Alternating Currents

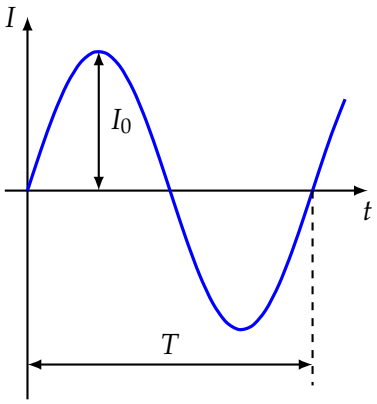
Name _____ Date _____

By the end of this topic I can…

- describe sinusoidal a.c. and use peak, r.m.s. and mean power
- read voltage, period and frequency from an oscilloscope trace
- explain the transformer and why electricity is transmitted at high voltage
- explain half-wave and full-wave rectification and smoothing

Sinusoidal alternating current

A direct current (d.c.) flows in one direction only. An alternating current (a.c.) reverses its direction repeatedly. Generators produce sinusoidal a.c., which is the main kind in this course.



The sinusoidal waveform

$$I = I_0 \sin \omega t \qquad V = V_0 \sin \omega t$$

- I_0, V_0 are the peak current and voltage (the amplitudes)
- ω is the angular frequency:

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{and} \quad T = \frac{1}{f}$$

- the mean current and mean voltage over a cycle are zero: the positive and negative halves cancel

Question 28.1 An alternating voltage has a peak value of 16 V and a period of 0.10 s. Write an equation for the voltage.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.10} = 20\pi \approx 63 \text{ rad s}^{-1}$, so $V = 16\sin(20\pi t)$ (volts, seconds)

Question 28.2 The rotation speed of the coil in a simple generator doubles. Describe the change in peak voltage and frequency.

$\mathcal{E}_{\max} = \omega NBA$ and $f = \dfrac{\omega}{2\pi}$ both **double**.

Question 28.3 A student says the mean current in a resistor is zero, so an alternating current produces no heating. Is this correct?

No. The heating power $P = I^2R \geq 0$ depends on I^2, not I. The mean of I^2 is not zero, so there is a non-zero mean power.

Power and r.m.s. values

• power in a resistor: $P = I^2R = \dfrac{V^2}{R}$, never negative, but it varies with time. We usually want the mean power :

$$\langle P \rangle = \langle I^2 \rangle R = \frac{\langle V^2 \rangle}{R}$$

• the root-mean-square values are $I_{\text{rms}} = \sqrt{\langle I^2 \rangle}$ and $V_{\text{rms}} = \sqrt{\langle V^2 \rangle}$, so

$$\langle P \rangle = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

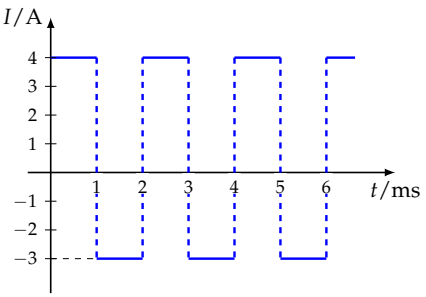
The r.m.s. value of an alternating current (voltage) is the steady d.c. current (voltage) that delivers the same mean power to a resistive load.

• for a **sine wave** only:

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \qquad V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

• mains voltages (e.g. 230 V) are r.m.s. values

Why $1/\sqrt{2}$? $\langle P\rangle = \dfrac{1}{T}\displaystyle\int_0^T I_0^2 R\sin^2\omega t\,\mathrm{d}t$ and
$\displaystyle\int_0^T \sin^2\omega t\,\mathrm{d}t = \tfrac{1}{2}\int_0^T (1-\cos 2\omega t)\mathrm{d}t = \frac{T}{2}$, so $\langle P\rangle = \tfrac{1}{2}I_0^2R = I_{\text{rms}}^2R$.



Current through the resistor.

Example 28.1 The alternating current in a 120 Ω resistor is shown (square wave: +4 A for half the period and −3 A for the other half). Find (a) the r.m.s. current and (b) the mean power.

