Current Electricity

AS Level Physics 9702

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By the end of this topic I can. . .

- state and use Kirchhoff's laws
- combine resistors in series and parallel
- use e.m.f. and internal resistance
- use potential dividers, a potentiometer and bridges

Kirchhoff's first law

First law

Current into a point = current out of it (conservation of **charge**).

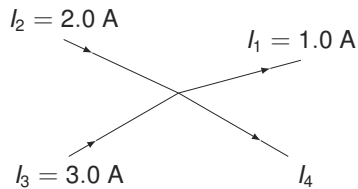
$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Example: finding a current

The diagram shows part of an electric circuit. The values of I_1, I_2 and I_3 are labelled. Calculate the current I_4.

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applying the first law: $I_2 + I_3 = I_1 + I_4$

$$\Rightarrow \quad I_4 = 2.0 + 3.0 - 1.0 = 4.0 \text{ A}$$

Kirchhoff's second law

Second law

Around a closed loop, sum of e.m.f.s = sum of p.d.s (conservation of **energy**).

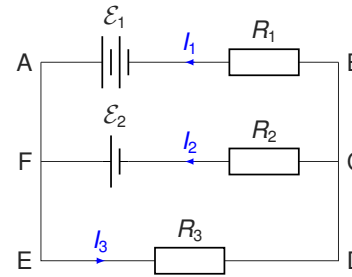
$$\sum \mathcal{E} = \sum V$$

Example: two-loop circuit

The diagram shows a circuit where e.m.f. of the batteries and the values of resistance are all known. Write down the equations for the currents I_1, I_2 and I_3 using Kirchhoff's laws.

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apply KCL for point C or F: $I_1 + I_2 = I_3$ ①

apply KVL for loop $AEDBA$: $\mathcal{E}_1 = I_1 R_1 + I_3 R_3$ ②

apply KVL for loop $FEDCF$: $\mathcal{E}_2 = I_2 R_2 + I_3 R_3$ ③

in principle, I_1, I_2 and I_3 can be solved from the three equations

one can also apply KVL for loop $AFCBA$ to write:
$\mathcal{E}_1 - \mathcal{E}_2 = I_1 R_1 - I_2 R_2$

this equation looks complex but is simply ②−③, which is not an independent equation

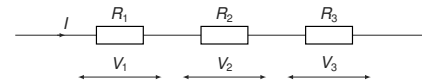
when writing down equations using KVL, choosing

Resistors in series

Series

Same **current**; p.d.s add.

$$R_{\text{total}} = R_1 + R_2 + \cdots + R_n$$



Resistors in parallel

Parallel

Same **p.d.**; currents add.

- adding in parallel: R_{total} decreases

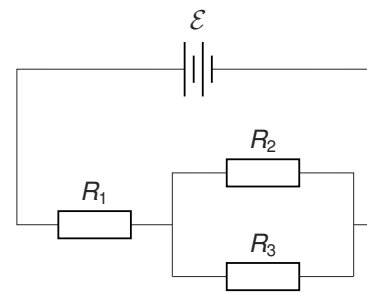
$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

Example: series-parallel network

In the circuit shown, the battery has an e.m.f. $\mathcal{E} = 18$ V, and the resistors have $R_1 = 6.0\ \Omega$, $R_2 = 4.0\ \Omega$ and $R_3 = 12\ \Omega$. (a) Find the total resistance of all external resistors. (b) Find the current through the battery. (c) Find the p.d. across R_1, R_2 and R_3. (d) Find the total power dissipated in R_2 and R_3.

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total resistance: $R = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_2}\right)^{-1} =$

$6.0 + \left(\frac{1}{4.0} + \frac{1}{12}\right)^{-1} = 6.0 + 3.0 = 9.0\ \Omega$

current through battery: $I = \frac{\mathcal{E}}{R} = \frac{18}{9.0} = 2.0$ A

p.d across R_1: $V_1 = IR_1 = 2.0 \times 6.0 = 12$ V

p.d across R_2 and R_3:

$V_2 = V_3 = \mathcal{E} - V_1 = 18 - 12 = 6.0$ V

power dissipated in R_2: $P_2 = \frac{V_2^2}{R_2} = \frac{6.0^2}{4.0} = 9.0$ W

Internal resistance

Internal resistance

Real cells have internal resistance r.

- terminal p.d. $V = \mathcal{E} - Ir$
- lost volts $= Ir$
- $I_{\text{max}} = \mathcal{E}/r$

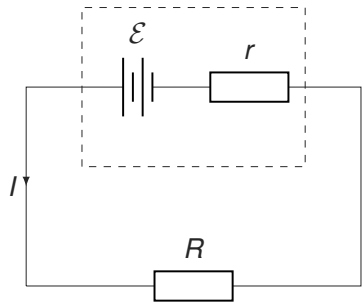
$$\mathcal{E} = V_R + V_r = I(R + r)$$

Example: car battery

A car battery has an e.m.f. of 20 V and an internal resistance of 0.50 Ω. When a starter motor of resistance of 3.50 Ω is connected to the battery, find (a) the current supplied to the motor, (b) the p.d. across the battery terminals, (c) the power at which the motor operates.

