Current Electricity

Name _____ Date _____

By the end of this topic I can...

- state and use Kirchhoff's two laws
- combine resistors in series and parallel
- use e.m.f., internal resistance, terminal p.d. and lost volts
- use potential dividers, a potentiometer and bridge circuits

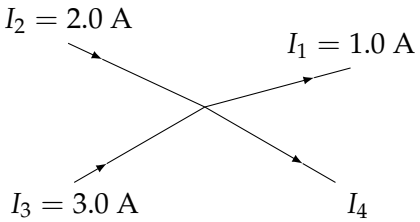
Kirchhoff's laws

Kirchhoff's first law: at any point in a circuit the sum of currents ___into___ the point equals the sum of currents ___out of___ it:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

- a consequence of conservation of ___charge___

Example 12.1 The diagram shows part of an electric circuit. The values of I_1, I_2 and I_3 are labelled. Calculate the current I_4.



applying the first law: $I_2 + I_3 = I_1 + I_4$

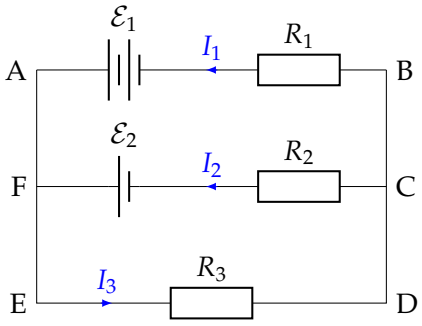
$\Rightarrow \quad I_4 = 2.0 + 3.0 - 1.0 = 4.0$ A

Kirchhoff's second law: around any closed loop the sum of the e.m.f.s equals the sum of the p.d.s:

$$\sum \mathcal{E} = \sum V$$

- a consequence of conservation of ___energy___; choose a loop direction and be consistent with signs

Example 12.2 The diagram shows a circuit where e.m.f. of the batteries and the values of resistance are all known. Write down the equations for the currents I_1, I_2 and I_3 using Kirchhoff's laws.



apply KCL for point C or F: $I_1 + I_2 = I_3$ ①
apply KVL for loop $AEDBA$: $\mathcal{E}_1 = I_1R_1 + I_3R_3$ ②
apply KVL for loop $FEDCF$: $\mathcal{E}_2 = I_2R_2 + I_3R_3$ ③
in principle, I_1, I_2 and I_3 can be solved from the three equations
one can also apply KVL for loop $AFCBA$ to write: $\mathcal{E}_1 - \mathcal{E}_2 = I_1R_1 - I_2R_2$
this equation looks complex but is simply ②−③, which is not an independent equation
when writing down equations using KVL, choosing simpler loops gives easier equations

Resistor networks

Series

• same ___current___ through each; p.d.s ___add___

$$R_{\text{total}} = R_1 + R_2 + \cdots + R_n$$

Parallel

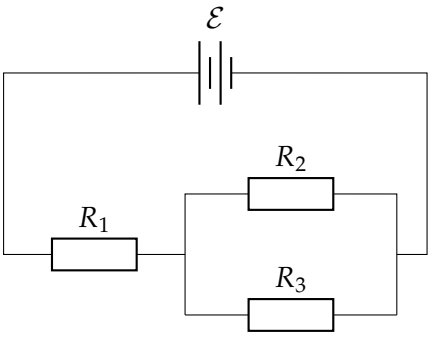
• same ___p.d.___ across each; currents ___add___

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

• adding a resistor in series ___increases___ R_{total}; adding one in parallel ___decreases___ it

• n identical resistors R_0 in parallel: $R_{\text{total}} =$ ___R_0/n___

Example 12.3 In the circuit shown, the battery has an e.m.f. $\mathcal{E} = 18$ V, and the resistors have $R_1 = 6.0\ \Omega$, $R_2 = 4.0\ \Omega$ and $R_3 = 12\ \Omega$. (a) Find the total resistance of all external resistors. (b) Find the current through the battery. (c) Find the p.d. across R_1, R_2 and R_3. (d) Find the total power dissipated in R_2 and R_3.



