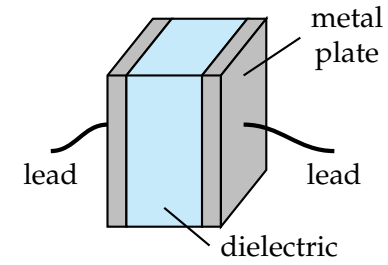
Capacitors

Name _____ Date _____

By the end of this topic I can…

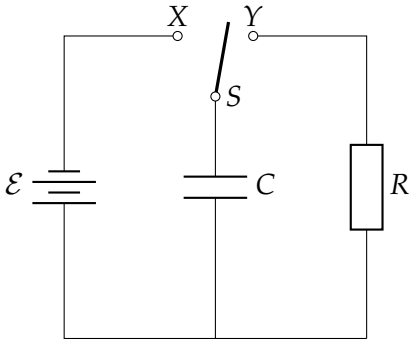
- define capacitance, $C = \frac{Q}{V}$, and use it with the energy stored $W = \frac{1}{2}QV$
- add capacitors in parallel and in series, and analyse simple networks
- compare capacitor networks with resistor networks
- (extension) describe charging and discharging through a resistor, and the time constant $\tau = RC$

Capacitors



An introduction

A _____ is a component with two _____ separated by an insulator called the _____. The circuit symbol is two short parallel lines.



• switch S at X: the capacitor is connected to the supply and _____; positive and negative charge are separated onto the plates and stay there even if the supply is removed

• switch S at Y: the capacitor _____ through R, acting as a temporary _____

Capacitors store and release _____ (and so energy). They are also used to smooth power-supply output and to block d.c. while passing a.c.

Capacitance

_____ of a parallel-plate capacitor is the ratio of the _____ to the _____:

• unit: the _____ (F), $1\ \text{F} = 1\ \text{C V}^{-1}$. Named after _____.

• the farad is very large, so we use sub-multiples: $1\ \mu\text{F} =$ _____, $1\ \text{nF} =$ _____, $1\ \text{pF} =$ _____

• the plates carry _____ charges $+Q$ and $-Q$, so the net charge on the capacitor is _____. In $C = Q/V$, Q is

the charge on **one** plate, not the difference.

• capacitance depends on the _____ (and dielectric), not on Q or V. For two metal plates in a vacuum:

$$\qquad$$

where A is the _____ and d is the _____. With a dielectric, ϵ_0 is replaced by its permittivity ϵ. (These formulae are not examined.)

Question 24.1 A capacitor stores 36 μC on each plate when the p.d. across it is 12 V. What is its capacitance? What is the *net* charge on it?

Analogy with a gas

• a container of volume V holds $n = \left(\frac{V}{RT}\right) p$ of gas: pumping in more gas raises the _____

• a capacitor holds charge $Q = CV$: adding more charge raises the _____, and C is the charge _____

• a container has a maximum pressure; a capacitor has a _____ beyond which _____ can occur across it

Energy stored in a capacitor

Charging means pushing electrons from one plate to the other, which needs _____, so energy is stored.

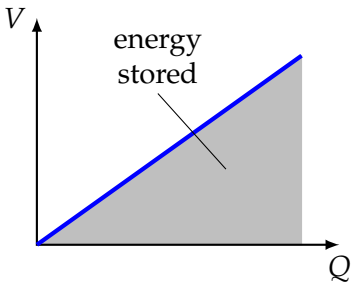
• the V–Q graph is a _____, so the work done is the _____:

$$\qquad$$

• substituting $Q = CV$:

$$\qquad$$

Strictly the energy is stored in the _____ between the plates.



The area under the V–Q graph is the energy stored.

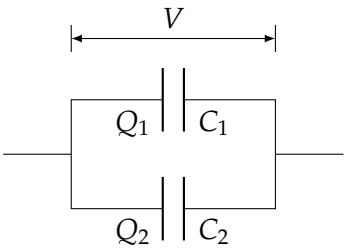
Example 24.1 When the p.d. across a capacitor of 1.8×10^{-4} F is increased from 10 V to 20 V, how much additional energy is stored?

