Electrical Quantities

Name _____ Date _____

By the end of this topic I can...
- use charge, current and the microscopic equation $I = nAvq$
- define p.d., e.m.f., resistance and resistivity and use them
- calculate electrical power
- sketch and interpret I–V graphs for wires, lamps, thermistors and diodes

Charge and current

• electric charge is measured in coulombs (C); the elementary charge is $e =$ 1.60×10^{-19} C

• charge is **quantised**: $Q = Ne$ for a whole number N of electrons

• **conservation of charge**: the total charge of an isolated system is constant

Example 11.1 A metal sphere carries a net charge of $+2.4 \times 10^{-9}$ C. Suggest whether the sphere has gained additional electrons or lost electrons, and also calculate how many electrons has the sphere gained or lost?

The sphere is positively-charged, so it has lost some of its electrons.
The number of lost electrons: $N = \frac{Q}{e} = \frac{2.4\times10^{-9}}{1.60\times10^{-19}} \quad \Rightarrow \quad N = 1.5 \times 10^{10}$

Electric current is the charge flow per unit time :

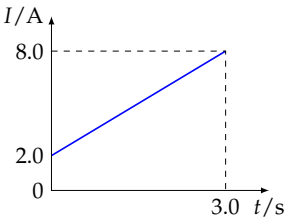
$$I = \frac{\Delta Q}{\Delta t} \qquad [I] = \text{A}$$

• **conventional current** flows from positive to negative, opposite to the electron flow

• $Q = It$ only for a constant current; otherwise use the area under an I–t graph

Example 11.2 A current of 25 mA flows in a wire. How many electrons have passed a point in the wire in one hour?

Charge flow: $Q = It = 25 \times 10^{-3} \times 3600 \quad \Rightarrow \quad Q = 90\,\text{C}$
Number of electrons: $N = \frac{Q}{e} = \frac{90}{1.60\times10^{-19}} \quad \Rightarrow \quad N \approx 5.6 \times 10^{20}$

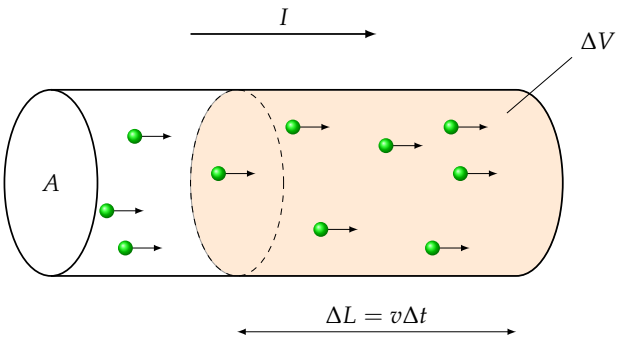


Example 11.3 The current in a lamp is increased uniformly from 2.0 A to 8.0 A over 3.0 s. What is the charge flow?

Average current during this time is: $\bar{I} = \frac{2.0+8.0}{2} = 5.0$ A
charge flow is then: $Q = \bar{I}t = 5.0 \times 3.0 = 15$ C
we can also use area under I-t graph

$$Q = \frac{1}{2} \times (2.0 + 8.0) \times 3.0 = 15 \text{ C}$$

Microscopic view of current



Let n be the number of charge carriers per unit volume, each with charge q and **mean drift velocity** v in a wire of cross-section A. In time Δt the charge passing a point is $nAv\Delta t\, q$, so

$$I = nAvq$$

- metals have a large n, semiconductors a small n

Example 11.4 A length of silver wire of a diameter of 1.2 mm carries a current of 2.0 A. The electron number density in silver is 5.9×10^{28} m^{-3}. What is the mean drift velocity of the electrons?

$$v = \frac{I}{nAe} = \frac{I}{n \cdot \frac{1}{4}\pi d^2 \cdot e} \Rightarrow$$

$$\frac{2.0}{5.9 \times 10^{28} \times \frac{1}{4}\pi \times (1.2 \times 10^{-3})^2 \times 1.60 \times 10^{-19}} \approx 1.9 \times 10^{-4} \text{ m s}^{-1}$$

This calculation shows that each free electron actually travel at very low speeds in a wire

Example 11.5 A metal conductor and a piece of semi-conductor of the same cross section are connected in series. When an electric current is driven through them, compare the mean drift speed of the charge carriers in the two materials.

The same current and same cross section, so mean drift speed $v = \frac{I}{nAe} \propto \frac{1}{n}$
The metal has larger n, so free electrons move at relatively lower speeds
The semi-conductor has smaller n, so charge carriers move at relatively higher speeds

Potential difference and e.m.f.

