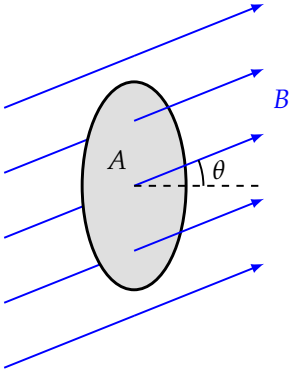
Electromagnetic Induction

Name _____ Date _____

By the end of this topic I can...
- define magnetic flux and flux linkage
- state and use Faraday's law and Lenz's law
- find the e.m.f. $\mathcal{E} = BLv$ of a conductor cutting field lines
- explain eddy currents, magnetic braking and the a.c. generator

Flux and the laws of induction

Magnetic flux



Magnetic flux is the product of a closed area and the flux density through it at right angles:

$$\phi = BA\cos\theta$$

where θ is the angle between B and the normal to the area.

- unit: the weber (Wb), 1 Wb = 1 T m^2
- if B is perpendicular to the area, $\phi =$ BA
- for a coil of N turns the flux linkage is

$$\Phi = N\phi = NBA$$

Think of it as the number of field lines passing through the coil.

- flux changes when B, A or θ changes: moving a magnet towards or away from a coil, varying the current in a solenoid, inserting an iron core, pushing a conductor through a field, rotating a coil in a field

Question 27.1 Explain why each process listed above gives a change in magnetic flux.

Magnet moved: B at the coil changes. Varying solenoid current: B changes. Iron core inserted: B increases (large permeability). Conductor pushed through a field: the area of the circuit enclosed changes (A changes). Coil rotated: the angle θ between B and the normal changes.

Faraday's law

Michael Faraday and Heinrich Lenz discovered the laws in the 1830s.

Faraday's law : the induced e.m.f. is proportional to the rate of change of flux linkage.

- in SI units: $\mathcal{E} = -\dfrac{\Delta\Phi}{\Delta t}$
- it gives the magnitude of the e.m.f.; what matters is the change in flux. No change means no e.m.f.

Example 27.1 A coil of 80 turns is wound tightly round a solenoid of cross-sectional area 35 cm^2. The flux density at the centre of the solenoid is 75 mT. (a) Find the flux linkage. (b) The solenoid current is reversed in 0.40 s. Find the average induced e.m.f.

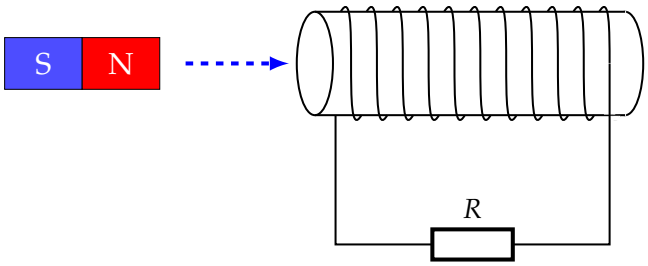
(a) $\Phi = NBA = 80 \times 75 \times 10^{-3} \times 35 \times 10^{-4} = 0.021$ Wb

(b) $\mathcal{E} = \dfrac{\Delta\Phi}{\Delta t} = \dfrac{0.021 - (-0.021)}{0.40} = 0.105 \text{ V} \approx 0.11 \text{ V}$

Lenz's law

Lenz's law : the induced current is in a direction so that its effects oppose the change in flux that produces it.

- it gives the direction (polarity); this is the minus sign in Faraday's law
- it follows from conservation of energy : induced current produces heat, so work must be done against the opposing force. If a motion causes the change, the magnetic force on the induced current resists the motion.



Example 27.2 A coil of 200 turns of diameter 10 cm is connected to a resistor $R = 4.0\ \Omega$. A bar magnet far away is pushed (N pole first) into the coil in 2.0 s until the field inside is 0.40 T. (a) Find the average induced e.m.f. (b) Find the average current. (c) In which direction does the current flow through R?