Question 24.2 A capacitor of 2500 μF is charged to a working voltage of 18 V. (a) What is the magnitude of the positive charge on its plate? (b) What is the energy stored?

Question 24.3 A capacitor initially charged to a p.d. of 16 V discharges and loses 40% of its energy. What is its new p.d.?

Question 24.4 For an isolated metal sphere of radius 30 cm situated in vacuum, what is the electric potential energy stored when charged to a potential of 120 kV?

Capacitor networks



Capacitors in parallel

Capacitors in parallel have the _____, $V = V_1 = V_2$, but the charge is _____: $Q_{\text{total}} = Q_1 + Q_2$.

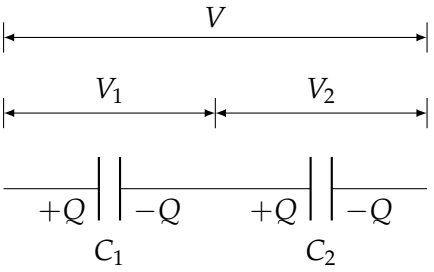
Dividing by V: $\frac{Q_{\text{total}}}{V} = \frac{Q_1}{V} + \frac{Q_2}{V}$, so

- adding a capacitor in parallel _____ the total capacitance (like one capacitor with larger _____)

Example 24.2 A capacitor C_0 is charged to a p.d. V_0. It is disconnected from the supply and connected across an identical capacitor. Discuss the change in p.d. and in the energy stored.

Question 24.5 Capacitor A of capacitance C and capacitor B of capacitance $3C$ are connected in parallel across a supply. Find the ratio of energy stored in A to that in B, and the ratio of charge.

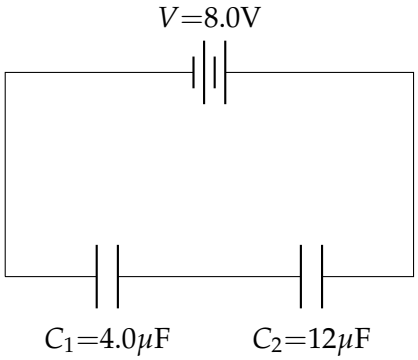
Capacitors in series



Capacitors in series carry the _____, $Q = Q_1 = Q_2$ (the isolated section between them has zero net charge), and the p.d. is _____: $V_{\text{total}} = V_1 + V_2$.

Dividing by Q: $\frac{V_{\text{total}}}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q}$, so

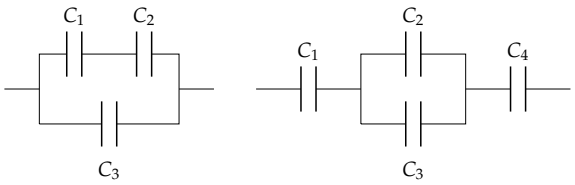
- adding a capacitor in series _____ the total capacitance (like one capacitor with a larger _____)



Example 24.3 Find the p.d. across each capacitor in the circuit shown ($C_1 = 4.0\ \mu$F, $C_2 = 12\ \mu$F, supply 8.0 V).

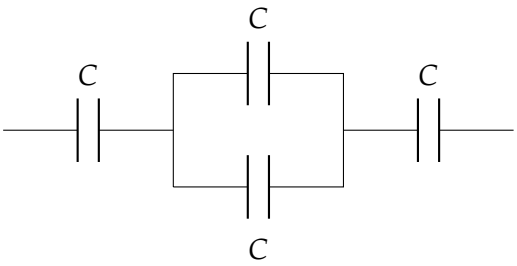
More complicated networks

Break a network into smaller parts that are in series or in parallel.



(a) (b)

Example 24.4 $C_1 = C_2 = C_3 = C_4 = 10\ \mu$F. Calculate the capacitance of networks (a) and (b).



Example 24.5 Four identical capacitors are arranged as shown. Each can withstand a maximum p.d. of 12 V. What is the maximum safe p.d. between the terminals?

Question 24.6 Using at most four capacitors of 24 μF, design a network with a combined capacitance of (a) 72 μF, (b) 8 μF, (c) 36 μF, (d) 18 μF.

