is the gravitational _____:

- unit: $[g] =$ _____, which is the same as m s^{-2}
- for an isolated source M, at distance r a test mass feels $F = \dfrac{GMm}{r^2}$, so

g depends on the _____ mass M, not on the test mass m.

- g is a _____ and points _____ the source (gravity is attractive).

With several sources, add the contributions as _____.



Example 19.6 Star A of mass 6.0×10^{30} kg and star B of mass 1.5×10^{30} kg are 2.0×10^{12} m apart. (a) What is the field strength at the mid-point P? (b) What force acts on a 4.0×10^{6} kg comet at P?

Acceleration of free fall

If gravity is the only force, $F_{\text{net}} = F_{\text{grav}}$, so $ma = mg$:

Gravitational field strength is the _____.[3]

[3] Strictly, the two m's are different: inertial mass (resistance to acceleration) and gravitational mass. No experiment has found any difference between them. Einstein's equivalence principle, the

Example 19.7 The Earth has radius 6370 km. (a) Find the mass of the Earth. (b) Find the acceleration of free fall at the top of Mount Everest (height 8.8 km).[4]

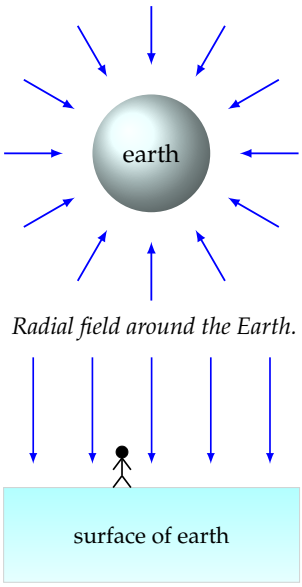
[4] Cavendish measured G in 1798 and could then 'weigh the world'.

Gravitational field lines

_____ show the pattern of field strength.

- the _____ of the lines is the direction of the field
- the _____ of the lines shows the strength: closer lines mean a _____ field
- gravitational field lines always _____ (gravity is attractive)

Around a planet the field is _____. Near its surface the lines are _____: a _____ field.



Radial field around the Earth.

Near the surface the field is almost uniform.

Gravitational potential and potential energy

Potential energy

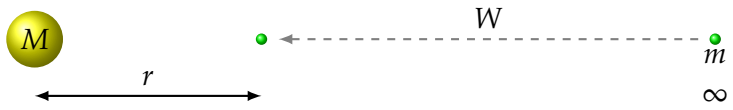
_____ is the energy an object has because of its position in a force field. Work done _____ the field _____ P.E.; work done _____ the field _____ P.E.:

To define P.E. at a point X: choose a position where $E_p = 0$, find the work W done by the field moving the object from there to X; then $E_{p,X} = -W$.

Near the Earth's surface take $E_p = 0$ at sea level. Lifting m through height h means work $-mgh$ is done by gravity, so

and $E_p =$ _____

Gravitational potential energy



We choose $E_p = 0$ at $r = \infty$ (no force, so no energy).

_____ is the
_____ bringing a mass to a point from
_____.

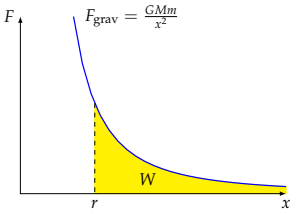
The force $\dfrac{GMm}{x^2}$ changes with x, so the work is the _____:[5]

$W = \dfrac{GMm}{r}$ and $\Delta E_p = -W$, with $E_p(\infty) = 0$, so

- $E_p \to 0$ as $r \to \infty$, matching our choice of zero
- E_p is a _____; it is always _____ because gravity is _____
- separating masses means doing work, so E_p _____ with r; the maximum, at infinity, is _____
- $E_p = mgh$ only works near the surface (uniform field); $E_p = -GMm/r$ is the _____ formula [6]

Example 19.8 A meteor is moving towards a planet of mass M. At distance r_1 its speed is v_1; at r_2 it is v_2. Only gravity acts. Relate these quantities.

[5] In general $W = \int F\,\mathrm{d}x$. Here $W = \int_r^\infty \frac{GMm}{x^2}\,\mathrm{d}x = \left[-\frac{GMm}{x}\right]_r^\infty = \frac{GMm}{r}$.



F–x graph: the shaded area is the work W.

[6] For $r_1 \approx r_2 \approx R$: $\Delta E_p = GMm\frac{r_1 - r_2}{r_1 r_2} \approx m\frac{GM}{R^2}\Delta r = mg\,\Delta h$.

Example 19.9 An object thrown fast enough from a planet's surface escapes its gravity. The minimum speed needed is the **escape velocity**. Find it for the Earth (use $R = 6.4 \times 10^6$ m, $M = 6.0 \times 10^{24}$ kg).

Question 19.5 A rock falls from rest from a height $3R$ above the surface of an airless planet of uniform density, radius R and mass M. Show that it hits the ground at speed $v = \sqrt{\dfrac{3GM}{2R}}$.

Question 19.6 A probe of mass m orbits a planet of mass M in a circle of radius r. (a) Show that its total mechanical energy is $E_{\text{total}} = -\dfrac{GMm}{2r}$. (b) If small resistive forces act, state the change in its orbital radius and speed.

Question 19.7 A *black hole* is a region where gravity is so strong that not even light can escape. (a) Show that a star of mass M must be compressed below the Schwarzschild radius $R_S = \dfrac{2GM}{c^2}$ to become one. (b) Show that R_S for the Sun ($M = 2.0 \times 10^{30}$ kg) is about 3 km.[7]

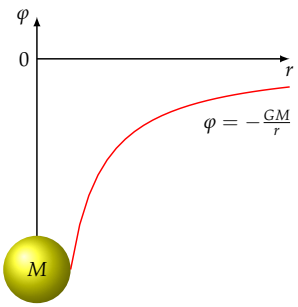
[7] For very strong fields Newtonian gravity fails and general relativity is needed. That the Newtonian and relativistic radii agree is a coincidence.

Gravitational potential

It is useful to define the potential energy _____: $\varphi = \dfrac{E_p}{m}$.

_____ at a point is the _____ bringing a _____ from _____ to that point.

- unit: $[\varphi]$ = _____
- for an isolated source M:



Potential against distance from an isolated mass.

- φ = ____ at infinity, and φ is always _____; it increases with distance from the source
- φ is a _____: for several sources, add the potentials as _____
- a mass moving through a change in potential $\Delta\varphi$ gains kinetic energy:

Example 19.10 Star A ($M_A = 1.5 \times 10^{30}$ kg) and planet B ($M_B = 6.0 \times 10^{26}$ kg) form an isolated system. Point P lies between them, $r_A = 2.0 \times 10^{12}$ m from A and $r_B = 8.0 \times 10^{10}$ m from B. (a) Find the potential at P. (b) A meteor starts at rest very far away. Find its speed at P.

Example 19.11 The Moon is an isolated sphere of radius 1.74×10^3 km, with potential -2.82×10^6 J kg^{-1} at its surface. (a) Find the Moon's mass. (b) A stone moves from $3R$ to $2R$ from the Moon's centre. Find the change in potential. (c) If it starts from rest, find its final speed.

Question 19.8 The Moon has radius 1700 km and mass 7.4×10^{22} kg. (a) Find the change in potential when an object rises from the surface to 800 km above it. (b) A rock is projected upwards at 1800 m s^{-1} from the surface. Find its speed at 800 km. (c) Can it escape completely?