(a) $\Delta\Phi = NBA = 200 \times 0.40 \times \pi \times 0.050^2 = 0.628$ Wb

$\mathcal{E} = \dfrac{\Delta\Phi}{\Delta t} = \dfrac{0.628}{2.0} = 0.314$ V

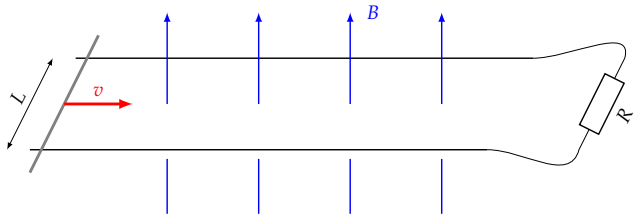
(b) $I = \dfrac{\mathcal{E}}{R} = \dfrac{0.314}{4.0} = 0.0785$ A

(c) Flux to the right is increasing. By Lenz's law the induced field points to the **left**, so the left end of the coil behaves like a north pole and repels the magnet. The right-hand grip rule then gives current through R **to the right**.

Question 27.2 The magnet is now pulled away. State and explain the direction of the induced current.

The flux to the right now decreases, so the induced field points **right**: the left end of the coil is a south pole and attracts the magnet, opposing its removal. The current through R is **reversed** (to the left).

Motional e.m.f.



Example 27.3 Two parallel tracks $L = 45$ cm apart are in a uniform field of 0.80 T (upwards), joined by a resistor $R = 6.0\ \Omega$. A metal rod is pushed along the tracks at $v = 3.0\ \text{m s}^{-1}$ towards R. (a) Find the induced current. (b) State its direction (seen from above). (c) Find the external force needed for constant speed.

In time Δt the area changes by $\Delta A = Lv\Delta t$, so $\Delta\phi = B\Delta A = BLv\Delta t$ and

$$\mathcal{E} = \frac{\Delta\phi}{\Delta t} = BLv$$

(a) $I = \dfrac{\mathcal{E}}{R} = \dfrac{BLv}{R} = \dfrac{0.80 \times 0.45 \times 3.0}{6.0} = 0.18\ \text{A}$
(b) The enclosed area (and upward flux) decreases, so the induced field is upwards: **anticlockwise** seen from above.
(c) Constant speed: $F_{\text{ext}} = F_B = BIL = 0.80 \times 0.18 \times 0.45 \approx 0.065\ \text{N}$

- for a straight conductor cutting field lines at right angles:

$$\mathcal{E} = BLv$$

Question 27.3 Find another way to determine the direction of the induced current in the rod above.

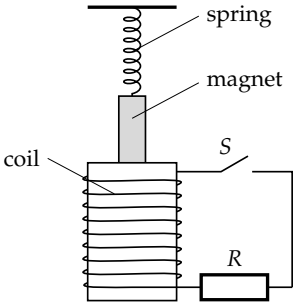
Use the force on the free charges in the rod, which move with velocity v through the field: left-hand rule (or $F = qv \times B$) gives the direction of the current in the rod. Alternatively, the magnetic force on the current must oppose the motion (to the left), and the left-hand rule then gives the current direction.

Applications

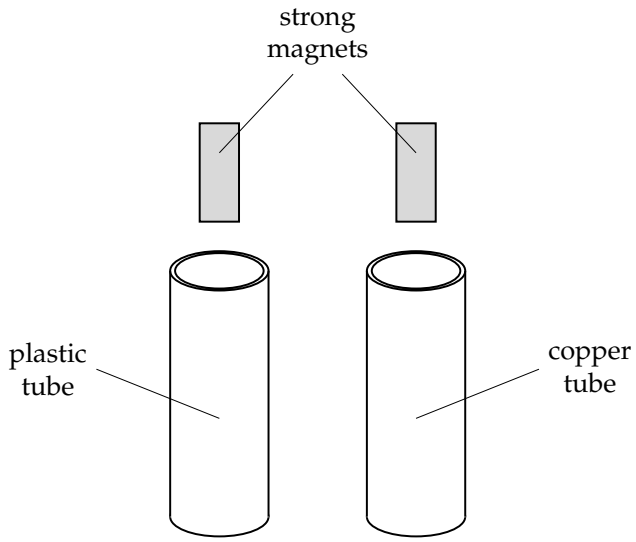
Magnetic braking

• **damped pendulum**: a metal disc swinging between the poles of an electromagnet stops quickly when the magnet is switched on. Different parts of the disc see different rates of change of flux, producing circulating eddy currents.

