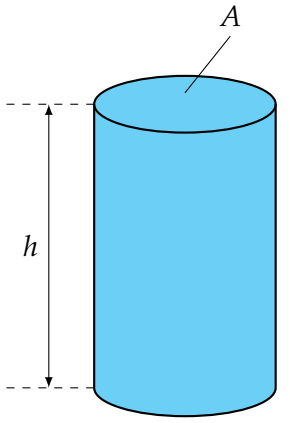
Fluids

Name _____ Date _____

By the end of this topic I can...
- derive and use $p = \rho g h$ for pressure in a fluid
- use manometers and barometers to find pressure
- explain the origin of upthrust and use Archimedes' principle



Column of fluid of area A and depth h.

Pressure in a fluid

A fluid (a liquid or a gas) has no fixed shape and can flow.

At depth h the weight of the fluid above produces a pressure:

$$p = \frac{F}{A} = \frac{mg}{A} = \frac{\rho V g}{A} \quad \Rightarrow \quad \boxed{p = \rho g h}$$

- pressure depends on depth and density, not on the shape or volume of the container
- points at the same depth in the same liquid have the same pressure
- the atmosphere also presses on the surface, so the total pressure is

$$\boxed{P = P_{\text{atm}} + \rho g h}$$

- the change in pressure is always $\boxed{\Delta p = \rho g \Delta h}$

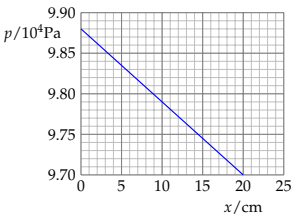
Example 8.1 The atmospheric pressure is 1.0×10^5 Pa and sea water has density 1020 kg m^{-3}. Find the total pressure 50 m below the surface.

$P = P_{\text{atm}} + \rho g h = 1.0 \times 10^5 + 1020 \times 9.81 \times 50 \approx 6.0 \times 10^5$ Pa.

Example 8.2 A 10 m vertical column of liquid contains oil (density 840 kg m^{-3}) on water (density 1000 kg m^{-3}). The pressure due to the liquids at the bottom is 89 kPa. Find the depth of the oil.

Let the oil depth be x: $840 \times 9.81\, x + 1000 \times 9.81\,(10 - x) = 89 \times 10^3$.
$10 - 0.16x = 9.07 \quad \Rightarrow \quad x \approx 5.8$ m.

Example 8.3 The pressure p of a liquid varies with height x above the base of its container as shown (total depth 20 cm). (a) Find the atmospheric pressure. (b) Find the density of the liquid.



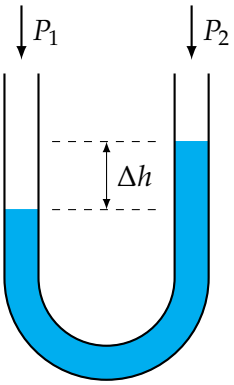
(a) At the surface ($x = 20$ cm) only the atmosphere acts: $P_{\text{atm}} = 9.70 \times 10^4$ Pa.
(b) $\rho = \dfrac{\Delta p}{g\Delta h} = \dfrac{(9.88 - 9.70) \times 10^4}{9.81 \times 0.20} \approx 9.2 \times 10^2$ kg m^{-3}.

Measuring pressure

Manometer

A **manometer** is a U-shaped tube of liquid. A pressure difference between the two ends gives a height difference Δh:

$$P_1 - P_2 = \rho g \Delta h$$



A U-tube manometer.

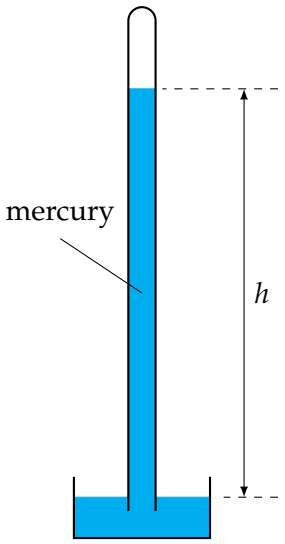
- mercury is often used because of its high density (1.36×10^4 kg m^{-3})

Example 8.4 Side A of a mercury manometer is connected to a gas supply and side B is open to the atmosphere (1.01×10^5 Pa). The mercury on side A is 14 cm higher than on B. Find the gas pressure.

$P_{\text{atm}} - P_{\text{gas}} = \rho g h \quad \Rightarrow \quad P_{\text{gas}} = 1.01 \times 10^5 - 1.36 \times 10^4 \times 9.81 \times 0.14 \approx 8.2 \times 10^4$ Pa.

Barometer

A tube of mercury inverted in a basin: the atmosphere supports a column of height h, so $P_{\text{atm}} =$ $\rho g h$.



A mercury barometer.

