Circular Motion

A Level Physics 9702

J. Rowing • Vijay International School Praslin

By the end of this topic I can...

- define angular displacement and the radian, and use $s = r\theta$
- define angular velocity and use $\omega = \Delta\theta/\Delta t$, $v = \omega r$ and $\omega = 2\pi/T$
- explain why uniform circular motion is accelerated and use $a = v^2/r = \omega^2 r$
- identify what provides the centripetal force and use $F = mv^2/r = m\omega^2 r$

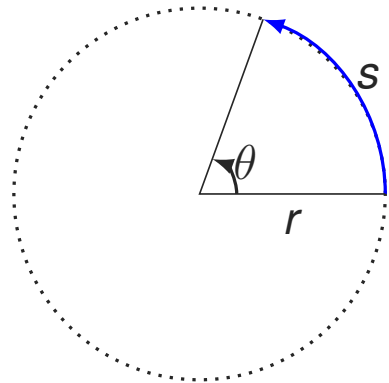
Angular displacement

Angular displacement

The **angle swept out** by an object moving along a circular path.

- unit: the **radian** (rad), 2π rad $= 360°$
- one radian: the angle formed when an arc is **the same length as the radius**

$$s = r\theta$$



Angular velocity

Angular velocity

The **angular displacement swept out per unit time**.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- unit: rad s^{-1}
- a vector in reality; at A Level treat it as a scalar (angular speed)

Linking linear and angular speed

In time Δt the distance moved along the arc is

$$\Delta s = v\,\Delta t = r\,\Delta\theta$$

Linking linear and angular speed

In time Δt the distance moved along the arc is

$$\Delta s = v\,\Delta t = r\,\Delta\theta$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r}$$

Linking linear and angular speed

In time Δt the distance moved along the arc is

$$\Delta s = v\,\Delta t = r\,\Delta\theta$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r}$$

$$\boxed{v = \omega r}$$

true at any instant

Uniform circular motion

Constant ω, so (compare $s = vt$):

$$\theta = \omega t$$

One period T sweeps 2π:

$$\omega = \frac{2\pi}{T}$$

Uniform circular motion

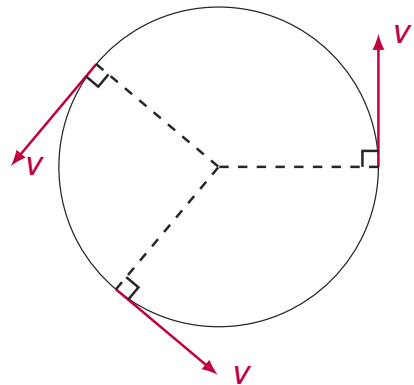
Constant ω, so (compare $s = vt$):

$$\theta = \omega t$$

One period T sweeps 2π:

$$\omega = \frac{2\pi}{T}$$

Still accelerating! The speed is constant, but the **velocity** changes: its direction is always tangential.



Example 18.1

An object moves at constant speed around a circular track of radius 2.5 m, taking 40 s for one lap. What are its angular speed and linear speed?

Example 18.1

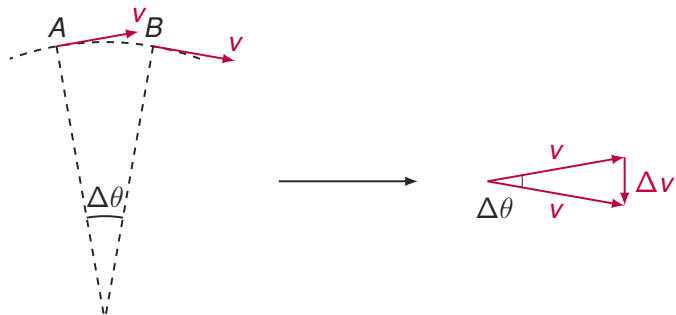
An object moves at constant speed around a circular track of radius 2.5 m, taking 40 s for one lap. What are its angular speed and linear speed?

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{40} \approx 0.157 \text{ rad s}^{-1} \qquad v = \omega r = 0.157 \times 2.5 \approx 0.39 \text{ m s}^{-1}$$

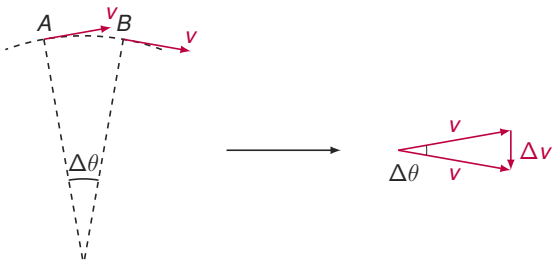
Centripetal acceleration

Centripetal acceleration

The acceleration due to the change in **direction** of the velocity. It points **towards the centre** of the circle.