total resistance: $R = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_2}\right)^{-1} = 6.0 + \left(\frac{1}{4.0} + \frac{1}{12}\right)^{-1} = 6.0 + 3.0 = 9.0\ \Omega$
current through battery: $I = \frac{\mathcal{E}}{R} = \frac{18}{9.0} = 2.0$ A
p.d across R_1: $V_1 = IR_1 = 2.0 \times 6.0 = 12$ V
p.d across R_2 and R_3: $V_2 = V_3 = \mathcal{E} - V_1 = 18 - 12 = 6.0$ V
power dissipated in R_2: $P_2 = \frac{V_2^2}{R_2} = \frac{6.0^2}{4.0} = 9.0$ W
power dissipated in R_3: $P_3 = \frac{V_3^2}{R_3} = \frac{6.0^2}{12} = 3.0$ W

Internal resistance

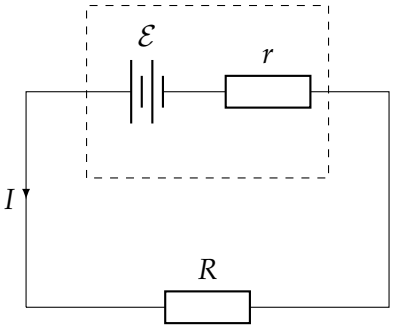
• a real source has **internal resistance** r: it behaves as an ideal cell of e.m.f. $\mathcal{E}$ in series with r

$$\mathcal{E} = V_R + V_r = I(R + r)$$

• $V_R = IR$ is the **terminal p.d.**; $V_r = Ir$ is the **lost volts**, so $V_R = \mathcal{E} - Ir$ is ___less than___ the e.m.f. when current flows

• maximum current (short circuit, $R = 0$): $I_{\max} =$ ___$\mathcal{E}/r$___; open circuit: $I = 0$ and terminal p.d. = ___$\mathcal{E}$___

• powers: total = ___$I\mathcal{E}$___; to external circuit = ___I^2R___; lost in the cell = ___I^2r___



Example 12.4 A car battery has an e.m.f. of 20 V and an internal resistance of 0.50 Ω. When a starter motor of resistance of 3.50 Ω is connected to the battery, find (a) the current supplied to the motor, (b) the p.d. across the battery terminals, (c) the power at which the motor operates.

current in circuit: $I = \frac{\mathcal{E}}{R+r} = \frac{20}{3.50+0.50} = 5.0$ A
terminal p.d.: $V_R = IR = 5.0 \times 3.50 = 17.5$ V
power of motor: $P_R = IV_R = 5.0 \times 17.5 = 87.5$ W or $P_R = I^2R = 5.0^2 \times 3.50 = 87.5$ W

Example 12.5 A battery of e.m.f. 12 V is connected to a network of total resistance of 22 Ω. The current through the battery is 0.50 A. Find (a) the internal resistance of the battery, (b) the total power produced by battery, (c) the power dissipated in battery.

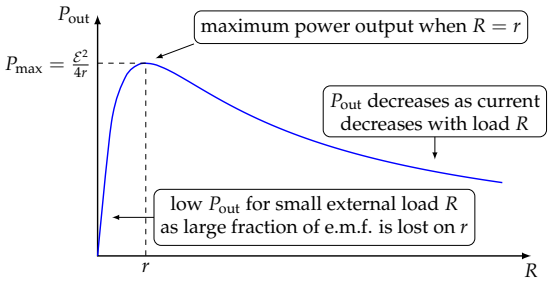
lost volts: $V_r = \mathcal{E} - IR = 12 - 0.50 \times 22 = 1.0$ V
internal resistance: $r = \frac{V_r}{I} = \frac{1.0}{0.50} = 2.0\ \Omega$
power produced by battery: $P_{\text{total}} = I\mathcal{E} = 0.50 \times 12 = 6.0$ W
power dissipated in battery: $P_r = IV_r = 0.50 \times 1.0 = 0.50$ W or $P_r = I^2r = 0.50^2 \times 2.0 = 0.50$ W

