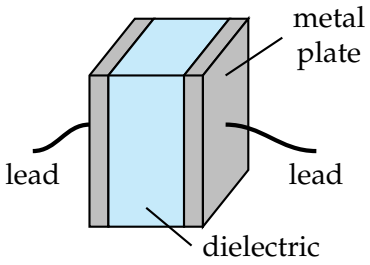
Capacitors

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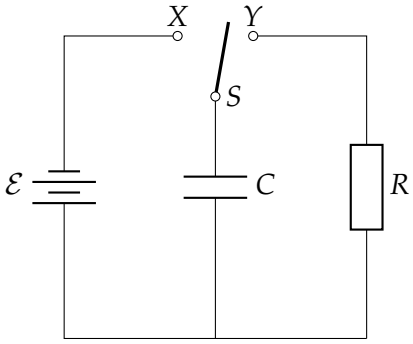
- define capacitance, $C = \frac{Q}{V}$, and use it with the energy stored $W = \frac{1}{2}QV$
- add capacitors in parallel and in series, and analyse simple networks
- compare capacitor networks with resistor networks
- (extension) describe charging and discharging through a resistor, and the time constant $\tau = RC$

Capacitors



An introduction

A capacitor is a component with two conducting plates separated by an insulator called the dielectric. The circuit symbol is two short parallel lines.



- switch S at X: the capacitor is connected to the supply and charges; positive and negative charge are separated onto the plates and stay there even if the supply is removed
- switch S at Y: the capacitor discharges through R, acting as a temporary power source

Capacitors store and release charge (and so energy). They are also used to smooth power-supply output and to block d.c. while passing a.c.

Capacitance

Capacitance of a parallel-plate capacitor is the ratio of the charge on one plate to the potential difference across the plates:

$$C = \frac{Q}{V}$$

- unit: the farad (F), $1\ \text{F} = 1\ \text{C V}^{-1}$. Named after Michael Faraday.
- the farad is very large, so we use sub-multiples: $1\ \mu\text{F} =$ 10^{-6} F, $1\ \text{nF} =$ 10^{-9} F, $1\ \text{pF} =$ 10^{-12} F
- the plates carry equal and opposite charges $+Q$ and $-Q$, so the net charge on the capacitor is zero. In $C = Q/V$, Q is

the charge on **one** plate, not the difference.

• capacitance depends on the geometry (and dielectric), not on Q or V. For two metal plates in a vacuum:

$$C = \frac{\epsilon_0 A}{d}$$

where A is the area of a plate and d is the plate separation. With a dielectric, ϵ_0 is replaced by its permittivity ϵ. (These formulae are not examined.)

Question 24.1 A capacitor stores 36 μC on each plate when the p.d. across it is 12 V. What is its capacitance? What is the *net* charge on it?

$C = \frac{Q}{V} = \frac{36\ \mu\text{C}}{12\ \text{V}} = 3.0\ \mu\text{F}$
The net charge is $+36 - 36 = 0\ \mu$C.

Analogy with a gas

• a container of volume V holds $n = \left(\frac{V}{RT}\right) p$ of gas: pumping in more gas raises the pressure

• a capacitor holds charge $Q = CV$: adding more charge raises the p.d., and C is the charge capacity

• a container has a maximum pressure; a capacitor has a breakdown voltage beyond which sparking can occur across it

Energy stored in a capacitor

Charging means pushing electrons from one plate to the other, which needs work done, so energy is stored.

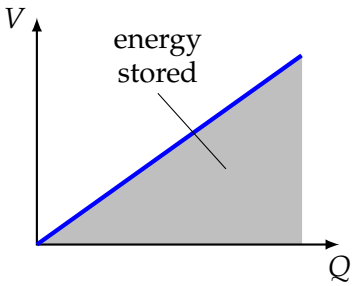
• the V–Q graph is a straight line through the origin, so the work done is the area under it:

$$W = \tfrac{1}{2} QV$$

• substituting $Q = CV$:

$$W = \tfrac{1}{2} CV^2 = \frac{Q^2}{2C}$$

Strictly the energy is stored in the electric field between the plates.



The area under the V–Q graph is the energy stored.

Example 24.1 When the p.d. across a capacitor of 1.8×10^{-4} F is increased from 10 V to 20 V, how much additional energy is stored?

