Electrostatics

Name _____ Date _____

By the end of this topic I can…

- use Coulomb's law, and define and use electric field strength E
- draw electric field lines and equipotentials
- use electric potential energy and potential: $E_p = \frac{Qq}{4\pi\epsilon_0 r}$, $V = \frac{Q}{4\pi\epsilon_0 r}$
- use $E = -\mathrm{d}V/\mathrm{d}r$, $E = V/d$, and compare electric and gravitational fields

Electric forces and fields

Coulomb's law

Charles Augustin de Coulomb published this law in 1785. It tells us how charged objects attract or repel through the _____.

_____: the electric force between two point charges is _____ and _____:

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}$ is the _____, and $k = \dfrac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ \mathrm{N\ m^2\ C^{-2}}$.

• Q and q are _____; a uniformly charged sphere acts as a point charge, with r measured between _____

• like charges: $Q_1 Q_2 > 0$, so $F > 0$: _____; opposite charges: $Q_1 Q_2 < 0$, so $F < 0$: _____

Example 23.1 A hydrogen atom has radius about 53 pm. Estimate the electric force between the proton and the electron.

Question 23.1 Two protons are a distance r apart. Find the ratio of the electric force to the gravitational force between them.

Electric fields

An _____ is a region of space where a _____.

Coulomb's law and Newton's law of gravitation are both _____, so electric and gravitational fields are very similar.

Electric field strength

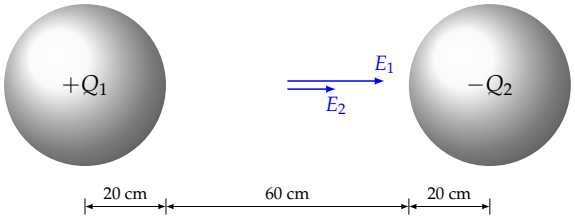
_____ is the electric _____:

- unit: $[E] =$ _____ $= \mathrm{V\ m^{-1}}$
- for a point charge Q, a test charge q at distance r feels $F = \dfrac{Qq}{4\pi\epsilon_0 r^2}$, so

The field depends on the _____ Q only. A charged sphere (outside, $r > R$) acts like a point charge at its _____.

- E is a _____: it points _____ a positive source and _____ a negative one. For several charges, add as _____.
- the force on a charge q is $F = Eq$: in the direction of E if $q > 0$, _____ to E if $q < 0$

Example 23.2 A Van de Graaff generator sparks when the surface field is 4.0×10^4 V cm^{-1}. Its sphere has diameter 40 cm. What is the charge on it?

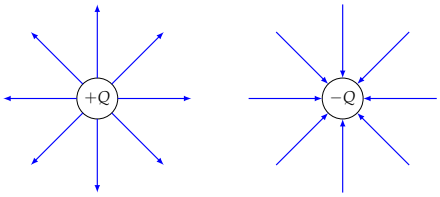


Example 23.3 Two identical metal spheres of radius 20 cm carry charges $+2.0\ \mu$C and $-1.0\ \mu$C, with a 60 cm gap between them. (a) Find the field strength midway between their centres. (b) A dust particle with charge -1.3×10^{-8} C is placed there. Find the force on it.

Question 23.2 At charge 4.0×10^{-7} C the field at the surface of a Van de Graaff sphere is 2.4×10^6 V m^{-1}. Find the extra charge needed to raise the surface field to 3.0×10^6 V m^{-1}.

Question 23.3 Two positively charged particles A and B are in a vacuum. P is on the line joining them, a distance x from A. Sketch how the field strength E varies with x.

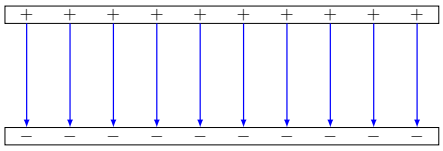
Electric field lines



Field of a positive and a negative sphere.