(a) $\langle I^2\rangle = \dfrac{4^2+(-3)^2}{2} = 12.5\ \text{A}^2$, so $I_{\text{rms}} = \sqrt{12.5} = 3.54\ \text{A}$
(b) $\langle P\rangle = I_{\text{rms}}^2 R = 12.5\times 120 = 1500\ \text{W}$

Example 28.2 A sinusoidal supply with maximum voltage 75 V is connected across a 30 Ω resistor. Find the energy dissipated in 2.0 minutes.

$V_{\text{rms}} = \dfrac{75}{\sqrt{2}} = 53.0\ \text{V}, \quad \langle P\rangle = \dfrac{53.0^2}{30} = 93.8\ \text{W}$
$W = \langle P\rangle\, t = 93.8\times 120 \approx 1.13\times 10^4\ \text{J}$

Question 28.4 $V = 310\sin(100\pi t)$ (SI units). Find (a) the peak voltage, (b) the r.m.s. voltage, (c) the frequency, (d) the mean power across a 250 Ω resistor.

(a) 310 V (b) $310/\sqrt{2} = 219$ V (c) $f = \dfrac{100\pi}{2\pi} = 50$ Hz
(d) $\langle P\rangle = \dfrac{219^2}{250} = 192$ W

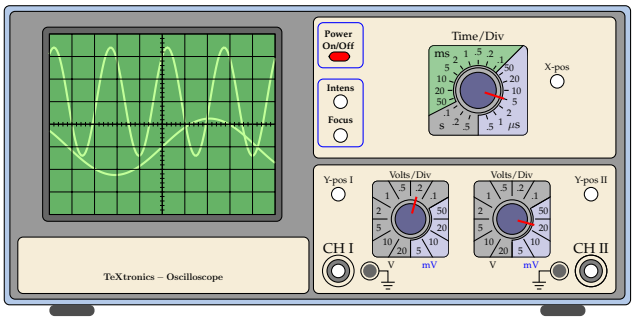
Question 28.5 What is the mean power in a resistor when the supply has peak current 5.0 A and peak voltage 8.0 V?

$\langle P\rangle = I_{\text{rms}}V_{\text{rms}} = \dfrac{5.0}{\sqrt{2}}\times\dfrac{8.0}{\sqrt{2}} = 20\ \text{W}$

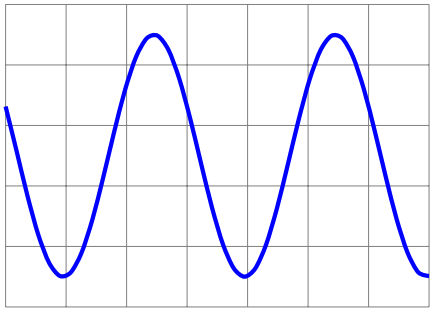
Question 28.6 The peak value of a sinusoidal current equals a steady d.c. current. Both are applied to the same load. Find $P_{\text{d.c.}}/P_{\text{a.c.}}$.

$P_{\text{d.c.}} = I_0^2 R$ and $\langle P_{\text{a.c.}} \rangle = \frac{1}{2} I_0^2 R$, so the ratio is **2**.

Measuring a.c. with an oscilloscope



A cathode-ray oscilloscope (c.r.o.) uses a beam of electrons. The time base moves the beam horizontally (time axis); the signal on the vertical plates deflects it up and down. It shows a graph of voltage against time.



- Y-gain : volts per vertical division
- time base : time per horizontal division

Example 28.3 The trace is shown (4 divisions peak to peak, 3 divisions per cycle). The time base is 10 ms/div and the Y-gain is 5 V/div. Find the peak voltage, period and frequency.

$V_0 = \frac{1}{2} \times 4 \times 5 = 10$ V

$T = 3 \times 10 = 30$ ms

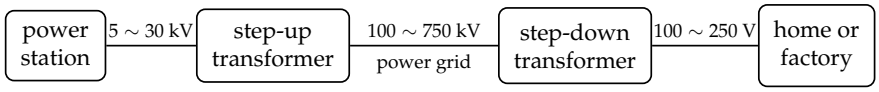
$f = \dfrac{1}{T} = \dfrac{1}{30 \times 10^{-3}} = 33.3$ Hz

Question 28.7 The time base is now 5 ms/div and the Y-gain is 2 V/div with the same trace. Write an equation for the voltage.