Deriving a_c



$$\Delta v = 2v \sin \tfrac{\Delta\theta}{2} \approx v\,\Delta\theta$$

$$a = \frac{\Delta v}{\Delta t} \approx v\frac{\Delta\theta}{\Delta t} = v\omega$$

Deriving a_c



$$\Delta v = 2v \sin \tfrac{\Delta\theta}{2} \approx v\,\Delta\theta$$

$$a = \frac{\Delta v}{\Delta t} \approx v\frac{\Delta\theta}{\Delta t} = v\omega$$

using $v = \omega r$:

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Centripetal force

Centripetal force F_c

The **resultant force** on an object moving in a circle; always directed **towards the centre**.

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

- not a new force: the **resultant** of weight, tension, friction…
- acts at **right angles** to the velocity: changes direction, not speed

What provides the centripetal force?

- planet orbiting a star

What provides the centripetal force?

- planet orbiting a star

  gravity

- athlete on a curved track

What provides the centripetal force?

- planet orbiting a star

  gravity

- athlete on a curved track

  friction between shoes and track

- aeroplane banking

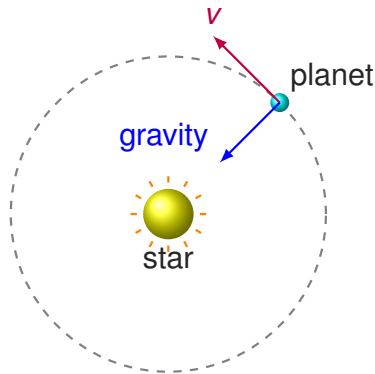
What provides the centripetal force?

- planet orbiting a star

  gravity
- athlete on a curved track

  friction between shoes and track
- aeroplane banking

  horizontal component of the lift



Example 18.3: a rock orbiting just above the Earth

Ignore air resistance, so only weight acts. $R = 6400$ km. Find the orbital speed and period.

Example 18.3: a rock orbiting just above the Earth

Ignore air resistance, so only weight acts. $R = 6400$ km. Find the orbital speed and period.

Weight provides the centripetal force: $mg = \dfrac{mv^2}{R}$

Example 18.3: a rock orbiting just above the Earth

Ignore air resistance, so only weight acts. $R = 6400$ km. Find the orbital speed and period.

Weight provides the centripetal force: $mg = \dfrac{mv^2}{R}$

$$v = \sqrt{gR} = \sqrt{9.81 \times 6.4 \times 10^6} \approx 7.9 \times 10^3 \text{ m s}^{-1}$$

$$T = \frac{2\pi R}{v} \approx 5.1 \times 10^3 \text{ s} \approx 85 \text{ min}$$

Example 18.4: the turntable

Object at $d = 40$ cm from the centre, $\mu = 0.30$. What is the maximum number of revolutions per minute before it slides?

Example 18.4: the turntable

Object at $d = 40$ cm from the centre, $\mu = 0.30$. What is the maximum number of revolutions per minute before it slides?

Friction provides F_c: $m\omega^2 d \leq \mu m g \quad \Rightarrow \quad \omega^2 \leq \dfrac{\mu g}{d}$

Example 18.4: the turntable

Object at $d = 40$ cm from the centre, $\mu = 0.30$. What is the maximum number of revolutions per minute before it slides?

Friction provides F_c: $m\omega^2 d \leq \mu m g \quad \Rightarrow \quad \omega^2 \leq \dfrac{\mu g}{d}$

$$\omega_{\max} = \sqrt{\frac{0.30 \times 9.81}{0.40}} \approx 2.71 \text{ rad s}^{-1} \quad \Rightarrow \quad T_{\min} = \frac{2\pi}{\omega_{\max}} \approx 2.32 \text{ s}$$

$$n_{\max} = \frac{60}{2.32} \approx 25.9 \text{ revolutions per minute}$$

Example 18.5: whirling in a vertical circle

$m = 0.40$ kg, $r = 0.80$ m, constant ω. (a)
Minimum ω if the string stays taut? (b)
Maximum ω if the string breaks above 20 N?

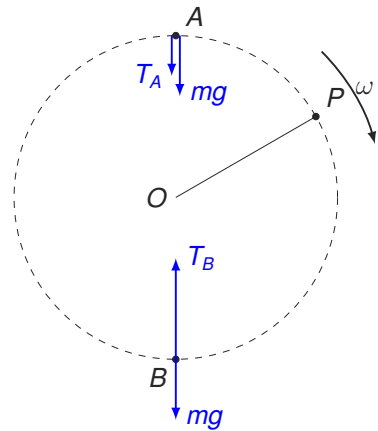
Example 18.5: whirling in a vertical circle

$m = 0.40$ kg, $r = 0.80$ m, constant ω. (a) Minimum ω if the string stays taut? (b) Maximum ω if the string breaks above 20 N?

