Capacitors

A Level Physics 9702

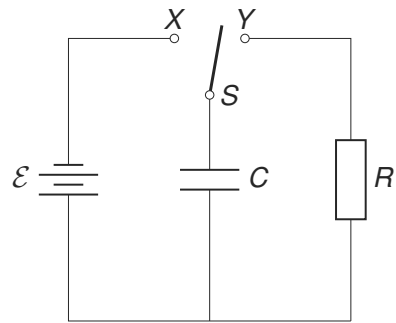
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By the end of this topic I can...

- define capacitance, $C = Q/V$, and use the energy stored $W = \frac{1}{2}QV$
- add capacitors in parallel and in series
- analyse simple capacitor networks
- (extension) charging, discharging and the time constant

What is a capacitor?

- two **conducting plates** with an insulating **dielectric** between
- switch to *X*: **charges**; switch to *Y*: **discharges** through *R*
- stores and releases **charge** and energy
- also used to smooth supplies and to block d.c.



Capacitance

Capacitance

The ratio of the **charge on one plate** to the **p.d. across the plates**.

- unit: farad, F = C V^{-1}
- 1 μF = 10^{-6} F, 1 nF = 10^{-9} F, 1 pF = 10^{-12} F
- net charge on a capacitor is **zero**
- C depends on **geometry**, not on Q or V
- parallel plates: $C = \epsilon_0 A/d$

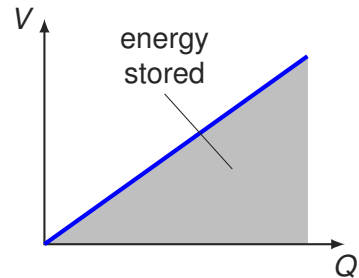
$$C = \frac{Q}{V}$$

Energy stored

Area under the V–Q graph:

$$W = \tfrac{1}{2}QV$$

$$W = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$



- stored in the **electric field** between the plates

Example: extra energy

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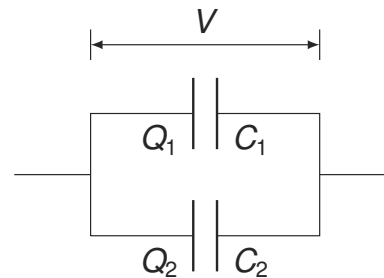
$$\Delta W = \frac{1}{2}CV_f^2 - \frac{1}{2}CV_i^2 = \frac{1}{2} \times 1.8 \times 10^{-4} \times (20^2 - 10^2) = 2.7 \times 10^{-2}\ \text{J}$$

Capacitors in parallel

Same **p.d.**; charge is **shared**:

$$C_{\text{total}} = C_1 + C_2 + C_3 + \cdots$$

- total capacitance **increases**
- like one capacitor with larger plates



Example: sharing charge

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Initially $Q = C_0 V_0$ and $W_0 = \frac{1}{2} C_0 V_0^2$.

Combined capacitance $= 2C_0$. Charge is conserved, so $V = \dfrac{Q}{2C_0} = \frac{1}{2} V_0$.

$W = \frac{1}{2}(2C_0)(\frac{1}{2} V_0)^2 = \frac{1}{4} C_0 V_0^2 = \frac{1}{2} W_0$

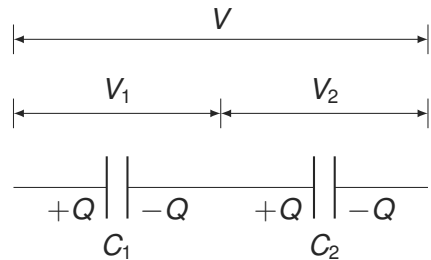
Half the energy is lost as heat when charge flows between the capacitors.

Capacitors in series

Same **charge**; p.d. is **shared**:

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

- total capacitance **decreases**
- like one capacitor with larger plate separation



Example: series capacitors

Find the p.d. across each capacitor in the circuit shown ($C_1 = 4.0\ \mu$F, $C_2 = 12\ \mu$F, supply 8.0 V).