Example 12.6 An electric heater is operating at a working p.d. of 200 V. A current of 5.0 A is driven by a voltage source of 230 V. The source has an internal resistance of 4.0 Ω and the resistance of the connecting wires is not negligible. Find (a) the lost volts in the source, (b) the resistance of the wires, (c) the useful power from the source, (d) the efficiency of this circuit.

lost volts: $V_r = Ir = 5.0 \times 4.0 = 20$ V
p.d. across wires: $V_{\text{wire}} = \mathcal{E} - V_{\text{heater}} - V_r = 230 - 200 - 20 = 10$ V
resistance of wires: $R_{\text{wire}} = \frac{V_{\text{wire}}}{I} = \frac{10}{5.0} = 2.0\ \Omega$
useful power: $P_{\text{useful}} = P_{\text{heater}} = IV_{\text{heater}} = 5.0 \times 200 = 1000$ W
efficiency: $\eta = \frac{P_{\text{useful}}}{P_{\text{total}}} = \frac{IV_{\text{heater}}}{I\mathcal{E}} = \frac{200}{230} \approx 87\%$

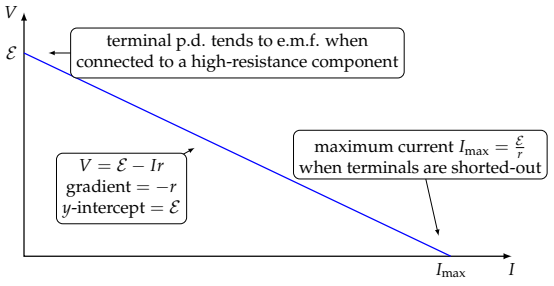
Example 12.7 A cell of e.m.f. $\mathcal{E}$ and internal resistance r is connected in series with a variable resistor R. R is gradually increased from zero. (a) Suggest how the p.d. across the battery terminals change when R is increased. (b) For larger values of R, power delivered to R decreases. Suggest the advantage, despite the low power output, of using this cell in a circuit of larger resistance.

as R increases, current in circuit decreases
less voltage is lost in cell, so terminal p.d. will increase
for same reason, less power is lost in cell, so higher efficiency for the circuit
more explicitly, terminal p.d. $V_R = IR = \frac{\mathcal{E}R}{R+r}$, and efficiency $\eta = \frac{P_R}{P_{\text{total}}} = \frac{I^2R}{I^2(R+r)} = \frac{R}{R+r}$
from these we can tell: if $R \uparrow$, then $\frac{R}{R+r} \uparrow$, so $V_R \uparrow$ and $\eta \uparrow$



• output power to the load is greatest when $R =$ __r__: $P_{\max} =$ $\frac{\mathcal{E}^2}{4r}$

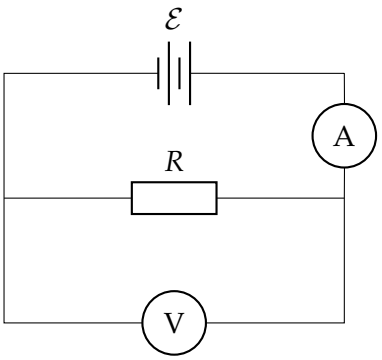
Measuring e.m.f. and internal resistance



• vary R and record V and I; since $V = \mathcal{E} - Ir$, a graph of V against I is a straight line with y-intercept = $\mathcal{E}$ and gradient = $-r$

Meters

- an ideal ammeter has ___zero___ resistance; an ideal voltmeter has ___infinite___ resistance
- real meters have a resistance: treat them as ordinary resistors in the circuit



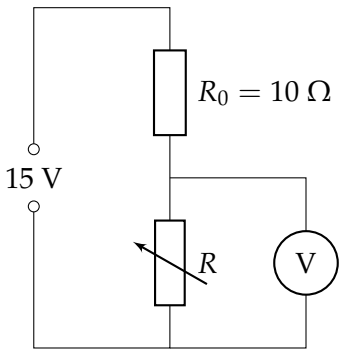
Example 12.8 The diagram shows a simple circuit. The battery has a negligible internal resistance and an e.m.f. of 8.0 V. The resistor has a fixed resistance of 20 Ω. (a) Find the readings shown on the ammeter and the voltmeter if both meters are ideal. (b) Instead, the ammeter has a non-zero resistance of 1.0 Ω and the voltmeter has a resistance of 60 Ω, what are the true readings displayed on the two meters?