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current in circuit: $I = \frac{\mathcal{E}}{R+r} = \frac{20}{3.50+0.50} = 5.0$ A
terminal p.d.: $V_R = IR = 5.0 \times 3.50 = 17.5$ V
power of motor: $P_R = IV_R = 5.0 \times 17.5 =$
87.5 W or $P_R = I^2R = 5.0^2 \times 3.50 = 87.5$ W

Example: finding *r*

Example: finding r

lost volts: $V_r = \mathcal{E} - IR = 12 - 0.50 \times 22 = 1.0$ V
internal resistance: $r = \frac{V_r}{I} = \frac{1.0}{0.50} = 2.0\ \Omega$
power produced by battery: $P_{\text{total}} = I\mathcal{E} = 0.50 \times 12 = 6.0$ W
power dissipated in battery:
$P_r = IV_r = 0.50 \times 1.0 = 0.50$ W or $P_r = I^2 r = 0.50^2 \times 2.0 = 0.50$ W

Example: heater and wires

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lost volts: $V_r = Ir = 5.0 \times 4.0 = 20$ V
p.d. across wires: $V_{\text{wire}} = \mathcal{E} - V_{\text{heater}} - V_r = 230 - 200 - 20 = 10$ V
resistance of wires: $R_{\text{wire}} = \frac{V_{\text{wire}}}{I} = \frac{10}{5.0} = 2.0\ \Omega$
useful power: $P_{\text{useful}} = P_{\text{heater}} = IV_{\text{heater}} = 5.0 \times 200 = 1000$ W
efficiency: $\eta = \frac{P_{\text{useful}}}{P_{\text{total}}} = \frac{IV_{\text{heater}}}{I\mathcal{E}} = \frac{200}{230} \approx 87\%$

Example: variable resistor

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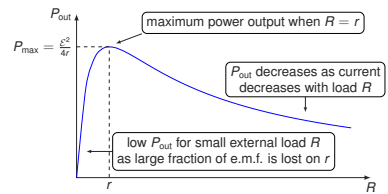
as R increases, current in circuit decreases
less voltage is lost in cell, so terminal p.d. will increase
for same reason, less power is lost in cell, so higher efficiency for the circuit
more explicitly, terminal p.d. $V_R = IR = \frac{\mathcal{E}R}{R+r}$, and efficiency $\eta = \frac{P_R}{P_{\text{total}}} = \frac{I^2R}{I^2(R+r)} = \frac{R}{R+r}$
from these we can tell: if $R \uparrow$, then $\frac{R}{R+r} \uparrow$, so $V_R \uparrow$ and $\eta \uparrow$

Maximum power

Max power

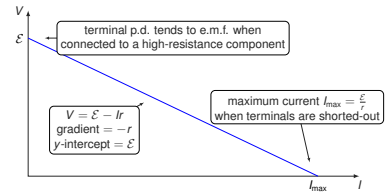
Output power is greatest when $R = r$.

$$P_{\text{max}} = \frac{\mathcal{E}^2}{4r}$$



Measuring e.m.f. and r

- vary R; record V and I
- $V = \mathcal{E} - Ir$: straight line
- intercept $= \mathcal{E}$; gradient $= -r$

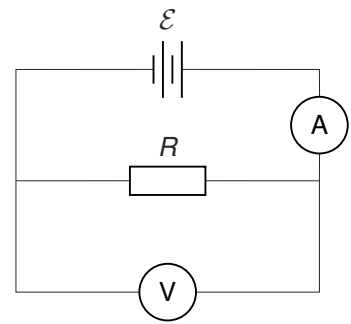


Example: real meters

The diagram shows a simple circuit. The battery has a negligible internal resistance and an e.m.f. of 8.0 V. The resistor has a fixed resistance of 20 Ω. (a) Find the readings shown on the ammeter and the voltmeter if both meters are ideal. (b) Instead, the ammeter has a non-zero resistance of 1.0 Ω and the voltmeter has a resistance of 60 Ω, what are the true readings displayed on the two meters?

