Electrostatics

A Level Physics 9702

J. Rowing • Vijay International School Praslin

By the end of this topic I can...

- use Coulomb's law, and define and use electric field strength E
- draw electric field lines and equipotentials
- use electric potential energy and potential
- use $E = -\mathrm{d}V/\mathrm{d}r$ and $E = V/d$; compare electric and gravitational fields

Coulomb's law

Coulomb's law

The force between two **point charges** is proportional to the **product of the charges** and inversely proportional to the **square of their separation**.

- $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C}^2\ \mathrm{N}^{-1}\ \mathrm{m}^{-2}$; $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9$
- like charges repel, opposite charges attract

$$F = \frac{Qq}{4\pi\epsilon_0 r^2}$$

Example: the hydrogen atom

Example: the hydrogen atom

$$F = \frac{Q_p Q_e}{4\pi\epsilon_0 r^2} = 8.99 \times 10^9 \times \frac{(1.60 \times 10^{-19})^2}{(53 \times 10^{-12})^2} \approx 8.2 \times 10^{-8}\ \text{N}$$

Electric field strength

Electric field strength

The electric **force per unit positive charge**.

- unit: N C^{-1} = V m^{-1}
- a **vector**: away from $+$, towards $-$
- force on a charge: $F = Eq$

$$E = \frac{F}{q}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Example: Van de Graaff generator

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$E = 4.0 \times 10^6$ V m^{-1}, $r = 0.20$ m:

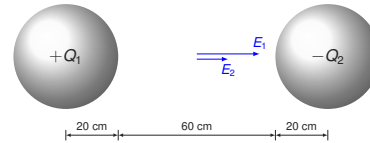
$$Q = 4\pi\epsilon_0 E r^2 = 4\pi \times 8.85 \times 10^{-12} \times 4.0 \times 10^6 \times 0.20^2 \approx 1.8 \times 10^{-5} \text{ C}$$

Example: two charged spheres

Two identical metal spheres of radius 20 cm carry charges $+2.0\ \mu$C and $-1.0\ \mu$C, with a 60 cm gap between them. (a) Find the field strength midway between their centres. (b) A dust particle with charge -1.3×10^{-8} C is placed there. Find the force on it.

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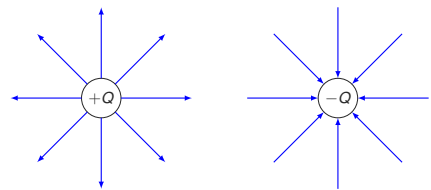
The midpoint is $20 + 30 = 50$ cm $= 0.50$ m from each centre. The fields point the same way (away from $+$, towards $-$):

$$E = E_1 + E_2 = 8.99 \times 10^9 \left(\frac{2.0 \times 10^{-6}}{0.50^2} + \frac{1.0 \times 10^{-6}}{0.50^2} \right) \approx 1.08 \times 10^5 \text{ N C}^{-1}$$

towards the negative sphere (to the right). (b)
$F = Eq = 1.08 \times 10^5 \times 1.3 \times 10^{-8} \approx 1.4 \times 10^{-3}$ N;
the charge is negative, so the force is opposite to E (to

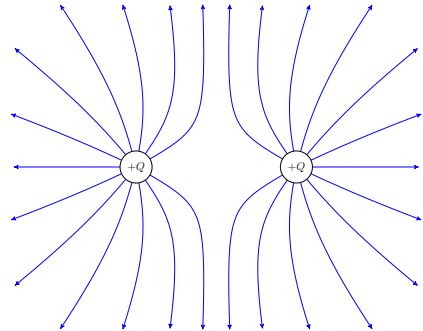
Electric field lines

- arrows show **direction**; spacing shows **strength**
- leave **positive**, end on **negative** charges
- charged sphere: **radial**, acts as a point charge
- between plates: **parallel**, equally spaced: **uniform**



Field of two charges

- two equal positive charges (left)
- two opposite charges (right)



Electric potential energy

Electric potential energy

The **work done by the electric force** bringing a charge from **infinity**.

- like charges: $E_p > 0$; opposite charges: $E_p < 0$
- a **scalar**, but the sign matters

$$E_p = \frac{Qq}{4\pi\epsilon_0 r}$$

Example: alpha particle and gold nucleus

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The α-particle slows as it approaches (repulsion): K.E. is converted to electric P.E. At closest approach $v = 0$:

$$\tfrac{1}{2}mu^2 = \frac{Qq}{4\pi\epsilon_0 r}$$

$$u^2 = \frac{2}{4 \times 1.66 \times 10^{-27}} \times \frac{79 \times 2 \times (1.60 \times 10^{-19})^2}{4\pi \times 8.85 \times 10^{-12} \times 10^{-14}}$$

$$u \approx 3.3 \times 10^7 \text{ m s}^{-1}$$

Electric potential

Electric potential

The **work done** bringing a **unit positive charge** from **infinity**.

- unit: V = J C^{-1}
- a **scalar**: add potentials from several charges

$$V = \frac{E_p}{q}$$

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Example: accelerating an electron

Example: accelerating an electron

Gain in K.E. = loss in electric P.E.: $\frac{1}{2}mv^2 = q\Delta V$

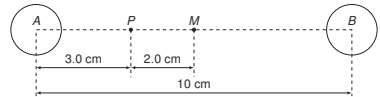
$$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 600}{9.11 \times 10^{-31}}} \approx 1.45 \times 10^7 \text{ m s}^{-1}$$

Example: potential at two points

Spheres A ($+20$ pC) and B ($+84$ pC) are 10 cm apart in a vacuum. P is 3.0 cm from A and M is the midpoint. (a) Find the potential at P and at M. (b) Find the work needed to move an α-particle from P to M.