Potential difference is the energy converted from electrical to other forms per unit charge passing between two points:

$$V = \frac{W}{Q}$$

E.m.f. is the energy converted from other forms to electrical per unit charge passing through the source:

$$\mathcal{E} = \frac{W}{Q}$$

- unit: the volt, 1 V = $1\,\text{J}\,\text{C}^{-1}$
- a voltmeter (in parallel) measures p.d.; an ammeter (in series) measures current

Example 11.6 How much electrical energy is transformed into thermal energy when a charge of 10 C flows through a heater whose p.d. across is 25 V?

$$V = \frac{W}{Q} \quad \Rightarrow \quad W = VQ = 25 \times 10 = 250\,\text{J}$$

Example 11.7 A fully-charged 12 V car battery can supply a total electrical energy of 0.90 MJ. The starter motor of the car requires an average current of 150 A for a period of 2.0s. The battery is not able to recharge due to a fault. How many times can the starter motor be used?

total charge that can be supplied: $Q_{\text{total}} = \frac{W}{\mathcal{E}} = \frac{0.90\times10^6}{12} = 75000$ C
charge needed for each start: $Q = It = 150 \times 2.0 = 300$ C
number of starts possible: $N = \frac{Q_{\text{total}}}{Q} = \frac{75000}{300} = 250$

Example 11.8 Electrons beams are accelerated between a heated cathode and an anode with a potential difference of 40 V. What is the speed of the electrons when they reach the anode?

electrical potential energy is transformed into kinetic energy of electrons:

$$qV = \frac{1}{2}mv^2 \quad \Rightarrow$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 40}{9.11 \times 10^{-31}}} \approx 3.75 \times 10^6 \text{ m s}^{-1}$$

Resistance and resistivity

Resistance is the p.d. across a component divided by the current through it:

$$R = \frac{V}{I}$$

$[R] =$ V A^{-1} $=$ Ω

- for a uniform wire $R \propto$ L and $R \propto$ 1/A :

$$R = \frac{\rho L}{A}$$

where ρ is the **resistivity** of the material (unit Ω m; metals about 10^{-8} Ω m)

Example 11.9 A current of 0.10 A is driven through a heater when connected to a supply voltage of 220 V. (a) Find the resistance of the heater. (b) If the heater is made from a wire of resistivity $1.8 \times 10^{-5}\ \Omega$ m and a diameter of 0.24 mm, find the length of the wire.

$$R = \frac{V}{I} = \frac{220}{0.10} \quad \Rightarrow \quad R = 2200\ \Omega$$

$$R = \frac{\rho L}{A} \quad \Rightarrow \quad L = \frac{RA}{\rho} = \frac{2200 \times \pi \times (0.12 \times 10^{-3})^2}{1.8 \times 10^{-5}} \quad \Rightarrow \quad L \approx 5.5\ \text{m}$$

Example 11.10 A uniform wire of resistance R has a length of L and a cross-sectional area of A. If the wire is stretched to twice the length while its volume remains constant, what is the new resistance R' in terms of R?

volume: $V = LA =$ constant, so $A \propto \frac{1}{L}$, hence doubling L means A would become halved

resistance: $R = \frac{\rho L}{A} \propto \frac{L}{A} \quad \Rightarrow \quad \frac{R'}{R} = \frac{L'}{L} \times \frac{A}{A'} = 2 \times 2 \quad \Rightarrow \quad R' = 4R$

Electrical power

From $P = \dfrac{\Delta W}{\Delta t}$ and $V = \dfrac{W}{Q}$:

$P = IV$ $P = I^2R$ $P = \dfrac{V^2}{R}$

- unit: the watt, $1\ \text{W} = 1\ \text{A} \times 1\ \text{V}$

Example 11.11 An electric toaster is labelled '220 V 900 W'. When it is operating normally, find (a) the current through it, (b) the resistance in the toaster.

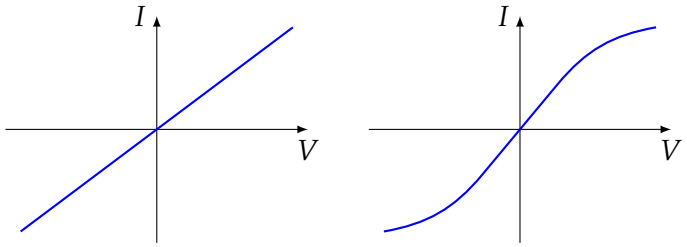
current: $I = \frac{P}{V} = \frac{900}{220} \quad \Rightarrow \quad I \approx 4.09$ A

resistance: $R = \frac{V}{I} = \frac{220}{4.09}$, or alternatively, $R = \frac{V^2}{P} = \frac{220^2}{900} \quad \Rightarrow \quad R \approx 53.8\ \Omega$

Example 11.12 Two conducting cylinders X and Y are of the same diameter and the same material. The length of the cylinder X is twice of that of the cylinder Y. If the two cylinders are connected in series to a power supply so that the current through them are equal, find the ratio of the electrical powers dissipated in the two cylinders $\frac{P_X}{P_Y}$.