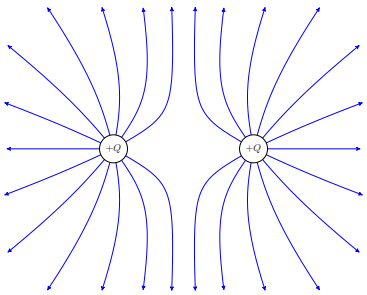
Field lines show direction (the arrows) and strength (the _____). They leave _____ charges and end on _____ charges.

Example 23.4 Sketch the field of a positive or a negative charged sphere, and explain why they act as point charges.

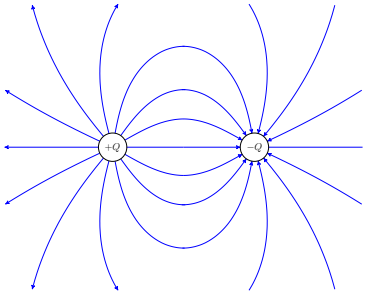


Field between oppositely charged plates.

Example 23.5 Sketch the field between two large oppositely charged metal plates.



Two equal positive charges.



Two opposite charges.

Electric potential and potential energy

Electric potential energy

As for gravity, the change in electric P.E. is the work done _____ (or by) the electric force, with $E_p = 0$ at $r = \infty$.

_____ is the _____ bringing a charge to a position from _____.

Bringing q from infinity to r from Q: $W = \int_{\infty}^{r} F\,\mathrm{d}x = -\dfrac{Qq}{4\pi\epsilon_0 r}$, and $\Delta E_p = -W$, so

- like charges: $Qq > 0$, $E_p > 0$; _____ to push them together (repulsion)
- opposite charges: $Qq < 0$, $E_p < 0$ (attraction); pulling them apart needs work, so E_p _____ towards zero
- E_p is a _____, but its sign matters: repulsion means positive, attraction means _____

Example 23.6 Alpha particles ($^4_2\alpha$) are fired at a gold foil ($^{197}_{79}\text{Au}$). A gold nucleus has radius about 10^{-14} m. What minimum initial speed lets an α-particle get this close?

Question 23.4 A metal sphere of radius 20 cm carries 5.0×10^{-7} C. A proton is fired at it at 1.8×10^6 m s^{-1}. Can it reach the surface?

Electric potential

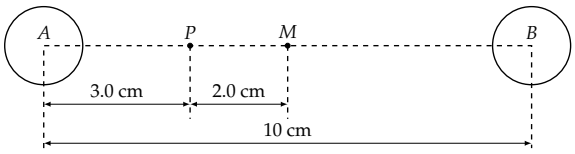
_____ at a point is the _____ bringing a _____ from _____ to that point:

- unit: $[V] = \text{J C}^{-1} =$ _____
- for a point charge Q:

- $V =$ _____ at infinity; $V > 0$ near a positive source, $V < 0$ near a negative one

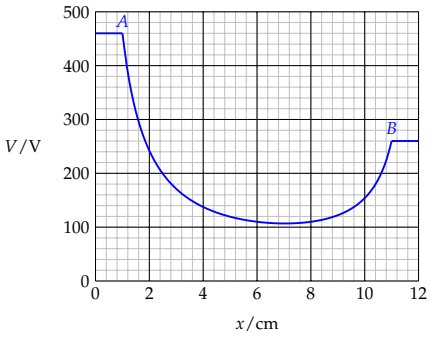
- V is a _____: add the contributions from several charges as _____

Example 23.7 An electron is accelerated from rest through a p.d. of 600 V. Find its final speed.



Example 23.8 Spheres A (+20 pC) and B (+84 pC) are 10 cm apart in a vacuum. P is 3.0 cm from A and M is the midpoint. (a) Find the potential at P and at M. (b) Find the work needed to move an α-particle from P to M.

Example 23.9 Spheres A and B (radius 1.0 cm) are positively charged. A proton starts at rest on the surface of A and moves along the line to B. The graph shows the potential V against distance x from the centre of A. (a) Find its maximum speed. (b) Find its speed on reaching the surface of B.