Example: series capacitors

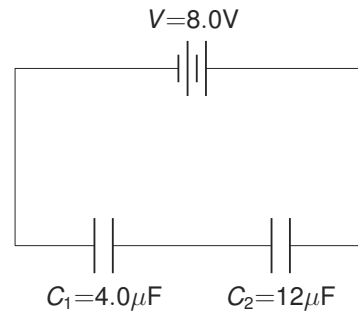
Find the p.d. across each capacitor in the circuit shown ($C_1 = 4.0\ \mu$F, $C_2 = 12\ \mu$F, supply 8.0 V).

$$C_{\text{total}} = \left(\frac{1}{4.0} + \frac{1}{12}\right)^{-1} = 3.0\ \mu\text{F}$$

$Q = C_{\text{total}} V = 3.0 \times 8.0 = 24\ \mu$C, the same on each capacitor.

$$V_1 = \frac{Q}{C_1} = \frac{24}{4.0} = 6.0\text{ V}, \quad V_2 = \frac{Q}{C_2} = \frac{24}{12} = 2.0\text{ V}$$

(Check: $6.0 + 2.0 = 8.0$ V.)

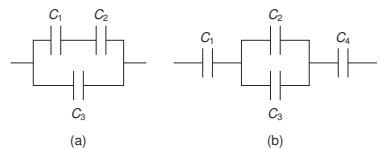


Example: two networks

$C_1 = C_2 = C_3 = C_4 = 10\ \mu$F. Calculate the capacitance of networks (a) and (b).

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(a) C_1 and C_2 in series:
$C_{12} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} = 5.0\ \mu$F; this is in parallel with C_3: $C_{\text{total}} = 5 + 10 = 15\ \mu$F

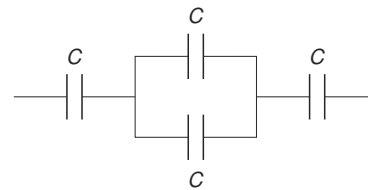
(b) C_2 and C_3 in parallel: $C_{23} = 20\ \mu$F; in series with C_1 and C_4: $C_{\text{total}} = \left(\frac{1}{10} + \frac{1}{20} + \frac{1}{10}\right)^{-1} = 4.0\ \mu$F

Example: maximum safe p.d.

Four identical capacitors are arranged as shown. Each can withstand a maximum p.d. of 12 V. What is the maximum safe p.d. between the terminals?

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Four identical capacitors are arranged as shown. Each can withstand a maximum p.d. of 12 V. What is the maximum safe p.d. between the terminals?



Let the leftmost capacitor carry charge Q. The rightmost also carries Q.
The two in parallel share this charge, so each carries $\frac{Q}{2}$ and has half the p.d.: 6 V when the end capacitors are at 12 V.
$V_{\text{max}} = 12 + 6 + 12 = 30$ V

Capacitors and resistors

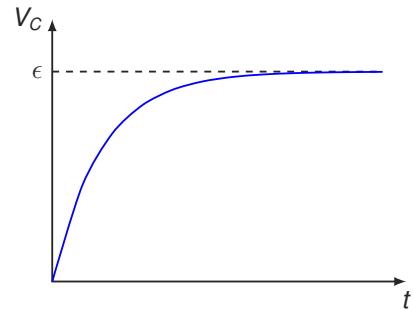
- **series**: capacitors share p.d., same Q; resistors share p.d., same I
- **parallel**: capacitors share Q, same V; resistors share I, same V
- C adds in parallel and as reciprocals in series: the **opposite** of R

Charging a capacitor

Extension (not on the 9702 syllabus):

$$V_C = \mathcal{E}\left(1 - \mathrm{e}^{-t/RC}\right)$$

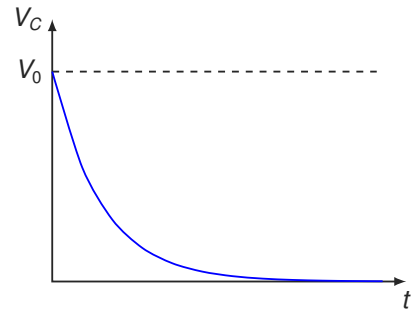
- V_C rises at a **decreasing** rate towards $\mathcal{E}$
- charging current falls to **zero**



Discharging a capacitor

$$V_C = V_0 \, \mathrm{e}^{-t/RC}$$

- V_C and the current decay **exponentially** to zero



The time constant

$$\tau = RC$$

- larger R: smaller current, **slower**
- larger C: more charge to move, **slower**
- almost complete after about 3–5τ

Key equations

capacitance $C = \dfrac{Q}{V}$

energy $W = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$

parallel $C_{\text{total}} = C_1 + C_2 + C_3 + \cdots$

series $\dfrac{1}{C_{\text{total}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \cdots$