Mechanical Equilibrium

AS Level Physics 9702

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By the end of this topic I can. . .

- define the moment of a force, the torque of a couple and the centre of gravity
- state and use the principle of moments
- state the conditions for equilibrium and apply them to beams, ladders and hanging objects
- use a closed force triangle for three forces in equilibrium

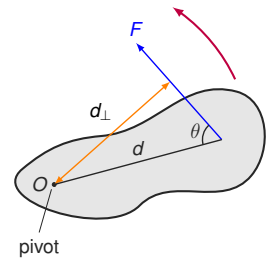
Moment of a force

Moment

Force × **perpendicular distance** from the pivot to the line of action.

- unit: N m; clockwise or anticlockwise
- depends on the pivot chosen

$$\text{moment} = F \times d_\perp$$



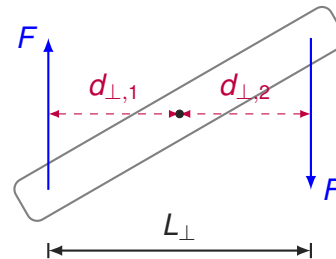
Torque of a couple

Couple

Two **equal, opposite, parallel** forces not on the same line: turning effect, no resultant force.

- $L_\perp$ is the perpendicular distance between the lines of action
- same torque about any point

$$\tau = F\,L_\perp$$

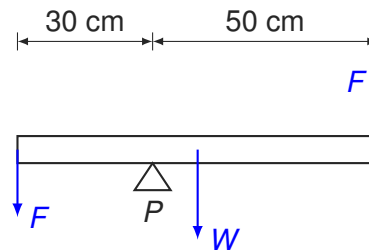


Example: moments on a beam

The diagram shows a uniform beam of weight $W = 20$ N and length 80 cm pivoted at point P. P is 30 cm from one end. Two equal but opposite forces of magnitude $F = 12$ N are acting at the two ends of the beam as shown. What is the resultant moment about point P?

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Moment of weight:
$\tau_w = Wd_w = 20 \times \left(0.50 - \frac{1}{2} \times 0.80\right) = 2.0$ N m (clockwise)

Torque of couple: $\tau_c = FL = 12 \times 0.80 = 9.6$ N m (anti-clockwise)

Resultant moment:

$\tau_{\text{net}} = \tau_c - \tau_W = 9.6 - 2.0 = 7.6$ N m (anti-clockwise)

Centre of gravity

Centre of gravity

The point where the **whole weight** of the object may be considered to act.

- uniform regular object: geometric centre
- hanging object: directly below the pivot
- stable if the weight acts inside the base when tilted

Principle of moments

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For a body in equilibrium, the sum of the **clockwise** moments equals the sum of the **anticlockwise** moments about any point.

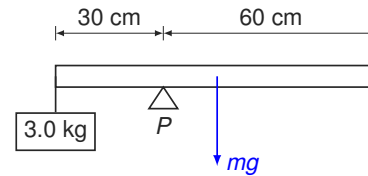
$$\sum \tau_{\text{clockwise}} = \sum \tau_{\text{anticlockwise}}$$

Example: balancing a rod

A uniform rod of length 90 cm is pivoted 30 cm from one end. It is balanced with a 3.0 kg load. Find the mass of the rod.

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Take moments about P:
$3.0 \times 9.81 \times 0.30 = m \times 9.81 \times \left(0.60 - \frac{1}{2} \times 0.90\right)$
so we find mass of rod: $m = 6.0$ kg

Example: the metre rule

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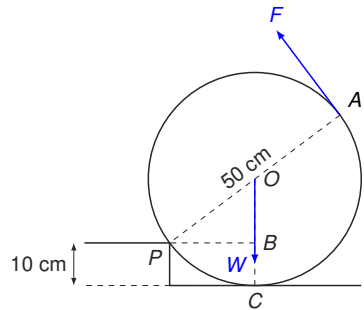
Moments about the support (the 40 cm mark), masses in grams, distances in cm:
anticlockwise: $m(40-15)+50(40-25)=25m+750$
clockwise: $120(50-40)+20(70-40)=1200+600=1800$
so $25m=1050$ and $m=42$ g.