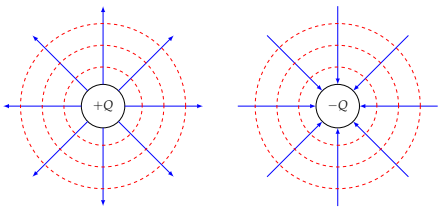


Question 23.5 Breakdown occurs when the surface field of a metal sphere of radius 16 cm exceeds 5.0×10^6 N C^{-1}. What is the surface potential at breakdown?

Question 23.6 A and B are 20 cm apart; A has charge $+7.2\ \mu$C. The potential is zero at the point P on AB with $AP = 5.0$ cm (between them). Find the charge of B.

Question 23.7 A particle of specific charge $+9.58 \times 10^7$ C kg^{-1} approaches a fixed metal sphere at potential $+500$ V, with initial speed 3.0×10^5 m s^{-1} from far away. Can it reach the surface?

Equipotentials



Field lines (blue) and equipotentials (red dashed).

Equipotential lines join points of the _____; they are the contour lines of potential.

- point charge: equipotentials are _____
- uniform field: equipotentials are _____
- equipotentials are always _____ to field lines, and moving along one needs _____

Further ideas

Comparison with gravitational fields

	gravitational	electric
force	$F = -\dfrac{GMm}{r^2}$	$F = \dfrac{Qq}{4\pi\epsilon_0 r^2}$
field strength	$g = -\dfrac{GM}{r^2}$	$E = \dfrac{Q}{4\pi\epsilon_0 r^2}$
potential energy	$E_p = -\dfrac{GMm}{r}$	$E_p = \dfrac{Qq}{4\pi\epsilon_0 r}$
potential	$\varphi = -\dfrac{GM}{r}$	$V = \dfrac{Q}{4\pi\epsilon_0 r}$

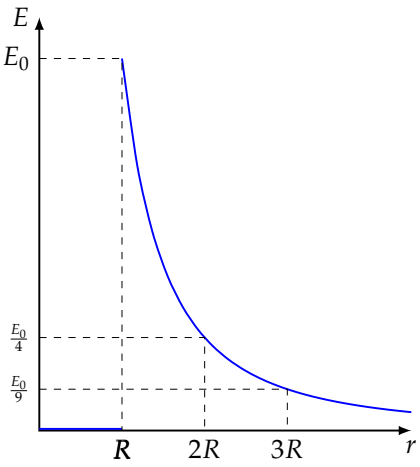
- *similar*: both are _____; E_p and potential vary as _____; both are zero at infinity
- *different*: mass is always positive but charge can be either sign, so electric force can be _____, gravity is _____; electric potential takes _____, gravitational potential is always _____

Fields inside conductors

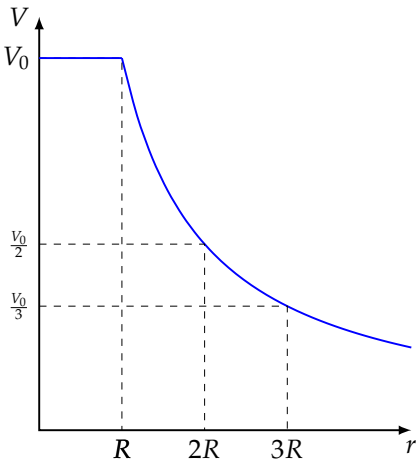
A charged conductor has free charges. They settle so that no net force acts on them.

Inside a conductor: field strength $E =$ ____ everywhere, and potential $V =$ _____ everywhere.

Example 23.10 Plot E and V against distance r from the centre of a metal sphere of radius R carrying charge Q.



E against r for a charged metal sphere.



V against r for the same sphere.

Question 23.8 Are the two spheres in the earlier V–x graph example conductors? What feature of the graph supports this?

Question 23.9 Why does a mobile phone signal weaken inside a lift?