if both meters are ideal, then $I = \frac{\mathcal{E}}{R} = \frac{8.0}{20} = 0.40$ A, and $V = \mathcal{E} = 8.0$ V
for non-ideal case, total resistance in circuit: $R_{\text{total}} = R_A + \left(\frac{1}{R} + \frac{1}{R_V}\right)^{-1} = 1.0 + \left(\frac{1}{20} + \frac{1}{60}\right)^{-1} = 16\ \Omega$
current through ammeter: $I = \frac{\mathcal{E}}{R_{\text{total}}} = \frac{8.0}{16} = 0.50$ A
p.d. across voltmeter: $V = \mathcal{E} - V_A = \mathcal{E} - IR_A = 8.0 - 0.50 \times 1.0 = 7.5$ V

Potential dividers

Two resistors in series share the supply p.d. in proportion to their resistances:

$$V_1 = \frac{R_1}{R_1 + R_2}\mathcal{E}$$

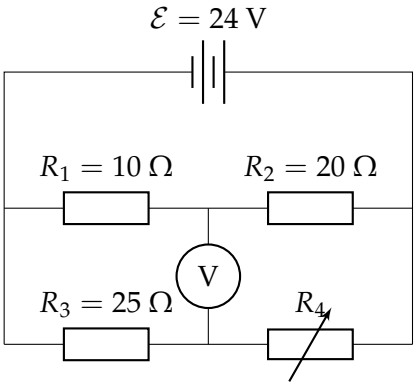


Example 12.9 The diagram shows an electric circuit incorporating a power supply of negligible internal resistance and a high-resistance voltmeter. Determine the range of voltage reading on the voltmeter as the variable resistor R is adjusted over its full range from 0 Ω to 50 Ω.

$V_{\text{min}} = \frac{R_{\text{min}}}{R_{\text{min}}+R_0} \times V_{\text{in}} = \frac{0}{0+10} \times 15 \quad \Rightarrow \quad V_{\text{min}} = 0$ V
$V_{\text{max}} = \frac{R_{\text{max}}}{R_{\text{max}}+R_0} \times V_{\text{in}} = \frac{50}{50+10} \times 15 \quad \Rightarrow \quad V_{\text{max}} = 12.5$ V
so voltage reading ranges from 0 to 12.5 V

Bridge circuit

• the bridge is ___balanced___ (no current in the galvanometer) when $\frac{R_1}{R_2} = \frac{R_x}{R}$



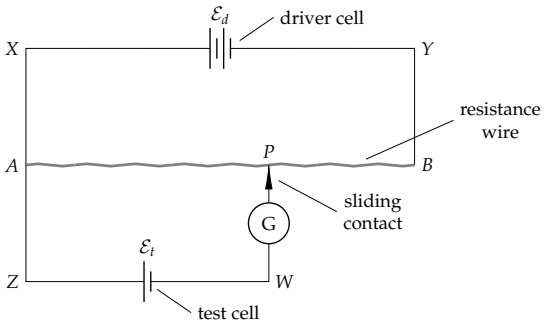
Example 12.10 Four resistors are connected to a battery as shown. (a) If the variable resistor R_4 is adjusted to have a resistance of 35 Ω, what is the reading on the voltmeter? (b) If the voltmeter reads zero, what is the resistance for R_4?