Example: rolling over a kerb

A cylinder of weight 100 N and diameter 50 cm rests against point *P* of a curb of height 10 cm. What is the minimum force required to cause the cylinder to roll to the left?

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To roll over the kerb the moment of F about P must at least equal the moment of the weight; F is smallest when its perpendicular distance from P is greatest ($|PA| = 50$ cm, the diameter).
Horizontal distance of the weight from P:
$|PB| = \sqrt{0.25^2 - 0.15^2} = 0.20$ m.
$F_{\min} \times 0.50 = 100 \times 0.20 \quad \Rightarrow \quad F_{\min} = 40$ N

Conditions for equilibrium

Equilibrium

Resultant force is **zero** and the resultant moment about any point is **zero**.

$$\sum F = 0$$

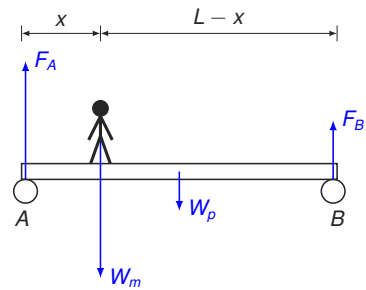
$$\sum \tau_{\text{clockwise}} = \sum \tau_{\text{anticlockwise}}$$

Example: a plank on two supports

A uniform plank of weight 100 N and length $L = 6.0$ m rests horizontally on two supports A and B. A man of weight 800 N stands a distance of $x = 1.5$ m from end A. Determine the forces acting at the two supports.

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Take moments about A: $W_m x + W_p \cdot \frac{L}{2} = F_B L$

$800 \times 1.5 + 100 \times 3.0 = F_B \times 6.0 \Rightarrow F_B = 250$ N

Take moments about B: $W_m(L - x) + W_p \frac{L}{2} = F_A L$

$800 \times 4.5 + 100 \times 3.0 = F_A \times 6.0 \Rightarrow F_B = 650$ N

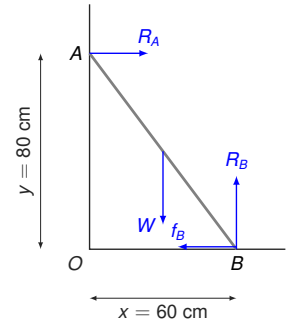
One can check that: $F_A + F_B = W_m + W_p$, there must be no resultant force in vertical direction

Example: a ladder

A uniform ladder of weight 120 N rests on a rough ground against a smooth wall as shown. The dimensions are labelled on the diagram. (a) What is the contact force acting at B? (b) What is the contact force acting at A? (c) What is the frictional force at B?

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Free-body diagram is drawn as shown

resolve vertically: $R_B = W \quad \Rightarrow \quad R_B = 120$ N

take moments about B: $R_A y = W\frac{x}{2}$

$$R_A = \frac{120 \times 0.30}{0.80} = 45 \text{ N}$$

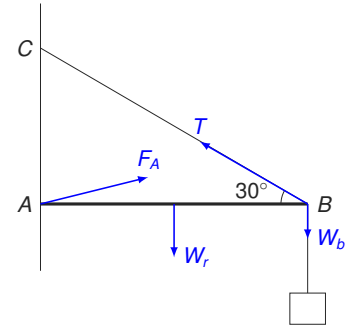
resolve horizontally: $f_B = R_A \quad \Rightarrow \quad f_B = 45$ N

Example: rod held by a string

The diagram shows a uniform rod AB of weight 60 N being held horizontally to a vertical wall by means of a light string. The string is attached to the rod at B, where a basket of weight 40 N is suspended. The other end of the string is fixed on the wall at C. The angle between the string and the rod is 30°. (a) Find the tension in the string. (b) Find the force acting on the rod at point A.

