Mechanical Equilibrium

Name _____ Date _____

By the end of this topic I can…

- define the moment of a force, the torque of a couple and the centre of gravity
- state and use the principle of moments
- state the two conditions for equilibrium and apply them to beams, ladders and hanging objects
- use a closed force triangle for three forces in equilibrium

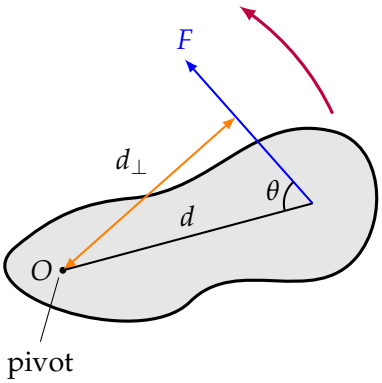
Moments and couples

For a point object zero resultant force is enough for equilibrium. For an extended (rigid) body a force can also have a _____.

Moment of a force

The **moment** of a force about a point is the _____ multiplied by the _____ from the point to the _____ of the force:

[moment] = _____



Moment of a force about a pivot O.

• resolve either the force or the distance so that the two are _____: a force F at angle θ to a lever of length d has moment $Fd\sin\theta$

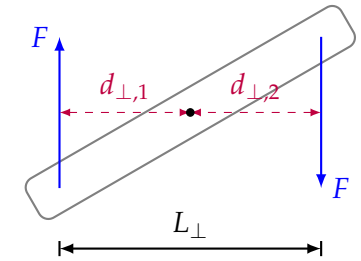
• a moment is _____ or _____

• its value depends on the _____ chosen: choose a pivot where an unknown force acts to remove it from the equation

Torque of a couple

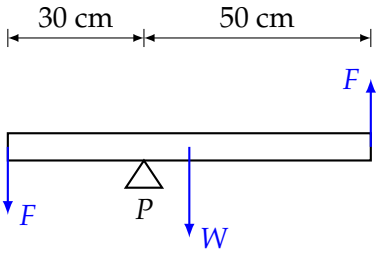
A **couple** is a pair of _____, _____ and _____ forces whose lines of action do not coincide. It produces a turning effect but _____.

The **torque of a couple** is one of the forces multiplied by the _____:



A couple: two equal and opposite forces.

The torque of a couple is the same about _____.



Example 4.1 The diagram shows a uniform beam of weight $W = 20$ N and length 80 cm pivoted at point P. P is 30 cm from one end. Two equal but opposite forces of magnitude $F = 12$ N are acting at the two ends of the beam as shown. What is the resultant moment about point P?

Centre of gravity

The **centre of gravity** is the point where the _____ of an object may be considered to act.

- a uniform regular object has its centre of gravity at its _____
- a freely suspended object comes to rest with its centre of gravity _____ the point of suspension
- to find it, suspend the object from two or more points and draw _____; they cross at the centre of gravity

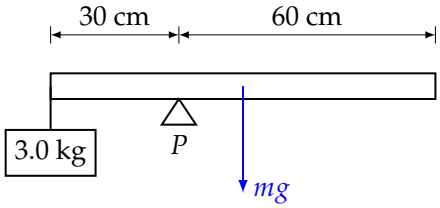
Stability

An object is **stable** if, when tilted slightly, the moment of its weight about the pivot edge turns it _____. The weight must act on the _____ of the base. Low centre of gravity and a wide base make an object more stable.

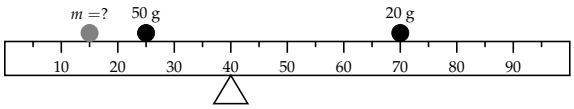
Equilibrium of rigid bodies

Principle of moments

For a body in equilibrium the sum of the _____ moments about any point equals the sum of the _____ moments about that point:

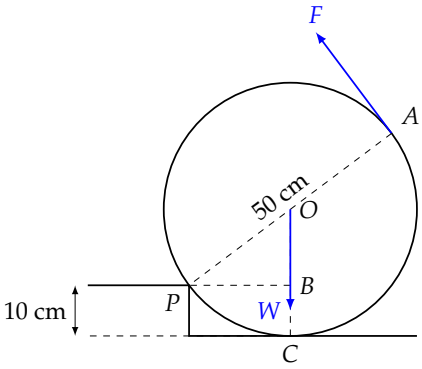


Example 4.2 A uniform rod of length 90 cm is pivoted 30 cm from one end. It is balanced with a 3.0 kg load. Find the mass of the rod.



Example 4.3 A student balances a metre rule of mass 120 g supported on a fulcrum at the 40 cm mark. She then places a 20 g mass on the 70 cm mark and a 50 g mass on the 25 cm mark as shown. To balance the rule, what mass should she place on the 15 cm mark?

Example 4.4 A cylinder of weight 100 N and diameter 50 cm rests against point P of a curb of height 10 cm. What is the minimum force required to cause the cylinder to roll to the left?