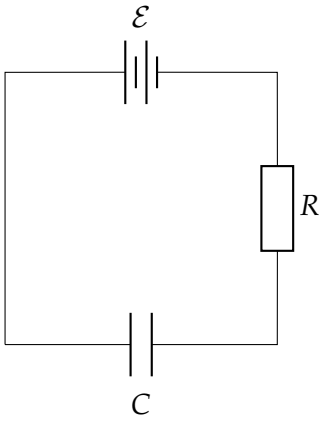
Question 24.7 Both networks in the example above are connected to a 15 V supply. Find the p.d. across each capacitor.

Capacitors and resistors compared

	capacitors	resistors
in series	same _____; $\frac{1}{C_{\text{total}}} = \sum \frac{1}{C_i}$	same _____; $R_{\text{total}} = \sum R_i$
in parallel	same _____; $C_{\text{total}} = \sum C_i$	same _____; $\frac{1}{R_{\text{total}}} = \sum \frac{1}{R_i}$

Charging and discharging (extension)

This section is **not** required by the 9702 syllabus.



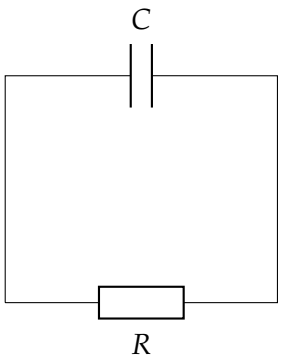
Charging

Initially $Q = 0$ and $V_C = 0$. At any instant $\mathrm{d}Q = C\,\mathrm{d}V_C$ and the current is $I = \dfrac{V_R}{R} = \dfrac{\mathcal{E} - V_C}{R}$.

So $\dfrac{\mathcal{E} - V_C}{R}\mathrm{d}t = C\,\mathrm{d}V_C$, which separates to $\displaystyle\int_0^t \frac{\mathrm{d}t}{RC} = \int_0^{V_C} \frac{\mathrm{d}V_C}{\mathcal{E} - V_C}$ and gives

- V_C rises at a _____ rate: it gets harder to push more charge onto plates that already repel it
- $V_C \to \mathcal{E}$ and the charging current falls to _____

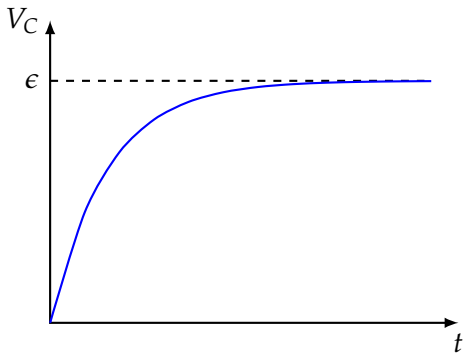
Discharging



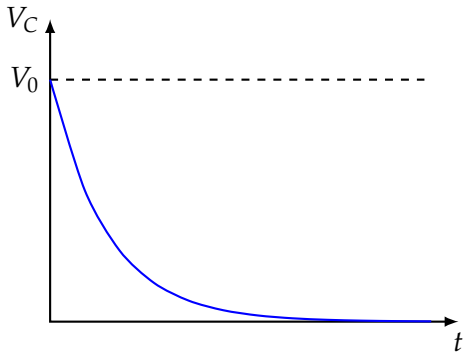
Starting from $V_C = V_0$, the p.d. across C and R is the same, and $dQ = -C\,dV_C$ (charge decreases). With $I = V_C/R$:

$$-\int_0^t \frac{dt}{RC} = \int_{V_0}^{V_C} \frac{dV_C}{V_C} \quad \Rightarrow$$

• both V_C and the discharge current decay _____ towards _____



Charging: V_C rises towards $\mathcal{E}$.



Discharging: V_C falls towards zero.

The time constant

The _____ $\tau = RC$ sets how quickly a capacitor charges or discharges:

• larger R means a _____ current, so charging or discharging takes _____

• larger C means more charge to move, so it takes _____

• after about 3–5τ the process is almost complete. After one τ, a discharging capacitor has fallen to $e^{-1} \approx 37\%$ of V_0.