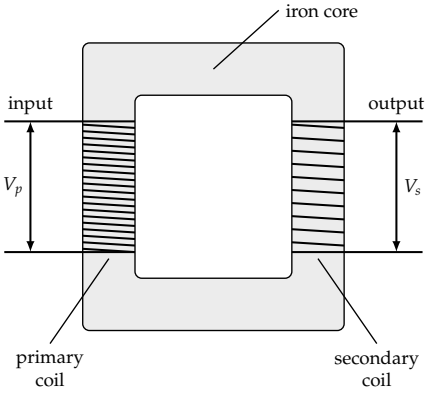
$V_0 = \frac{1}{2} \times 4 \times 2 = 4.0$ V, $T = 3 \times 5 = 15$ ms, $\omega = \dfrac{2\pi}{0.015} = 419$ rad s^{-1}
$V = 4.0 \sin(419t)$ (ignoring the offset and phase of the trace)

Power supply systems

High-voltage transmission



- cable loss is I^2R, so we want a low current
- power station output $P = IV$ is fixed, so low current means high voltage
- but low voltages are safer at generator and consumer, so we need transformers to step the voltage up and down



Transformers

A transformer changes an alternating voltage.

- a.c. in the primary coil gives a changing flux in the iron core; this links with the secondary coil and induces an a.c. voltage of the same frequency
- with no flux loss, Faraday's law gives $V_p = N_p \dfrac{d\phi}{dt}$ and $V_s = N_s \dfrac{d\phi}{dt}$, so

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

- $N_s > N_p$: step-up; $N_s < N_p$: step-down
- ideal transformer (100% efficient): input power = output power

$$I_p V_p = I_s V_s$$

- losses in a real transformer: eddy currents in the core (reduced by a laminated core); resistance of the coils (thick copper wire); flux leakage (continuous iron loop)

Example 28.4 An ideal transformer has 200 primary turns and 5000 secondary turns. The r.m.s. primary voltage is 8.0 V. What is the *peak* voltage across a resistor on the secondary?

$\frac{V_{s,0}}{V_{p,0}} = \frac{N_s}{N_p}$ and $V_{p,0} = \sqrt{2} \times 8.0 = 11.3$ V

$V_{s,0} = 11.3 \times \frac{5000}{200} = 283$ V

Question 28.8 An ideal transformer has 6000 primary turns. It converts a 220 V r.m.s. supply to an a.c. voltage with a peak value of 12.0 V. Find the number of secondary turns.

$V_s = \frac{12.0}{\sqrt{2}} = 8.49$ V r.m.s., so $N_s = 6000 \times \frac{8.49}{220} \approx 231$ turns

Question 28.9 What is the output if a steady d.c. voltage is applied to the input?

Zero. The flux is constant, so no e.m.f. is induced in the secondary.

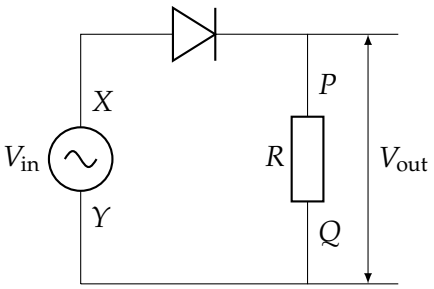
Question 28.10 Is the primary current in phase with the voltage induced in the secondary? Explain.

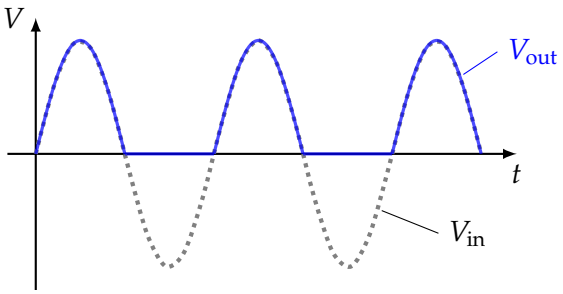
No. The flux is in phase with the primary current, but $V_s = N_s \frac{d\phi}{dt}$ depends on the *rate of change* of flux, so V_s is a quarter cycle (90°) out of phase with the primary current.

Rectification and smoothing

Many devices need d.c. ___Rectification___ converts a.c. to d.c. using ___diodes___, which allow current in one direction only.