$\Delta W = \frac{1}{2}CV_f^2 - \frac{1}{2}CV_i^2 = \frac{1}{2} \times 1.8 \times 10^{-4} \times (20^2 - 10^2) = 2.7 \times 10^{-2}$ J

Question 24.2 A capacitor of 2500 μF is charged to a working voltage of 18 V. (a) What is the magnitude of the positive charge on its plate? (b) What is the energy stored?

(a) $Q = CV = 2500 \times 10^{-6} \times 18 = 0.045$ C
(b) $W = \frac{1}{2}QV = \frac{1}{2} \times 0.045 \times 18 \approx 0.41$ J

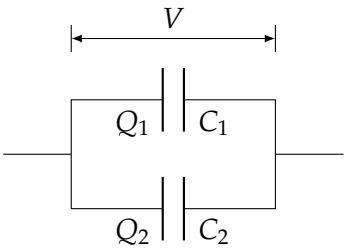
Question 24.3 A capacitor initially charged to a p.d. of 16 V discharges and loses 40% of its energy. What is its new p.d.?

$W \propto V^2$ for fixed C, and $W' = 0.60W$, so $V' = 16\sqrt{0.60} \approx 12$ V

Question 24.4 For an isolated metal sphere of radius 30 cm situated in vacuum, what is the electric potential energy stored when charged to a potential of 120 kV?

Self-capacitance $C = 4\pi\epsilon_0 R = 4\pi \times 8.85 \times 10^{-12} \times 0.30 \approx 3.3 \times 10^{-11}$ F
$W = \frac{1}{2}CV^2 = \frac{1}{2} \times 3.3 \times 10^{-11} \times (1.2 \times 10^5)^2 \approx 0.24$ J

Capacitor networks



Capacitors in parallel

Capacitors in parallel have the same p.d., $V = V_1 = V_2$, but the charge is shared: $Q_{\text{total}} = Q_1 + Q_2$.

Dividing by V: $\frac{Q_{\text{total}}}{V} = \frac{Q_1}{V} + \frac{Q_2}{V}$, so

$$C_{\text{total}} = C_1 + C_2 + C_3 + \cdots$$

- adding a capacitor in parallel increases the total capacitance (like one capacitor with larger plates)

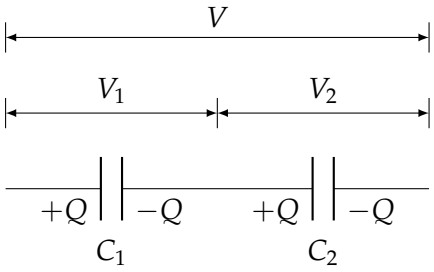
Example 24.2 A capacitor C_0 is charged to a p.d. V_0. It is disconnected from the supply and connected across an identical capacitor. Discuss the change in p.d. and in the energy stored.

Initially $Q = C_0V_0$ and $W_0 = \frac{1}{2}C_0V_0^2$.
Combined capacitance $= 2C_0$. Charge is conserved, so $V = \frac{Q}{2C_0} = \frac{1}{2}V_0$.
$W = \frac{1}{2}(2C_0)(\frac{1}{2}V_0)^2 = \frac{1}{4}C_0V_0^2 = \frac{1}{2}W_0$
Half the energy is lost as heat when charge flows between the capacitors.

Question 24.5 Capacitor A of capacitance C and capacitor B of capacitance $3C$ are connected in parallel across a supply. Find the ratio of energy stored in A to that in B, and the ratio of charge.

Same V: $W = \frac{1}{2}CV^2 \propto C$ and $Q = CV \propto C$.
Both ratios are $A : B = 1 : 3$.

Capacitors in series

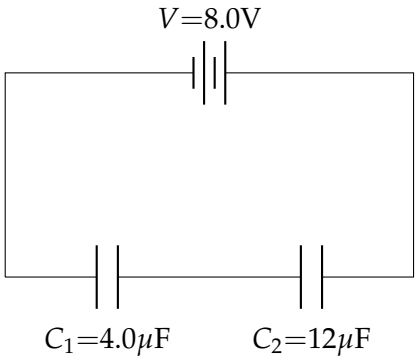


Capacitors in series carry the same charge, $Q = Q_1 = Q_2$ (the isolated section between them has zero net charge), and the p.d. is shared: $V_{\text{total}} = V_1 + V_2$.