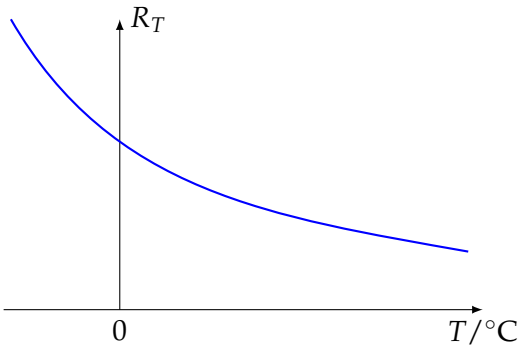
for either cylinder, electrical power: $P = I^2R = I^2 \cdot \frac{\rho L}{A} = I^2 \cdot \frac{4\rho L}{\pi d^2}$
same current I, same diameter d, same resistivity ρ, so power: $P \propto L \quad \Rightarrow$
$\frac{P_X}{P_Y} = \frac{L_X}{L_Y} = 2$

I–V characteristics

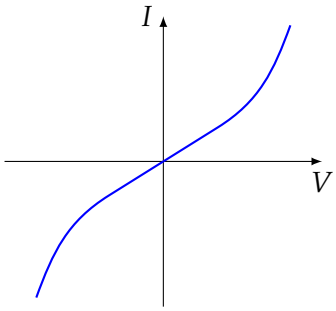


ohmic conductor filament lamp

- **ohmic conductor** (metal wire at constant temperature): $I \propto V$, a straight line through the origin; resistance is constant
- **filament lamp:** the filament heats up as current rises, its resistance increases, so the graph curves (gradient decreases)
- resistance at any point is V/I, not the gradient of the I–V graph

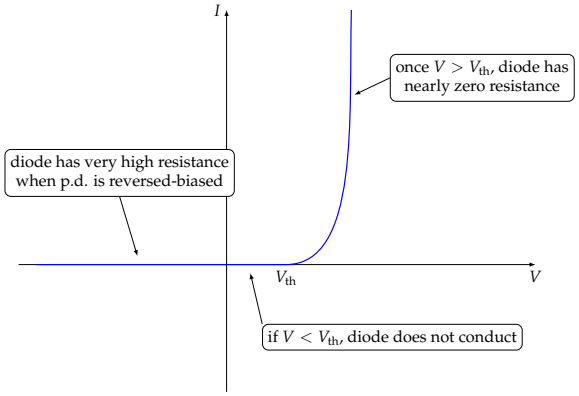


thermistor: R against θ



thermistor: I–V

• **NTC thermistor:** resistance ___decreases___ as temperature rises (more charge carriers are freed)



• **diode:** conducts in ___one direction___ only; forward-biased above the ___threshold voltage___ ($\approx$ 0.6 V for silicon) the resistance is very ___low___; reverse-biased the resistance is ___very high___

Practice questions

Question 11.1 Calculate the current that gives a charge flow of 300 C in one minute.

$$I = \frac{Q}{t} = \frac{300}{60} = 5.0 \text{ A}.$$

Question 11.2 A car battery can supply a steady 40 A for one hour. (a) What charge flows in this time? (b) For how long could it supply the 90 A needed to start the car?

(a) $Q = It = 40 \times 3600 = 1.44 \times 10^5$ C.
(b) $t = \dfrac{Q}{I} = \dfrac{1.44 \times 10^5}{90} = 1.6 \times 10^3$ s (about 27 minutes).

Question 11.3 A beam of 2.0×10^{10} electrons is stopped on an electrode in 5.0×10^{-8} s. Find (a) the charge collected, (b) the average current.

(a) $Q = Ne = 2.0 \times 10^{10} \times 1.60 \times 10^{-19} = 3.2 \times 10^{-9}$ C.
(b) $I = \dfrac{Q}{t} = \dfrac{3.2 \times 10^{-9}}{5.0 \times 10^{-8}} = 0.064$ A.

Question 11.4 The number density of free electrons in copper is 8.0×10^{28} m^{-3}. Find the drift speed in a wire of diameter 0.50 mm carrying 0.80 A.

$$v = \frac{I}{nAe} = \frac{0.80}{8.0 \times 10^{28} \times \pi(0.25 \times 10^{-3})^2 \times 1.60 \times 10^{-19}} \approx 3.2 \times 10^{-4} \text{ m s}^{-1}.$$

Question 11.5 Copper wires A (diameter d) and B (diameter $2d$) are joined and carry a current. Find v_A / v_B.

Same I and n, so $v \propto \dfrac{1}{A} \propto \dfrac{1}{d^2}$. $\dfrac{v_A}{v_B} = \dfrac{(2d)^2}{d^2} = 4$.

Question 11.6 What is the current in a 40 W light bulb on a 200 V supply?

$I = \dfrac{P}{V} = \dfrac{40}{200} = 0.20$ A.

Question 11.7 A 12 V battery supplies 5.0 A for 3.0 hours. Find (a) the charge supplied, (b) the energy supplied.

(a) $Q = It = 5.0 \times 3.0 \times 3600 = 5.4 \times 10^4$ C.
(b) $W = \mathcal{E}Q = 12 \times 5.4 \times 10^4 = 6.5 \times 10^5$ J.

Question 11.8 An electric cooker is labelled '240 V, 900 W'. Find (a) the resistance of its element, (b) the current when it works normally.

(a) $R = \dfrac{V^2}{P} = \dfrac{240^2}{900} = 64\ \Omega$.
(b) $I = \dfrac{P}{V} = \dfrac{900}{240} = 3.75$ A.