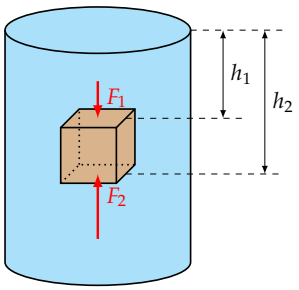
Example 8.5 A mercury barometer reads 760 mm. (a) Find the atmospheric pressure. (b) What minimum tube length is needed if water is used instead?

(a) $P_{\text{atm}} = 1.36 \times 10^4 \times 9.81 \times 0.760 \approx 1.01 \times 10^5$ Pa.
(b) $h' = \dfrac{P_{\text{atm}}}{\rho g} = \dfrac{1.01 \times 10^5}{1000 \times 9.81} \approx 10.3$ m.

Upthrust

The top and bottom faces of an immersed block are at different depths, so the pressure forces differ. The net upward force is the **upthrust**:

$$F_{\text{U}} = F_2 - F_1 = \rho g (h_2 - h_1) A \quad \Rightarrow \quad \boxed{F_{\text{U}} = \rho g V}$$



Forces on a submerged block.

Archimedes' principle: the upthrust on an immersed object equals the weight of fluid displaced.

- origin of upthrust: the pressure difference between the bottom and top surfaces
- object density ρ_{o}, liquid density ρ_{l}: resultant (down) $= (\rho_{\text{o}} - \rho_{\text{l}}) g V$
- $\rho_{\text{o}} > \rho_{\text{l}}$: the object sinks; $\rho_{\text{o}} < \rho_{\text{l}}$: it rises; equal: it stays at that level

Example 8.6 A block of mass 80 g and volume 50 cm^3 is held at rest, fully immersed in water, by a string. Find the tension.

$T + F_{\text{U}} = W \quad \Rightarrow \quad T = mg - \rho g V = 0.080 \times 9.81 - 1000 \times 9.81 \times 50 \times 10^{-6} \approx 0.29$ N.

Practice questions

Question 8.1 A dam holds a depth of 50 m of water. (a) Find the water pressure at its base. (b) Why are dam walls thicker near the bottom?

(a) $p = \rho g h = 1000 \times 9.81 \times 50 \approx 4.9 \times 10^5$ Pa.
(b) Pressure increases with depth, so the force on the wall is greater lower down.

Question 8.2 Estimate the pressure at the bottom of the Mariana Trench (depth 11 km) taking sea water density as 1030 kg m^{-3}.

$p = \rho g h = 1030 \times 9.81 \times 1.1 \times 10^4 \approx 1.1 \times 10^8$ Pa (plus atmospheric pressure, which is negligible here).

Question 8.3 A cylinder of trapped air is attached to a U-tube mercury manometer, open to the atmosphere (1.0×10^5 Pa) on the other side. The mercury on the open side is 45 mm higher. Find the pressure of the trapped air.

The gas pressure is greater: $P = P_{\text{atm}} + \rho g h = 1.0 \times 10^5 + 1.36 \times 10^4 \times 9.81 \times 0.045 \approx 1.06 \times 10^5$ Pa.

Question 8.4 A water manometer is connected to a flexible container with equal water levels. A girl stands on a 30 cm by 30 cm platform on the container and a height difference of 50 cm results. Find (a) the pressure she creates, (b) her mass.

(a) $p = \rho g h = 1000 \times 9.81 \times 0.50 \approx 4.9 \times 10^3$ Pa.
(b) $F = pA = 4905 \times (0.30 \times 0.30) = 441$ N, so $m = \dfrac{F}{g} = \dfrac{441}{9.81} = 45$ kg.

Question 8.5 A lead block and an aluminium block of identical size are fully immersed in water. On which is the upthrust greater?

The upthrust is the same on both: $F_{\text{U}} = \rho g V$ depends only on the fluid density and the volume displaced.

Question 8.6 A diver holds a cube of side 0.30 m and density 800 kg m^{-3} underwater (take water density 1000 kg m^{-3}). Find (a) the upthrust, (b) the initial acceleration when released.

(a) $F_U = \rho g V = 1000 \times 9.81 \times 0.30^3 \approx 2.6 \times 10^2$ N.
(b) $W = 800 \times 9.81 \times 0.027 = 212$ N. Net upward force $= 265 - 212 = 53$ N, $m = 21.6$ kg, so $a = \dfrac{53}{21.6} \approx 2.5$ m s^{-2} upwards.

Question 8.7 A sphere of radius 18 cm and density 2.4 g cm^{-3} is held fully submerged in water by a string. Find (a) the tension, (b) the acceleration just after the string is cut.

$V = \frac{4}{3}\pi(0.18)^3 = 0.0244$ m^3.
(a) $T = W - F_U = (2400 - 1000) \times 9.81 \times 0.0244 \approx 3.4 \times 10^2$ N.
(b) $a = \dfrac{T}{m} = \dfrac{(\rho_o - \rho_l)g}{\rho_o} = \dfrac{1400 \times 9.81}{2400} \approx 5.7$ m s^{-2} downwards.