Dividing by Q: $\dfrac{V_{\text{total}}}{Q} = \dfrac{V_1}{Q} + \dfrac{V_2}{Q}$, so

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

• adding a capacitor in series decreases the total capacitance (like one capacitor with a larger plate separation)



Example 24.3 Find the p.d. across each capacitor in the circuit shown ($C_1 = 4.0\ \mu$F, $C_2 = 12\ \mu$F, supply 8.0 V).

$C_{\text{total}} = \left(\dfrac{1}{4.0} + \dfrac{1}{12}\right)^{-1} = 3.0\ \mu\text{F}$

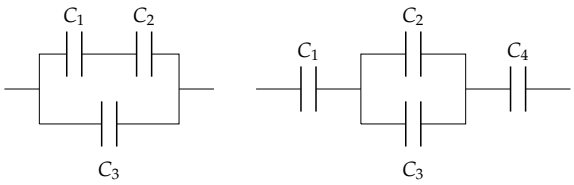
$Q = C_{\text{total}} V = 3.0 \times 8.0 = 24\ \mu\text{C}$, the same on each capacitor.

$V_1 = \dfrac{Q}{C_1} = \dfrac{24}{4.0} = 6.0\text{ V}, \quad V_2 = \dfrac{Q}{C_2} = \dfrac{24}{12} = 2.0\text{ V}$

(Check: $6.0 + 2.0 = 8.0$ V.)

More complicated networks

Break a network into smaller parts that are in series or in parallel.

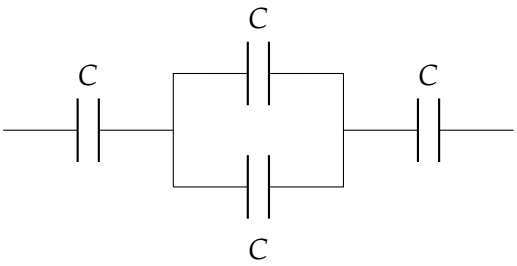


(a) (b)

Example 24.4 $C_1 = C_2 = C_3 = C_4 = 10\ \mu$F. Calculate the capacitance of networks (a) and (b).

(a) C_1 and C_2 in series: $C_{12} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} = 5.0\ \mu$F; this is in parallel with C_3: $C_{\text{total}} = 5 + 10 = 15\ \mu$F

(b) C_2 and C_3 in parallel: $C_{23} = 20\ \mu$F; in series with C_1 and C_4: $C_{\text{total}} = \left(\frac{1}{10} + \frac{1}{20} + \frac{1}{10}\right)^{-1} = 4.0\ \mu$F



Example 24.5 Four identical capacitors are arranged as shown. Each can withstand a maximum p.d. of 12 V. What is the maximum safe p.d. between the terminals?

Let the leftmost capacitor carry charge Q. The rightmost also carries Q.
The two in parallel share this charge, so each carries $\frac{Q}{2}$ and has half the p.d.: 6 V when the end capacitors are at 12 V.
$V_{\text{max}} = 12 + 6 + 12 = 30$ V

Question 24.6 Using at most four capacitors of 24 μF, design a network with a combined capacitance of (a) 72 μF, (b) 8 μF, (c) 36 μF, (d) 18 μF.

(a) three in parallel: $3 \times 24 = 72$ μF
(b) three in series: $24/3 = 8$ μF
(c) one capacitor in parallel with a series pair: $24 + 12 = 36$ μF
(d) three in parallel (72) in series with one (24): $\left(\frac{1}{72} + \frac{1}{24}\right)^{-1} = 18$ μF

Question 24.7 Both networks in the example above are connected to a 15 V supply. Find the p.d. across each capacitor.

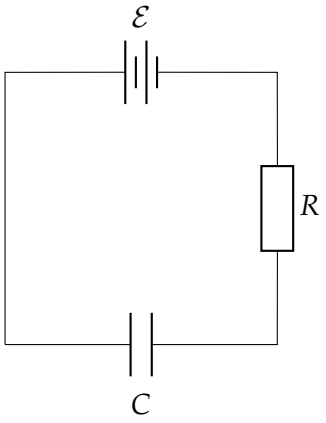
(a) The series pair C_1, C_2 has 5 μF across 15 V: $Q = 75$ μC each, so $V_1 = V_2 = 7.5$ V. C_3 has the full 15 V.
(b) $Q = C_{\text{total}} V = 4.0 \times 15 = 60$ μC. C_1: $60/10 = 6.0$ V; C_4: 6.0 V; the parallel pair C_{23}: $60/20 = 3.0$ V across each of C_2 and C_3.