Top A: $T_A = m\omega^2 r - mg$ Bottom B: $T_B = m\omega^2 r + mg$

(a) $T_A \geq 0$: $\omega_{\min} = \sqrt{g/r} \approx 3.5$ rad s^{-1}

(b) $T_B \leq 20$: $\omega_{\max} = \sqrt{\frac{T_{\max}}{m} - \frac{g}{r}} \approx 6.1$ rad s^{-1}



Example 18.6: the conical pendulum

Bob of mass 120 g in a horizontal circle,
$r = 10$ cm, string at $\theta = 25^\circ$ to the vertical.
Find the tension and the speed.

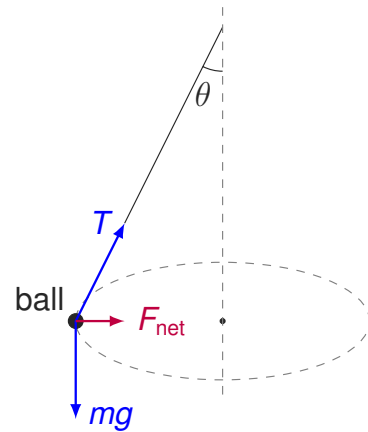
Example 18.6: the conical pendulum

Bob of mass 120 g in a horizontal circle, $r = 10$ cm, string at $\theta = 25^\circ$ to the vertical. Find the tension and the speed.

Vertical: $T\cos\theta = mg \quad \Rightarrow \quad T \approx 1.3$ N

Horizontal: $T\sin\theta = \dfrac{mv^2}{r}$

Divide: $v^2 = rg\tan\theta \quad \Rightarrow \quad v \approx 0.68$ m s^{-1}



Example 18.7: released from horizontal

Ball of mass m on a string of length l, released from rest with the string horizontal. Speed and tension at the lowest point?

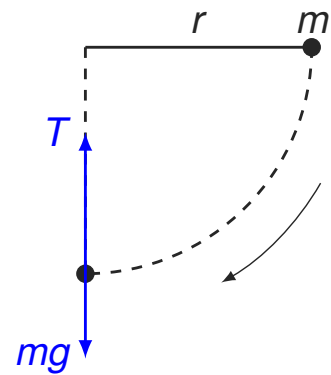
Example 18.7: released from horizontal

Ball of mass m on a string of length l, released from rest with the string horizontal. Speed and tension at the lowest point?

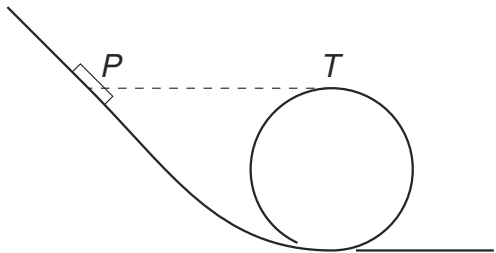
Energy: $mgl = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gl}$

Lowest point: $T - mg = \dfrac{mv^2}{l}$

$$T = mg + \frac{m(2gl)}{l} = 3mg$$

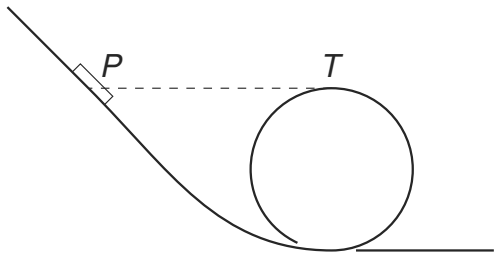


Challenge: design a roller-coaster



A slider starts from rest at P, level with the top of the loop T, on a frictionless track. (a) Show it cannot reach T. (b) What is the minimum release height?

Challenge: design a roller-coaster



A slider starts from rest at P, level with the top of the loop T, on a frictionless track. (a) Show it cannot reach T. (b) What is the minimum release height?

(a) Energy: $v_T = 0$ at T, but staying on the track needs $v_T \geq \sqrt{gr}$. (b) $mgh = 2mgr + \frac{1}{2}mgr \quad \Rightarrow \quad h_{\min} = 2.5\,r$

Key equations

arc length	$s = r\theta$	angular velocity	$\omega = \dfrac{\Delta\theta}{\Delta t}$
linear speed	$v = \omega r$	period	$\omega = \dfrac{2\pi}{T}$
centripetal acceleration	$a_c = \dfrac{v^2}{r} = \omega^2 r$	centripetal force	$F_c = \dfrac{mv^2}{r} = m\omega^2 r$