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if both meters are ideal, then $I = \frac{\mathcal{E}}{R} = \frac{8.0}{20} = 0.40$ A,
and $V = \mathcal{E} = 8.0$ V
for non-ideal case, total resistance in circuit:
$R_{\text{total}} = R_A + \left(\frac{1}{R} + \frac{1}{R_V}\right)^{-1} = 1.0 + \left(\frac{1}{20} + \frac{1}{60}\right)^{-1} = 16\ \Omega$
current through ammeter: $I = \frac{\mathcal{E}}{R_{\text{total}}} = \frac{8.0}{16} = 0.50$ A
p.d. across voltmeter:

Potential divider

Potential divider

Resistors share the supply p.d. in proportion to their resistance.

$$V_1 = \frac{R_1}{R_1 + R_2}\mathcal{E}$$

Example: variable divider

The diagram shows an electric circuit incorporating a power supply of negligible internal resistance and a high-resistance voltmeter. Determine the range of voltage reading on the voltmeter as the variable resistor R is adjusted over its full range from 0 Ω to 50 Ω.

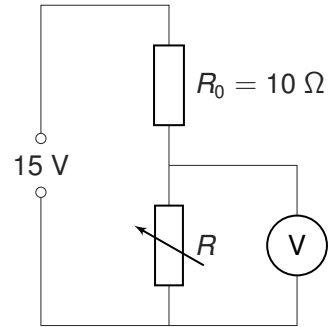
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$V_{\text{min}} = \frac{R_{\text{min}}}{R_{\text{min}}+R_0} \times V_{\text{in}} = \frac{0}{0+10} \times 15 \quad \Rightarrow \quad V_{\text{min}} = 0$ V

$V_{\text{max}} = \frac{R_{\text{max}}}{R_{\text{max}}+R_0} \times V_{\text{in}} = \frac{50}{50+10} \times 15 \quad \Rightarrow \quad V_{\text{max}} = 12.5$ V

so voltage reading ranges from 0 to 12.5 V

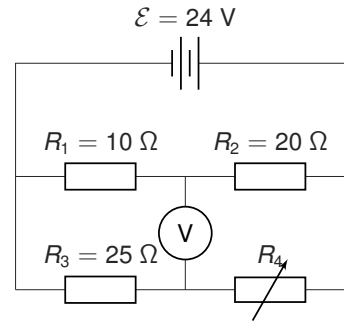


Example: bridge circuit

Four resistors are connected to a battery as shown. (a) If the variable resistor R_4 is adjusted to have a resistance of 35 Ω, what is the reading on the voltmeter? (b) If the voltmeter reads zero, what is the resistance for R_4?

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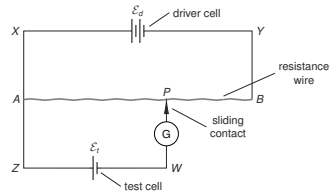
(a) p.d. across R_1: $V_1 = \frac{10}{20+10} \times 24 = 8.0$ V
p.d. across R_3: $V_3 = \frac{25}{35+25} \times 24 = 10.0$ V
voltmeter reading:
$V = V_3 - V_1 = 10.0 - 8.0 = 2.0$ V

(b) $V = 0$ means p.d. share between R_1 and R_2 is same as p.d. share between R_3 and R_4

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \Rightarrow \quad \frac{10}{20} = \frac{25}{R_4} \quad \Rightarrow \quad R_4 = 50\ \Omega$$

Potentiometer

- balance: no current in galvanometer
- no lost volts in the test cell
- $\mathcal{E}_t = \dfrac{AP}{AB}\mathcal{E}_d$

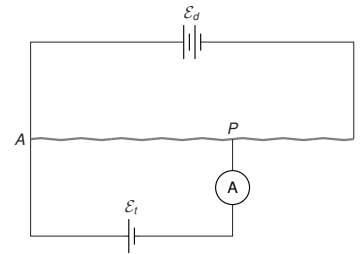


Example: potentiometer

The driver cell has e.m.f. 17 V and internal resistance 5.0 Ω. Wire AB has resistance 80 Ω and length 80 cm. The galvanometer reads zero when $AP = 50$ cm. Find (a) the current in the wire, (b) the p.d. across AP, (c) the e.m.f. of the test cell.

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(a) $I = \dfrac{\mathcal{E}}{R_{AB} + r} = \dfrac{17}{80 + 5.0} = 0.20$ A.

(b) $R_{AP} = \dfrac{50}{80} \times 80 = 50\ \Omega$, so

$V_{AP} = IR_{AP} = 0.20 \times 50 = 10$ V.

(c) No current flows in the test cell, so there are no lost volts: $\mathcal{E}_t = V_{AP} = 10$ V.

Key equations

KCL $\sum I_{\text{in}} = \sum I_{\text{out}}$

KVL $\sum \mathcal{E} = \sum V$

series $R_{\text{total}} = R_1 + R_2 + \cdots + R_n$

internal r $\mathcal{E} = V_R + V_r = I(R + r)$

divider $V_1 = \dfrac{R_1}{R_1 + R_2}\,\mathcal{E}$