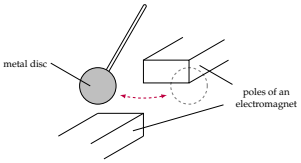
• the eddy currents dissipate energy as heat and the magnetic force on them opposes the motion: the oscillation is damped



• **falling magnet**: dropped through a plastic tube the magnet falls freely ($a = g$). Through a copper tube eddy currents are induced and oppose the motion, so $a < g$ and the magnet falls more slowly.

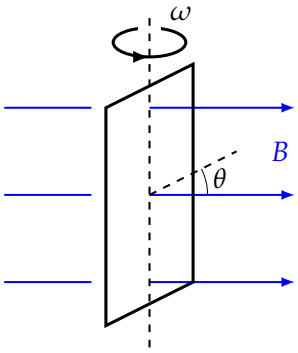


Question 27.4 A magnet hangs from a spring and oscillates in and out of a coil. With switch S open the amplitude hardly changes; with S closed it dies away quickly. Explain.



With S open no current can flow, so there is no magnetic force. With S closed, the induced e.m.f. drives a current which, by Lenz's law, produces a force **opposing** the magnet's motion. Energy of oscillation is dissipated as heat in the circuit.

The generator



A coil of N turns and area A rotates at constant angular speed ω in a uniform field B. Taking $\theta = \omega t$ as the angle between the coil's plane and the field ($\theta = 0$ at $t = 0$):

$$\Phi = NBA\sin\omega t$$

$$\mathcal{E} = \frac{\mathrm{d}\Phi}{\mathrm{d}t} = \boxed{\mathcal{E} = \omega NBA\cos\omega t}$$

a sinusoidal (alternating) e.m.f. with maximum $\mathcal{E}_{\max} =$ ωNBA.

- in practice, a strong electromagnet rotates inside fixed coils

Question 27.5 State and explain the effect on the voltage if the coil rotates faster.

$\mathcal{E}_{\max} = \omega NBA$ increases, and the frequency increases too, since the flux changes faster.

Question 27.6 Is the flux in phase with the e.m.f.? If not, what is the phase difference?

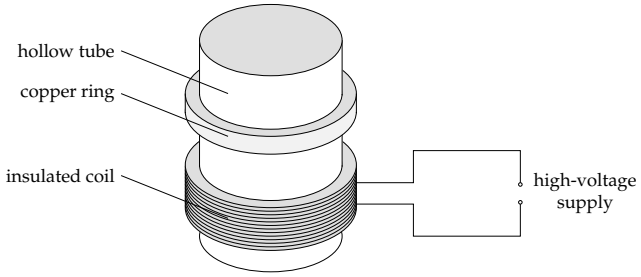
No. $\Phi \propto \sin\omega t$ and $\mathcal{E} \propto \cos\omega t$: the e.m.f. leads the flux by 90° ($\pi/2$ rad).

Question 27.7 Is the output maximum when the coil is parallel to the field or at right angles to it?

Maximum when the coil is **parallel** to the field: the flux is zero but changing fastest. At right angles the flux is a maximum and the e.m.f. is zero.

Electromagnetic gun

A light copper ring sits on a tube wound with a coil. When a high-voltage supply is switched on, the flux in the coil rises very quickly and induces a large e.m.f. and current in the ring. By Lenz's law the force on the ring is repulsive, much greater than its weight, so the ring jumps up.



Question 27.8 The coil is connected to a steady d.c. supply. An iron core is quickly inserted into the tube. What might happen to the copper ring?

Inserting iron greatly increases the flux through the ring, so a current is induced in the ring in the opposite sense to the coil's. The force is **repulsive**, so the ring jumps up (briefly).

Further applications to research: wireless charging, induction cooking, contactless payment, stylus pens for tablets.