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Take moments about A: $TL\sin\theta = W_b L + W_r \frac{1}{2} L$

$T\sin 30^\circ = 40 + 60 \times \frac{1}{2} \quad \Rightarrow \quad T = 140$ N

resolve horizontally:

$F_{A,x} = T\cos\theta \quad \Rightarrow \quad F_{A_x} = 140\cos 30^\circ \approx 121$ N

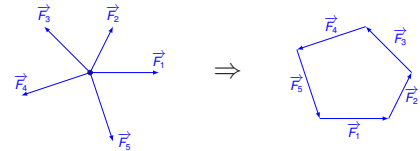
resolve vertically: $F_{A,y} + T\sin\theta = W_r + W_b \quad \Rightarrow$

$F_{A,y} = 60 + 40 - 140\sin 30^\circ = 30$ N

force at A: $F_A = \sqrt{F_{A,x}^2 + F_{A,y}^2} \quad \Rightarrow \quad F_A =$

Three forces in equilibrium

- zero resultant force: a closed **force triangle**
- zero resultant moment: lines of action are **concurrent**
- two forces: equal, opposite and along the same line

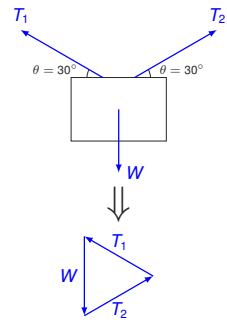


Example: a hanging painting

A painting of weight $W = 20$ N is supported by two strings as shown. Both strings form an angle $\theta = 30^\circ$ to the horizontal. Find the tension in the strings.

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By resolving forces, we have:

$$\begin{cases} T_1 \cos\theta = T_2 \cos\theta \\ T_1 \sin\theta + T_2 \sin\theta = W \end{cases}$$

we solve the equations to obtain:

$$T_1 = T_2 = \frac{W}{2\sin\theta} = \frac{20}{2\sin 30^\circ} = 20 \text{ N}$$

alternatively, we can construct the force triangle as shown

T_1, T_2 and W form an equilateral triangle, so

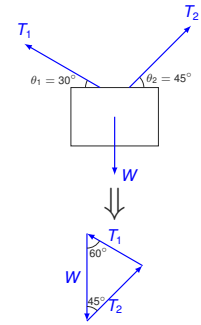
$$T_1 = T_2 = W = 20 \text{ N}$$

Example: unequal angles

The same painting of weight $W = 20$ N is supported by two strings at different angles $\theta_1 = 30^\circ$ and $\theta_2 = 45^\circ$ as shown. Find the forces in the two strings.

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The same painting of weight $W = 20$ N is supported by two strings at different angles $\theta_1 = 30°$ and $\theta_2 = 45°$ as shown. Find the forces in the two strings.



By resolving forces, we have:

$$\begin{cases} T_1 \cos\theta_1 = T_2 \cos\theta_2 \\ T_1 \sin\theta_1 + T_2 \sin\theta_2 = W \end{cases} \Rightarrow \begin{cases} \frac{\sqrt{3}}{2} T_1 = \frac{\sqrt{2}}{2} T_2 \\ \frac{1}{2} T_1 + \frac{\sqrt{2}}{2} T_2 = 20 \end{cases}$$

solving this, we find: $T_1 \approx 14.6$ N , $T_2 \approx 17.9$ N

alternatively, we construct the force triangle as shown

T_1 and T_2 are related to W by the the law of sine:

$$\frac{W}{\sin 75°} = \frac{T_1}{\sin 45°} = \frac{T_2}{\sin 60°}$$

from this we get the same result:

$T_1 \approx 14.6$ N , $T_2 \approx 17.9$ N

Key equations

moment	$\text{moment} = F \times d_\perp$	forces	$\sum F = 0$
couple	$\tau = F\, L_\perp$	moments	$\sum \tau_{\text{clockwise}} = \sum \tau_{\text{anticlockwise}}$