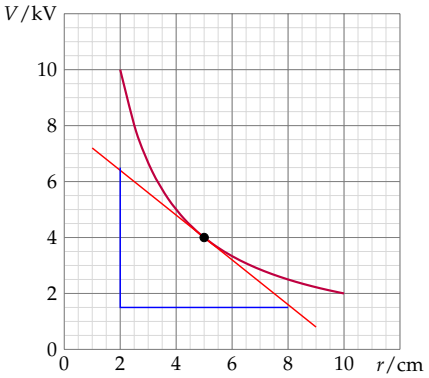
Field strength and potential

For a small displacement Δr: $\Delta V = \dfrac{\Delta E_p}{q} = -\dfrac{F\Delta r}{q} = -E\,\Delta r$, so

Field strength is the _____ of the potential with respect to displacement.

Conversely

- on a V–r graph the _____ gives E; on an E–r graph the _____ gives ΔV
- similarly $F = -\mathrm{d}E_p/\mathrm{d}r$: the force acts to _____ the potential energy



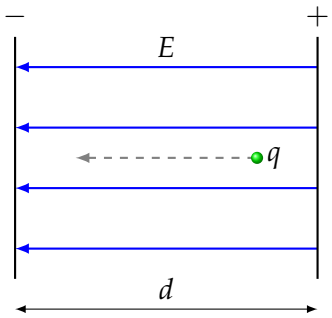
V (kV) against r (cm) near a charged object.

Example 23.11 The graph shows the potential near a charged object, with the tangent at $r = 5.0$ cm drawn. Find the field strength at 5.0 cm.

Question 23.10 Show that the object in the previous example behaves as a point charge, find its charge, and hence E at 5.0 cm.

Example 23.12 Differentiate $V = \dfrac{Q}{4\pi\epsilon_0 r}$ to find E for a point charge.

Question 23.11 Does 'field strength equals minus the potential gradient' also hold for gravitational fields?

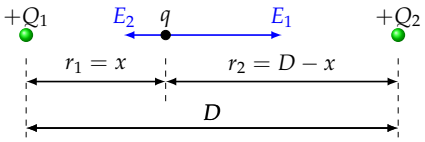


Uniform field between plates, and the linear V–r graph.

Uniform fields

For plates a distance d apart with p.d. V: moving q through the field, $W = Fd = Eqd$, so $\Delta E_p = -Eqd$ and $\Delta V = -Ed$ in magnitude:

The V–r graph is a straight line with gradient $-E$. The field points from _____ to _____ potential.



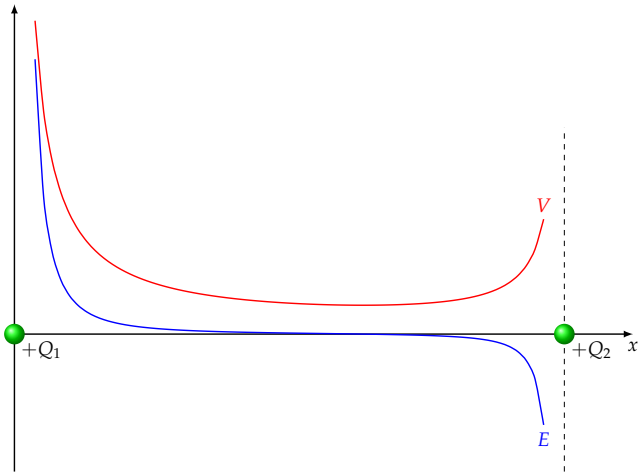
Charges $+Q_1$ and $+Q_2$, distance D apart.

Two positive point charges

Along the line joining them, at distance x from Q_1:

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1}{x} + \frac{Q_2}{D-x}\right) \qquad E = \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1}{x^2} - \frac{Q_2}{(D-x)^2}\right)$$

V is a _____, E a _____ (the two fields point in opposite directions between the charges).



Question 23.12 Discuss the field between (a) two negative charges, (b) two opposite charges.