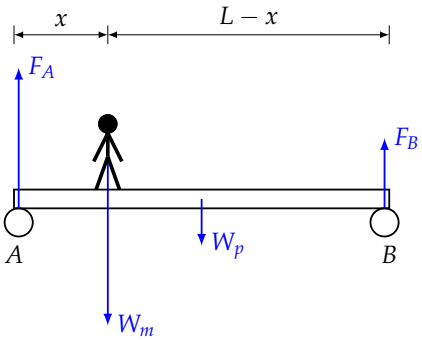


The conditions for equilibrium

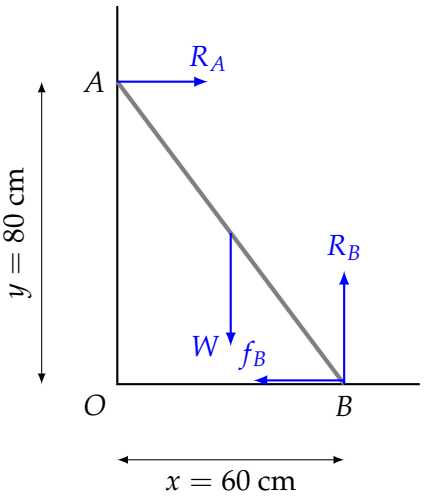
A body is in equilibrium when

- the resultant force is _____: in any direction
- the resultant moment about any point is _____

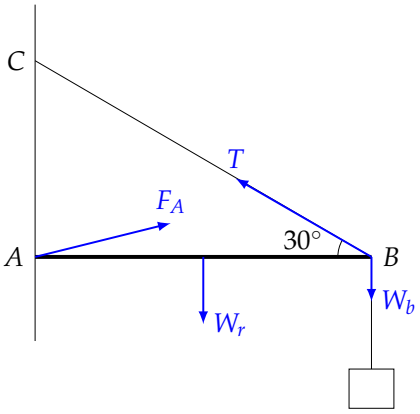
Example 4.5 A uniform plank of weight 100 N and length $L = 6.0$ m rests horizontally on two supports A and B. A man of weight 800 N stands a distance of $x = 1.5$ m from end A. Determine the forces acting at the two supports.



Example 4.6 A uniform ladder of weight 120 N rests on a rough ground against a smooth wall as shown. The dimensions are labelled on the diagram. (a) What is the contact force acting at B? (b) What is the contact force acting at A? (c) What is the frictional force at B?



Example 4.7 The diagram shows a uniform rod AB of weight 60 N being held horizontally to a vertical wall by means of a light string. The string is attached to the rod at B, where a basket of weight 40 N is suspended. The other end of the string is fixed on the wall at C. The angle between the string and the rod is 30°. (a) Find the tension in the string. (b) Find the force acting on the rod at point A.



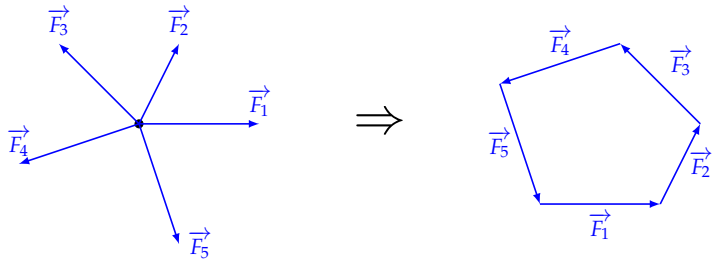
Forces in equilibrium

Two forces

Two forces in equilibrium must be _____ (zero resultant force) and act along the _____ (otherwise they form a couple).

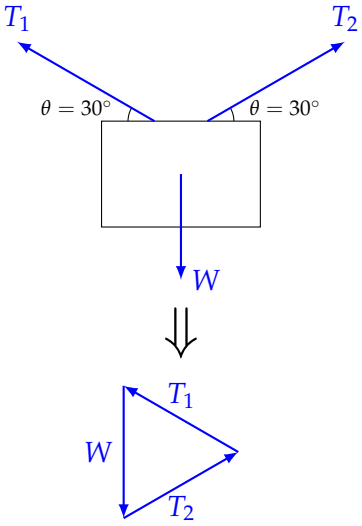
Three forces

- zero resultant force: the three force vectors form a closed **force triangle** when joined _____
- zero resultant moment: the three lines of action must pass through _____ (they are **concurrent**)

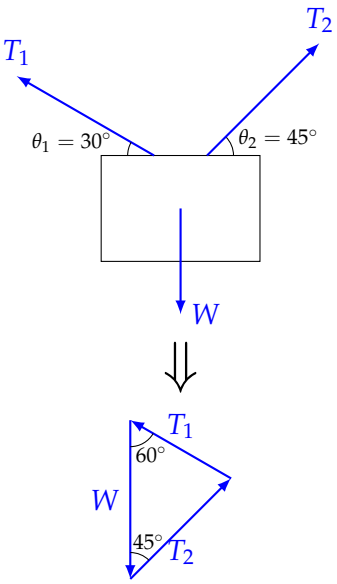


A set of forces in equilibrium forms a closed polygon.

Example 4.8 A painting of weight $W = 20$ N is supported by two strings as shown. Both strings form an angle $\theta = 30°$ to the horizontal. Find the tension in the strings.



Example 4.9 The same painting of weight $W = 20$ N is supported by two strings at different angles $\theta_1 = 30°$ and $\theta_2 = 45°$ as shown. Find the forces in the two strings.



Practice questions

Question 4.1 Is it possible for an object to be in equilibrium if only one force is acting on it?

Question 4.2 If three forces are in equilibrium, suggest and explain whether the lines of action must lie in the same plane.

Question 4.3 Two children sit on a uniform see-saw balanced on its centre. A child of mass 25 kg sits 1.8 m from the pivot. Where must a child of mass 30 kg sit to balance it?

Question 4.4 A driver turns a steering wheel of diameter 0.40 m by applying a force of 15 N with each hand, in opposite directions on opposite sides of the wheel. What is the torque applied?