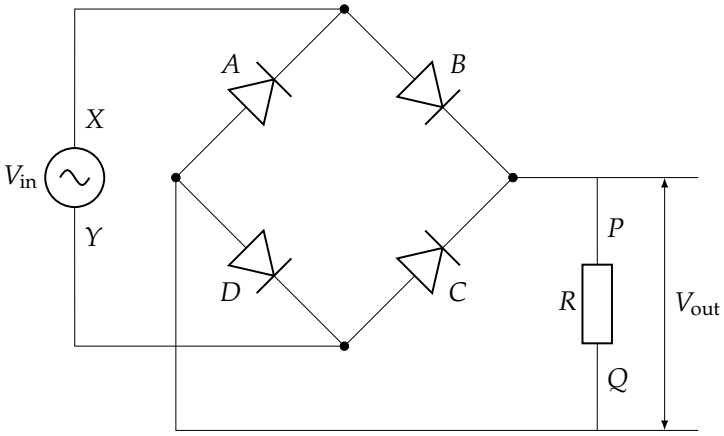
Half-wave rectification



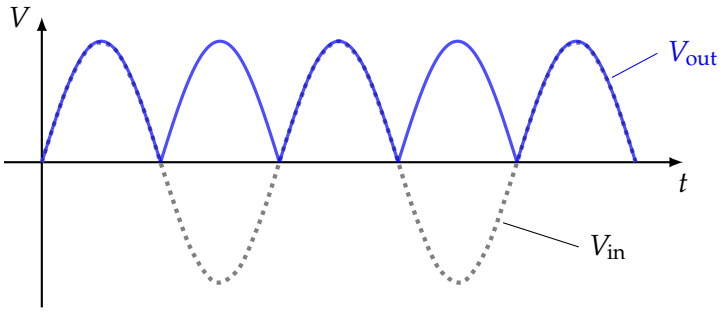


A ___single diode___ conducts only when X is positive, so V_{out} is in one direction, with zero output for half of each cycle. Only half of the supply power is available.

Full-wave rectification



A ___bridge___ of four diodes: when X is positive, the current path is $X \to B \to R \to D \to Y$; when Y is positive, it is $Y \to C \to R \to A \to X$. In both cases the current in R is in the ___same direction___, so P is always positive relative to Q.

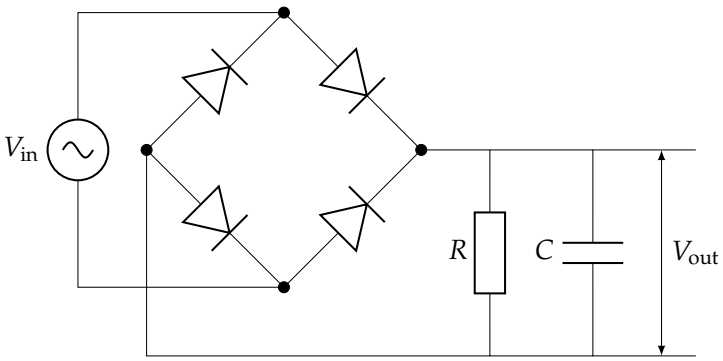


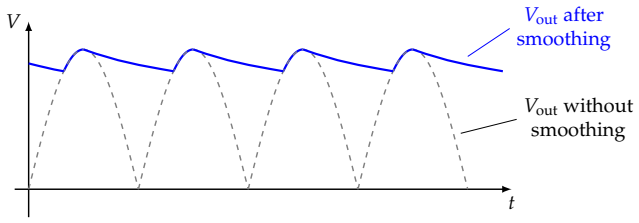
Question 28.11 How should the four diodes be rearranged so that Q is positive with respect to P?

Reverse the direction of **all four diodes** (the bridge conducts the other way).

Smoothing

The rectified d.c. still varies. A smoothing capacitor connected in parallel with the load stores energy as the voltage rises and discharges through the load as it falls, reducing the ripple.





- the time constant $\tau = RC$ matters: larger RC means slower discharge and less ripple
- if RC is too large the capacitor may not fully charge on each cycle

Question 28.12 A second capacitor is added in *series* with the smoothing capacitor. Sketch and explain the change in the output voltage.

The total capacitance **decreases** (series), so RC is smaller. The capacitor discharges faster between peaks: the ripple is **larger** and the mean output voltage is lower.