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$$V = V_A + V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{Q_A}{r_A} + \frac{Q_B}{r_B}\right)$$

$$V_P = 8.99\times10^9\left(\frac{20\times10^{-12}}{0.030} + \frac{84\times10^{-12}}{0.070}\right) \approx 16.8 \text{ V}$$

$$V_M = 8.99\times10^9\left(\frac{20\times10^{-12}}{0.050} + \frac{84\times10^{-12}}{0.050}\right) \approx 18.7 \text{ V}$$

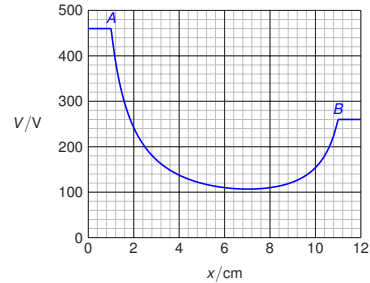
$$W = q\Delta V = 2\times1.60\times10^{-19}\times(18.7-16.8) \approx 6.2\times10^{-19} \text{ J}$$

Example: a proton between two spheres

Spheres A and B (radius 1.0 cm) are positively charged. A proton starts at rest on the surface of A and moves along the line to B. The graph shows the potential V against distance x from the centre of A. (a) Find its maximum speed. (b) Find its speed on reaching the surface of B.

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Increase in K.E. = loss in P.E.: $\frac{1}{2}mv^2 = q(V_A - V_P)$

(a) Fastest where V_P is smallest (107 V at $x = 7.2$ cm):

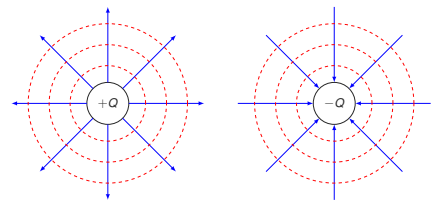
$$\tfrac{1}{2}\times 1.67\times 10^{-27} v_{\max}^2 = 1.60\times 10^{-19}\times(460-107) \quad \Rightarrow \quad v_{\max} \approx 2.60\times 10^5 \text{ m s}^{-1}$$

(b) At the surface of B, $V = 260$ V:

$$\tfrac{1}{2}\times 1.67\times 10^{-27} v_B^2 = 1.60\times 10^{-19}\times(460-260) \quad \Rightarrow \quad v_B \approx 1.96\times 10^5 \text{ m s}^{-1}$$

Equipotentials

- join points of the **same potential**
- point charge: concentric circles
- uniform field: parallel lines
- always **perpendicular** to field lines; no work along them



Gravitational and electric fields

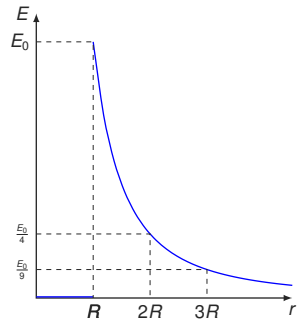
- both **inverse square laws**; E_p and potential $\propto 1/r$
- gravity: always attractive, potential always negative
- electric: attractive or repulsive, potential either sign

Inside a conductor

Inside a conductor

$E = 0$ everywhere and V is **constant**.

- metal sphere: outside acts as a point charge at the centre



Field strength and potential

Field strength

The **negative gradient** of the potential.

- V–r graph: gradient gives E
- E–r graph: area gives ΔV

$$E = -\frac{\mathrm{d}V}{\mathrm{d}r}$$

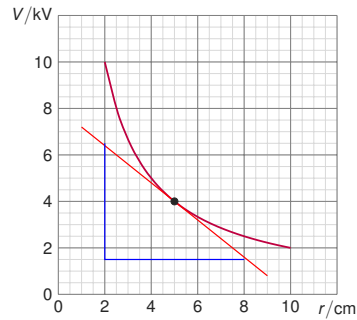
$$\Delta V = -\int E \,\mathrm{d}r$$

Example: gradient of a potential graph

The graph shows the potential near a charged object, with the tangent at $r = 5.0$ cm drawn. Find the field strength at 5.0 cm.

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Gradient of the tangent (from $(2.0\text{ cm}, 6.4\text{ kV})$ to $(8.0\text{ cm}, 1.6\text{ kV})$):

$$\frac{\Delta V}{\Delta r} = \frac{(1.6 - 6.4) \times 10^3}{(8.0 - 2.0) \times 10^{-2}} = -8.0 \times 10^4 \text{ V m}^{-1}$$

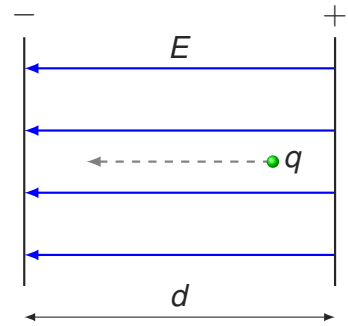
$$E = -\frac{\Delta V}{\Delta r} = 8.0 \times 10^4 \text{ V m}^{-1}$$

Uniform fields

Between plates a distance d apart with p.d. V:

$$E = \frac{V}{d}$$

- the V–r graph is a straight line, gradient $-E$
- field points from high to low potential



Key equations

Coulomb	$F = \dfrac{Qq}{4\pi\epsilon_0 r^2}$	potential	$V = \dfrac{Q}{4\pi\epsilon_0 r}$
field	$E = \dfrac{Q}{4\pi\epsilon_0 r^2}$	gradient	$E = -\dfrac{\mathrm{d}V}{\mathrm{d}r}$
P.E.	$E_p = \dfrac{Qq}{4\pi\epsilon_0 r}$	uniform	$E = \dfrac{V}{d}$