Capacitors and resistors compared

	capacitors	resistors
in series	same ___charge___ ; $\frac{1}{C_{\text{total}}} = \sum \frac{1}{C_i}$	same ___current___ ; $R_{\text{total}} = \sum R_i$
in parallel	same ___p.d.___ ; $C_{\text{total}} = \sum C_i$	same ___p.d.___ ; $\frac{1}{R_{\text{total}}} = \sum \frac{1}{R_i}$

Charging and discharging (extension)

This section is **not** required by the 9702 syllabus.



Charging

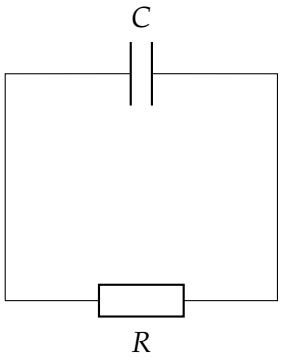
Initially $Q = 0$ and $V_C = 0$. At any instant $\mathrm{d}Q = C\,\mathrm{d}V_C$ and the current is $I = \dfrac{V_R}{R} = \dfrac{\mathcal{E} - V_C}{R}$.

So $\dfrac{\mathcal{E} - V_C}{R}\mathrm{d}t = C\,\mathrm{d}V_C$, which separates to $\displaystyle\int_0^t \frac{\mathrm{d}t}{RC} = \int_0^{V_C} \frac{\mathrm{d}V_C}{\mathcal{E} - V_C}$ and gives

$$V_C = \mathcal{E}\left(1 - \mathrm{e}^{-t/RC}\right)$$

- V_C rises at a ___decreasing___ rate: it gets harder to push more charge onto plates that already repel it
- $V_C \to \mathcal{E}$ and the charging current falls to ___zero___

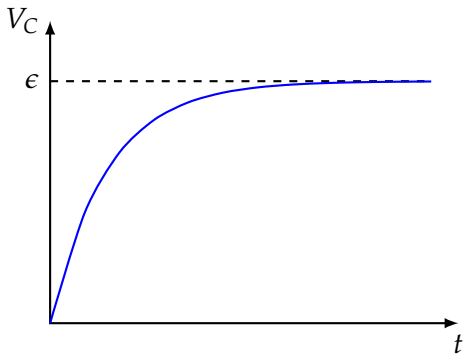
Discharging



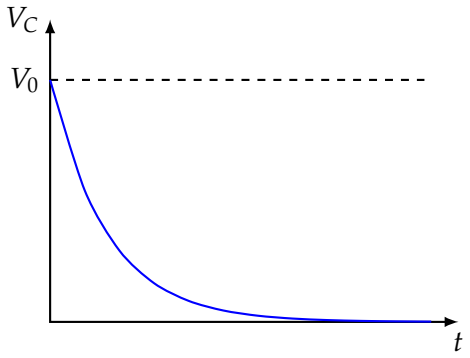
Starting from $V_C = V_0$, the p.d. across C and R is the same, and $dQ = -C\,dV_C$ (charge decreases). With $I = V_C/R$:

$$-\int_0^t \frac{dt}{RC} = \int_{V_0}^{V_C} \frac{dV_C}{V_C} \quad \Rightarrow \quad \boxed{V_C = V_0\,e^{-t/RC}}$$

• both V_C and the discharge current decay exponentially towards zero



Charging: V_C rises towards $\mathcal{E}$.



Discharging: V_C falls towards zero.

The time constant

The time constant $\tau = RC$ sets how quickly a capacitor charges or discharges: $\boxed{\tau = RC}$

• larger R means a smaller current, so charging or discharging takes longer

• larger C means more charge to move, so it takes longer

• after about 3–5τ the process is almost complete. After one τ, a discharging capacitor has fallen to $e^{-1} \approx 37\%$ of V_0.