(a) p.d. across R_1: $V_1 = \frac{10}{20+10} \times 24 = 8.0$ V
p.d. across R_3: $V_3 = \frac{25}{35+25} \times 24 = 10.0$ V
voltmeter reading: $V = V_3 - V_1 = 10.0 - 8.0 = 2.0$ V

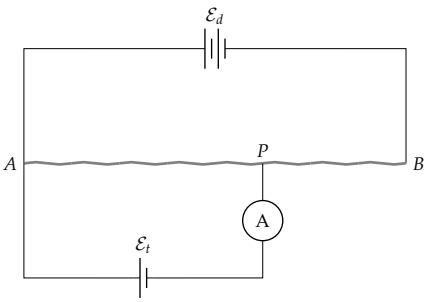
(b) $V = 0$ means p.d. share between R_1 and R_2 is same as p.d. share between R_3 and R_4

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \Rightarrow \quad \frac{10}{20} = \frac{25}{R_4} \quad \Rightarrow \quad R_4 = 50\ \Omega$$

Potentiometer



• at balance no current flows through the ___test cell___, so its internal resistance causes ___no lost volts___; $\mathcal{E}_t = \frac{AP}{AB} \times \mathcal{E}_d$



Example 12.11 The driver cell has e.m.f. 17 V and internal resistance 5.0 Ω. Wire AB has resistance 80 Ω and length 80 cm. The galvanometer reads zero when $AP = 50$ cm. Find (a) the current in the wire, (b) the p.d. across AP, (c) the e.m.f. of the test cell.

(a) $I = \dfrac{\mathcal{E}}{R_{AB} + r} = \dfrac{17}{80 + 5.0} = 0.20$ A.
(b) $R_{AP} = \dfrac{50}{80} \times 80 = 50\ \Omega$, so $V_{AP} = IR_{AP} = 0.20 \times 50 = 10$ V.
(c) No current flows in the test cell, so there are no lost volts: $\mathcal{E}_t = V_{AP} = 10$ V.

Practice questions

Question 12.1 Explain, using resistors in series or parallel, (a) why a long wire has more resistance than a short wire, (b) why a thick wire has less resistance than a thin one.

(a) A long wire is like many short wires in ___series___: resistances add, so R increases.
(b) A thick wire is like many thin wires in ___parallel___: more paths for the current, so R decreases.

Question 12.2 A 12 V battery of negligible internal resistance is connected to a 4.0 Ω resistor in series with a parallel pair of 6.0 Ω and 3.0 Ω resistors. Find (a) the total resistance, (b) the battery current, (c) the current in the 3.0 Ω resistor.

(a) Parallel pair: $\left(\dfrac{1}{6} + \dfrac{1}{3}\right)^{-1} = 2.0\ \Omega$. Total $= 4.0 + 2.0 = 6.0\ \Omega$.
(b) $I = \dfrac{12}{6.0} = 2.0$ A.
(c) P.d. across the pair $= 2.0 \times 2.0 = 4.0$ V, so $I_3 = \dfrac{4.0}{3.0} = 1.3$ A.

Question 12.3 A cell of e.m.f. 1.5 V and internal resistance 0.50 Ω is connected to a 2.5 Ω resistor. Find the current and the terminal p.d.

$I = \dfrac{\mathcal{E}}{R+r} = \dfrac{1.5}{2.5+0.50} = 0.50$ A. $V_R = IR = 0.50 \times 2.5 = 1.25$ V.

Question 12.4 A student measures a terminal p.d. of 1.50 V with no current and 1.20 V when the current is 0.50 A. Find the e.m.f. and internal resistance.

$\mathcal{E} = 1.50$ V (no current, no lost volts). Lost volts $= 1.50 - 1.20 = 0.30$ V, so $r = \dfrac{0.30}{0.50} = 0.60\ \Omega$.

Question 12.5 A 12 V supply of negligible internal resistance is connected across a 4.0 kΩ resistor in series with an 8.0 kΩ resistor. What is the p.d. across the 8.0 kΩ resistor?

$V = \dfrac{8.0}{4.0+8.0} \times 12 = 8.0$ V.

Question 12.6 A cell has e.m.f. 6.0 V and internal resistance 2.0 Ω. For what external resistance is the output power greatest, and what is that power?

Maximum power when $R = r = 2.0\ \Omega$: $P_{\max} = \dfrac{\mathcal{E}^2}{4r} = \dfrac{6.0^2}{4 \times 2.0} = 4.5$ W.